19 Coordinate geometry

Question 1 Sketch the following lines on the given number plane by first finding the x and y intercepts:

(a) $2x + y = 4$

$y = 0 \Rightarrow x =$
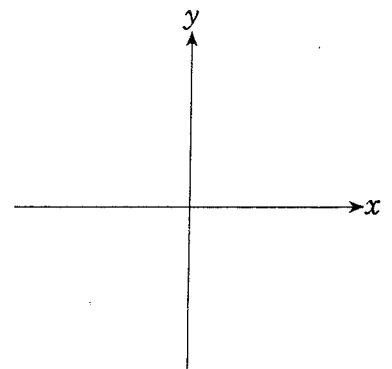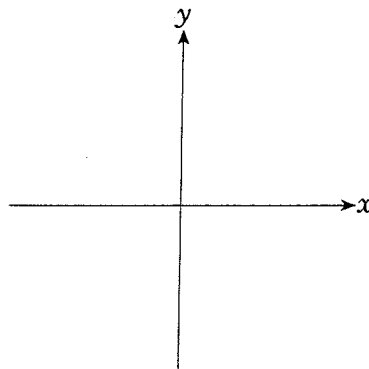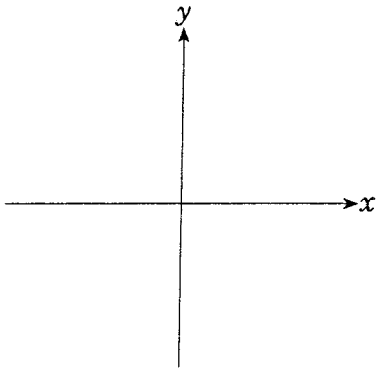
$x = 0 \Rightarrow y =$

(b) $3x + 2y = 6$

$y = 0 \Rightarrow x =$

$x = 0 \Rightarrow y =$

(c) $x - 4y = 12$

$y = 0 \Rightarrow x =$

$x = 0 \Rightarrow y =$

(d) $2x - 3y - 12 = 0$

$y = 0 \Rightarrow x =$

$x = 0 \Rightarrow y =$

(e) $y - 3x + 5 = 0$
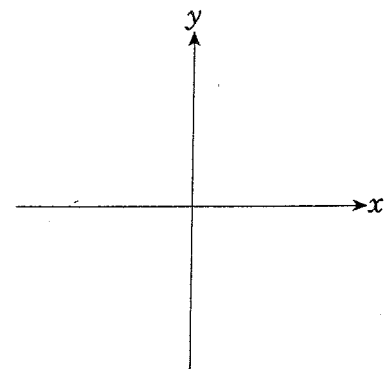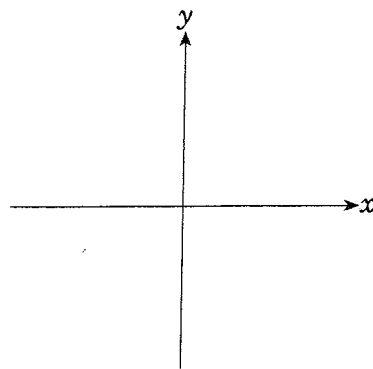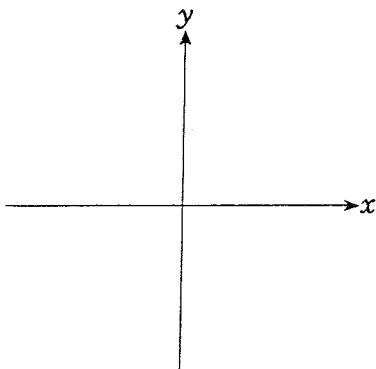
$y = 0 \Rightarrow x =$

$x = 0 \Rightarrow y =$

(f) $4x - 3y + 6 = 0$

$y = 0 \Rightarrow x =$

$x = 0 \Rightarrow y =$

Question 2 For each of the following pairs of points, find:
  (i) the distance between them
  (ii) the gradient of the line joining them
  (iii) the midpoint

(a) $(1, 3)$ and $(5, 9)$

(b) $(-3, 4)$ and $(7, 2)$

(c) $(-5, -3)$ and $(-2, 1)$

(d) $(0, 5)$ and $(-2, 7)$

(e) $(-7, 1)$ and $(3, -5)$

(f) $(3, 4)$ and $(-9, -3)$

Question 3   Use the gradient–intercept form to find the equations of the lines in the following:

(a) gradient $= -1$, y-intercept $= 4$

(b) $m = \dfrac{1}{2}$ $b = \dfrac{1}{2}$

(c) $m = -2$ $b = 3$

(d) $m = -\dfrac{3}{2}$ $b = \dfrac{5}{4}$

Question 4   Use the point–gradient form to find the equations of the lines in the following:

(a) gradient $= -1$, through $(2, -1)$

(b) $m = 4$, point is $(-1, -2)$

(c) $m = \dfrac{1}{2}$, through $(3, -2)$

(d) $m = -\dfrac{3}{2}$, through $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$

(e) $m = \dfrac{3}{5}$, through $(0, 5)$

(f) $m = -\dfrac{2}{7}$, through $(-3, 0)$

Question 5   Use either the two-point form or the point–gradient form (by first finding the gradient) to find the equation of the line joining the two points given.

(a) $(1, 3)$ and $(2, 5)$

(b) $(-1, -1)$ and $(3, 2)$

(c) $(-3, 2)$ and $(2, -3)$

(d) $(3, -5)$ and $(6, 3)$

Question 6   In each case, find the equation of the line from the given information, using the appropriate formula.

(a) gradient = 3, y-intercept = −7

(b) gradient = 2, through $(-3, 1)$

(c) through $(-3, -3)$ and $(7, 5)$

(d) $m = -2$,   through $(1, 7)$

(e) $m = 7$, $b = -1$

(f) through $(7, 3)$ and $(-2, -3)$

(g) $m = -1$,   through $(-1, -1)$

(h) gradient = 5, through $(0, 0)$

(i) gradient = −2, y-intercept = 9

(j) through $(4, -2)$ and $(-3, -7)$

(k) gradient = −2, through $(2, -5)$

(l) $m = \dfrac{2}{5}$,   through $(-3, -4)$

Question 7   In each of the following, find the equation of the line:

(a) through $(1, 2)$, parallel to $2x + y - 1 = 0$

(b) through $(-3, 2)$, parallel to $x - 3y + 5 = 0$

(c) through $(-1, -1)$, ∥ to $y = 5x + 3$

(d) through $(7, 2)$, ∥ to $7x - 4y + 3 = 0$

(e) through $(4, 1)$, perpendicular to $3x + 2y + 1 = 0$

(f) through $(2, 4)$, ⊥ to $3x - 4y - 11 = 0$

(g) through $\left(\frac{1}{2}, -\frac{1}{2}\right)$, ∥ to $4x - y - 3 = 0$

(h) through $(5, -5)$, ⊥ to $x - y - 7 = 0$

Question 8 Triangle ABC has vertices $A(-1, 4)$, $B(3, 6)$, $C(9, -4)$

(a) Find the coordinates of K, L, M, which are the midpoints of AB, BC and CA respectively.

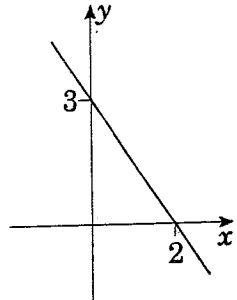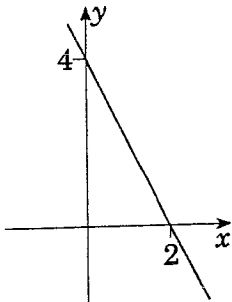**(b)** Find the equations of AL, BM and CK, which are the medians of the triangle.

(c) Find the coordinates of G, the intersection of AL and BM, and show that CK also passes through G.
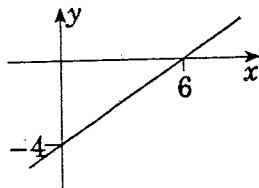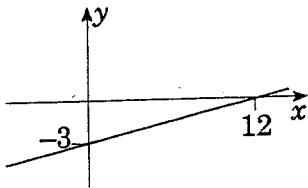
19 Coordinate geometry

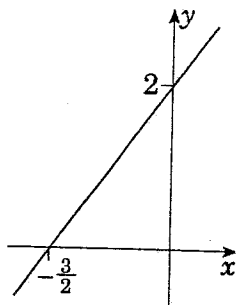**1** (a) $y = 0 \implies x = 2$ (b) $y = 0 \implies x = 2$

     $x = 0 \implies y = 4$ $x = 0 \implies y = 3$

(c) $y = 0 \implies x = 12$ (d) $y = 0 \implies x = 6$

     $x = 0 \implies y = -3$ $x = 0 \implies y = -4$

(e) $y = 0 \implies x = \frac{5}{3}$ (f) $y = 0 \implies x = -\frac{3}{2}$

     $x = 0 \implies y = -5$ $x = 0 \implies y = 2$
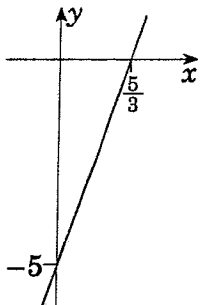
2 (a) (i) 7·21   (ii) $1\frac{1}{2}$ (iii) $(3, 6)$

   (b) (i) 10·2   (ii) $-\frac{1}{5}$ (iii) $(2, 3)$

   (c) (i) 5   (ii) $1\frac{1}{3}$ (iii) $\left(-3\frac{1}{2}, -1\right)$

   (d) (i) 2·83   (ii) -1 (iii) $(-1, 6)$

   (e) (i) 11·66   (ii) $-\frac{3}{5}$ (iii) $(-2, -2)$

   (f) (i) 13·89   (ii) $\frac{7}{12}$ (iii) $\left(-3, \frac{1}{2}\right)$

3 (a) $y = -x + 4$ (b) $y = \frac{1}{2}x + \frac{1}{2}$

   (c) $y = -2x + 3$ (d) $y = -\frac{3}{2}x + \frac{5}{4}$

4 (a) $y = -x + 1$ (b) $y = 4x + 2$

   (c) $y = \frac{1}{2}x - \frac{7}{2}$ (d) $y = -\frac{3}{2}x + \frac{5}{4}$

   (e) $y = \frac{3}{5}x + 5$ (f) $y = -\frac{2}{7}x - \frac{6}{7}$

5 (a) $y = 2x + 1$ (b) $y = \frac{3}{4}x - \frac{1}{4}$

   (c) $y = -x - 1$ (d) $y = \frac{8}{3}x - 13$

6 (a) $y = 3x - 7$ (b) $y = 2x + 7$

   (c) $y = \frac{4}{5}x - \frac{3}{5}$ (d) $y = -2x + 9$

   (e) $y = 7x - 1$ (f) $y = \frac{2}{3}x - \frac{5}{3}$

   (g) $y = -x - 2$ (h) $y = 5x$

   (i) $y = -2x + 9$ (j) $y = \frac{5}{7}x - \frac{34}{7}$

   (k) $y = -2x - 1$ (l) $y = \frac{2}{5}x - \frac{14}{5}$

7 (a) $y = -2x + 4$ (b) $y = \frac{1}{3}x + 3$

   (c) $y = 5x + 4$ (d) $y = \frac{7}{4}x - \frac{41}{4}$

   (e) $y = \frac{2}{3}x - \frac{5}{3}$ (f) $y = -\frac{4}{3}x + \frac{20}{3}$

   (g) $y = 4x - \frac{3}{2}$ (h) $y = -x$

8 (a) $K(1, 5)$ $L(6, 1)$ $M(4, 0)$

   (b) BM: $y = -6x + 24$ CK: $y = -\frac{9}{8}x + \frac{49}{8}$

       AL: $y = -\frac{3}{7}x + \frac{25}{7}$

   (c) $\left(3\frac{2}{3}, 2\right)$ Proof: $y = -\frac{9}{8} \times 3\frac{2}{3} + \frac{49}{8}$; $y = 2$