Compound interest

The amount to which an investment will grow under compound interest can be found using the formula $A = P\left(1 + \frac{r}{100}\right)^n$. Consider an investment of \$10 000 at an interest rate of 8% p.a.

1. If interest is compounded annually, the amount to which the investment will grow can be given by the function $A = 10\,000(1.08)^n$, where n is the number of years. Graph this function using graphing software or a calculator.
2. If interest is compounded six-monthly, the function becomes $A = 10\,000(1.04)^{2n}$. On the same set of axes graph this function.
3. Write a function that will show the amount to which the investment will grow, if interest is compounded quarterly and graph this function on the same set of axes.
4. Use the graphs drawn to describe the overall effect of shortening the compounding period.

Variations

From our work on measurement we know that the area of a circle is given by the formula $A = \pi r^2$, where A is the area and r is the radius of the circle.

This is an example of a quantity (area) that varies in proportion with the power of another quantity (radius). This can be written as $A \propto r$. The symbol $\propto$ means 'in proportion to'. In this example π is the constant of variation, that is, the amount by which r^2 must be multiplied to calculate the area.

An equation of the form $y = ax^2$ or $y = ax^3$ can be used to model several variations. In such cases we may need to calculate the constant of variation from some known or given information.

WORKED Example 11

It is known that y varies directly with the cube of x. It is known that $y = 24$ when $x = 2$. Write an equation connecting the variables x and y.

THINK	WRITE
1 Write a proportion statement.	$y \propto x^3$
2 Insert a constant of variation (k) to form an equation.	$y = kx^3$
3 Substitute the known values of x and y to find the value of k.	When $x = 2$, $y = 24$. $24 = k \times 2^3$ $= 8k$ $k = 3$
4 Replace the known value of k in the equation.	$y = 3x^3$

Once we have calculated the constant of variation, we are able to calculate one quantity given the other.

WORKED Example 12

The surface area of a cube varies directly with the square of the length of the cube's edge.
a A cube of edge length 5.5 cm has a surface area of 181.5 cm^2. Find the constant of variation.
b Find the surface area of a cube with an edge length of 7.2 cm.

THINK	WRITE
a 1 Write a proportion statement choosing pronumerals s and e.	**a** $s \propto e^2$
2 Insert the constant of variation, k, to form an equation.	$s = ke^2$
3 Substitute known information.	When $e = 5.5$, $s = 181.5$ $181.5 = k \times 5.5^2$
4 Calculate 5.5^2.	$181.5 = k \times 30.25$
5 Solve the equation (divide by 30.25).	$k = 6$
b 1 Rewrite the proportion statement with $k = 6$.	**b** $s = 6e^2$
2 Substitute $e = 7.2$.	When $e = 7.2$, $s = 6 \times 7.2^2$
3 Calculate s.	$s = 311.04$
4 Give a written answer.	The surface area of a cube with an edge of 7.2 cm is 311.04 cm^2.

Hyperbolic functions represent inverse variations. These variations occur when one quantity decreases as the other increases. An inverse variation is of the form $y = \frac{a}{x}$.

WORKED Example 13

It is known that y varies inversely with x and that when $y = 8$, $x = 4$. Write an equation connecting y with x.

THINK	WRITE
1 Write an inverse proportion statement.	$y \propto \frac{1}{x}$
2 Insert a constant of variation (k) to form an equation.	$y = \frac{k}{x}$
3 Substitute the known values of x and y to find the value of k.	When $x = 4$, $y = 8$. $8 = \frac{k}{4}$ $k = 32$
4 Replace the known value of k in the equation.	$y = \frac{32}{x}$

Such variations can also be applied to practical situations.

WORKED Example 14

It is known that the time taken for a journey varies inversely with speed. The trip takes 6 hours at 60 km/h.
a Find the constant of variation.
b How long will it take at 90 km/h?

THINK	WRITE
a 1 Write a proportion statement choosing pronumerals t and s.	**a** $t \propto \frac{1}{s}$
2 Insert the constant of variation, k, to form an equation.	$t = \frac{k}{s}$
3 Substitute known information.	When $t = 6$, $s = 60$ $6 = \frac{k}{60}$
4 Solve the equation (multiply by 60).	$k = 360$
b 1 Rewrite the equation with $k = 360$.	**b** $t = \frac{360}{s}$
2 Substitute $s = 90$.	When $s = 90$, $t = \frac{360}{90}$
3 Calculate t.	$t = 4$
4 Give a written answer.	The trip will take 4 hours at 90 km/h.

remember

1. A variation can be expressed as a function.
2. If one quantity varies as the square of another, the variation is of the form $y = ax^2$.
3. If one quantity varies as the cube of another, the variation is of the form $y = ax^3$.
4. If one quantity varies inversely as another, the variation is of the form $y = \frac{a}{x}$.
5. An inverse variation occurs when one quantity decreases while the other decreases.

EXERCISE 9D Variations

1 ...place t... is known that y varies directly with the square of x. If $y = 88$ when $x = 4$, write an ...tion connecting y with x.

Once known that b varies directly with the cube of a. When $a = 6$, $b = 108$. Write an give... n connecting b with a.

3 It is known that the distance, d, an object will fall, in metres, varies directly with the square of the time, t, it has been falling, in seconds. An object that has been falling for 2 seconds falls a distance of 19.6 metres.

a Write an equation connecting d with t.

b Graph the relationship between d and t.

WORKED Example 12

4 The surface area of a cube varies directly with the square of its side length.

a A cube of side length 15 cm has a surface area of 1350 cm^2. Find the constant of variation.

b What is the surface area of a cube that has a side length of 6.2 cm?

5 The area of a circle varies directly with the square of its radius.

a If the area of a circle with side length 6 cm is 113.1 cm^2, find the constant of variation. (Give your answer correct to 2 decimal places.)

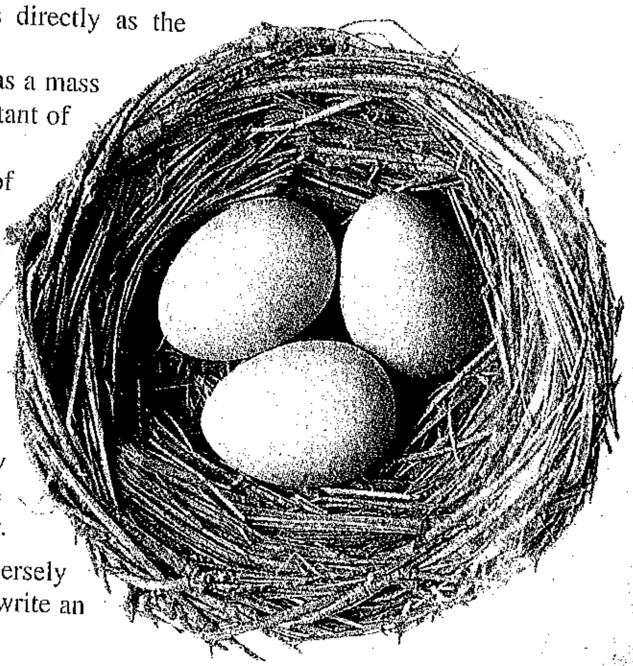
b What is the area of a circle with a radius of 12 cm?

6 The mass of an egg varies directly as the cube of the egg's length.

a An egg of length 5 cm has a mass of 31.25 g. Find the constant of variation.

b What will be the mass of an egg with a length of 6 cm?

c If an egg has a mass of 70 g, what would be the egg's length? (Give your answer correct to 1 decimal place.)



WORKED Example 13

7 It is known that y varies inversely with x. When $y = 10$, $x = 5$; write an equation connecting y with x.

8 It is known that m varies inversely with n. When $m = 0.5$, $n = 2$; write an equation connecting m and n.

9 The time taken, t, to travel between two points varies inversely with the average speed, s, for the trip. If the journey takes 2.5 hours at 60 km/h:

a write an equation that connects t with s

b graph the relationship between t and s.

WORKED Example 14

10 The time, t, taken to dig a trench varies inversely with the number of workers, n, digging. It takes four workers 5 hours to dig the trench.

a Find the constant of variation.

b How long would it take 10 workers to dig the same trench?

11 The fuel economy, f, of a car varies inversely with the speed, s, at whi... A car that averages 40 km/h has a fuel economy of 15 L/100 km. What w... fuel economy of a car that averages 50 km/h?

12 In an electricity circuit, the current (measured in amps, a) is inversely pr... the resistance (measured in ohms, r). When the resistance is 40 ohms, ... measured at 3 amps. What will be the current when the resistance is 15 oh...

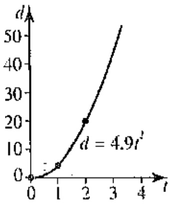
answers

Exercise 9D — Variations

1 $y = 5.5x^2$

2 $b = 0.5a^3$

3 a $d = 4.9t^2$

b 

4 a 6 b 230.64 cm^2

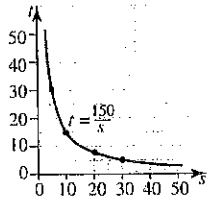
5 a 3.14 b 452.16 cm^2

6 a 0.25 b 54 g c 6.5 cm

7 $y = \frac{50}{x}$

8 $m = \frac{1}{n}$

9 a $t = \frac{150}{s}$

b 

10 a 20 b 2 hours

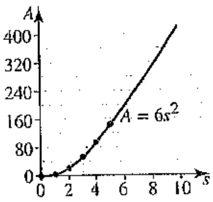
11 12 L/100 km

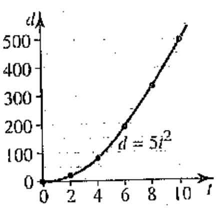
12 8 amps

Exercise 9E — Graphing physical phenomena

1 a

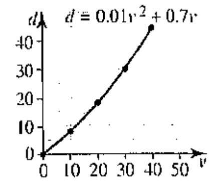
s	0	1	2	3	4	5
A	0	6	24	54	96	150

b 

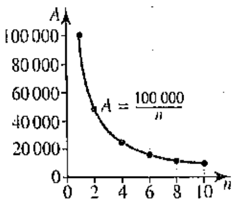
2 

3 a

v	0	10	20	30	40
d	0	8	18	30	44

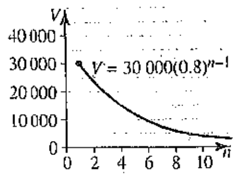
b 

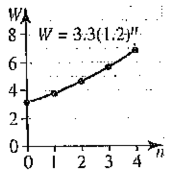
4 a $A = \frac{100\ 000}{n}$

b 

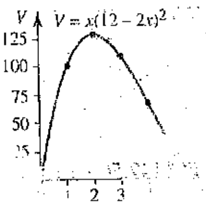
5 a

Age (years)	1	2	3	4	5
Value	$30 000	$24 000	$19 200	$15 360	$12 288

b 

6 

7 a 6 b Ch[illegible]k with your t[illegible]

c 

8 a

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