



MATHEMATICS

YEAR 10

HALF-YEARLY EXAMINATION

2006

Time Allowed: 75 minutes

INSTRUCTIONS:

- There are FIVE (5) Questions of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- Start each Part on a new page.
- Write on one side of paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write your name and Maths class clearly at the top of each question and clearly number each question.

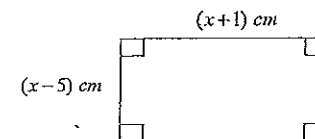
QUESTION 1 (20 Marks)

MARKS

1. Solve the following:

- | | | |
|-----|--|---|
| (a) | $(x-2)(x+5) = 0$ | 1 |
| (b) | $a^2 - 64 = 0$ | 1 |
| (c) | $y^2 + y - 20 = 0$ | 2 |
| (d) | $2x^2 + 7x + 6 = 0$ | 2 |
| (e) | $(m+5)^2 = 7$ (leave your answer in surd form) | 2 |
| (f) | $x^2 + 7 = 15 - 6x$ (answer to 2 decimal places) | 3 |

2. If the area of the rectangle drawn below is 27 cm^2 :



- | | | |
|-----|-----------------------------------|---|
| (a) | find the value of x . | 3 |
| (b) | find the length of the rectangle. | 1 |

3. A right-angled triangle is drawn so that the hypotenuse is twice the length of the shortest side, and the other side is 1cm longer than the shortest side. Draw a diagram and then form an algebraic equation to represent the information given. (Let x = the shortest side). Solve the equation and find the length of the hypotenuse to 2 decimal places. 5

QUESTION 2 (20 Marks)

MARKS

1. Jenny places \$620 in a term deposit account. If the bank pays her 5.2% p.a. simple interest for 6 months, how much interest does Jenny receive at the end of 6 months? 3

2. Catherine buys a new car valued at \$24 500 but decided to buy it on terms. She pays a \$5000 deposit and then pays \$700 monthly instalments over three years. 3
 - (a) How much has Catherine paid for the car?
 - (b) What is the amount of interest she paid on the car?

3. If Selina paid \$2350 in interest over two years on a loan of \$29 000, what was the rate per annum of simple interest charged? 4

4. \$35 000 is invested and earns compound interest at a rate of 5.4% p.a. Find the interest earned after two years if it compounded monthly. 4

5. A new plasma TV depreciates by 25% per year. If it costs \$1900 new, what will the TV be worth in five years? 3

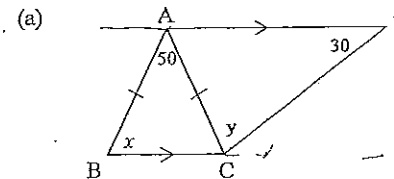
6. A yacht now worth \$120 000 has been depreciating at a rate of 8% p.a. for the last five years. What was its value five years ago? 3

QUESTION 3 (20 Marks)

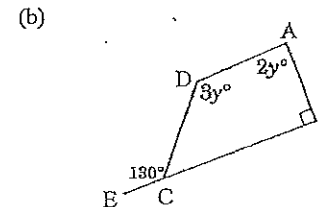
MARKS

1. When comparing the properties of a rectangle and a rhombus, state one way in which the diagonals are different for both quadrilaterals. 1

2. Determine the value of the pronumeral(s) in each of the following. Give reasons for your answers.



5



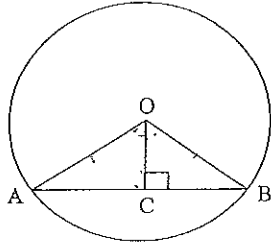
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QUESTION 3 continued

MARKS

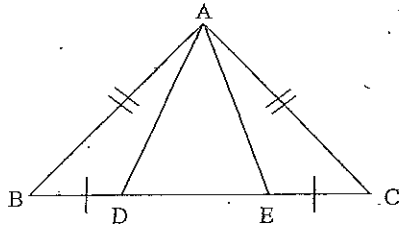
3. Prove \triangle s AOC and BOC are congruent.

Note: O is the centre of the circle and OC is drawn perpendicular to AB.



4

4. \triangle ABC is an isosceles triangle. $AB = AC$ and $BD = CE$.



(a) Prove \triangle s ABD and ACE are congruent.

4

(b) Hence show that \triangle ADE is isosceles.

2

QUESTION 4 (20 Marks)

MARKS

1. A traffic light shows red for 1 minute 20 seconds, amber for 20 seconds and green for 1 minute. At any given time, what is the probability that the light is:

8

- (a) amber?
- (b) not red?
- (c) amber or red?
- (d) green or blue?

2. Four cards, [one black (B), one red (R), one yellow (Y), one green (G)], are placed in a bag. Two cards are chosen at random without replacing the cards.

- (a) Construct a tree diagram and list all the possible pairs chosen.
- (b) What is the probability of getting a black card?
- (c) What is the probability of getting a red and a blue card?

3

1

1

3. The following data of blood pressure readings was collected for a number of children.

Blood Pressure	Frequency (f)
115-119	0
120-124	2
125-129	9
130-134	6
135-139	2
140-145	0

If a child is selected at random, find the probability that the child has blood pressure:

2

- (a) less than 125
- (b) between 130-139.

4. There are 80 employees in a certain large supermarket. 35 are check-out operators, 20 are shelf stackers. Of the employees, 11 work as both check-out operator and shelf stacker.

- (a) Draw a Venn diagram to display this information.
- (b) If an employee is chosen at random, what is the probability that the employee:
 - (i) is a check-out operator but not a shelf stacker?
 - (ii) is neither a check-out operator nor a shelf stacker?

3

1

1

QUESTION 5 (20 Marks)

1. For the parabola $y = x^2 + 4x - 5$, find the following information:

- (a) the y-intercept
- (b) the x-intercepts
- (c) the vertex.
- (d) Hence or otherwise, sketch the parabola showing all relevant features.

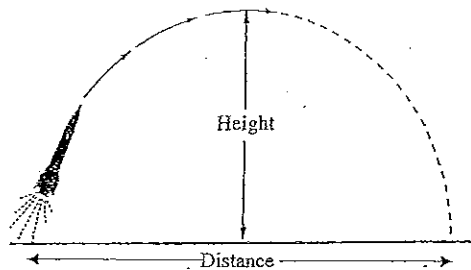
2. Find the equation of the parabola with x-intercepts of (-2,0) and (4,0) and y-intercept of (0,-16).

3. Sketch the following on separate graphs, showing all relevant features

- (a) $y = 3^x$
- (b) $xy = 5$
- (c) $x^2 + y^2 = 9$

4. A rocket fired from Earth travels in a parabolic path represented by the equation $y = -\frac{x^2}{20} + 3x$, where y is the vertical height in kilometres above the Earth's surface and x is the horizontal distance travelled in kilometres.

Find the maximum height that the rocket reaches.



End of Paper

MARKS

1

2

3

2

3

6

3

Year 10 Half-Yearly 2006 (Solutions)

Question 1 1. a) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 - 4(-5)}}{2(1)} = \frac{-2 \pm \sqrt{24}}{2} = -1 \pm \sqrt{6}$ b) $a = \pm 8$ (4 each) c) $(y+5)(y-4) = 0$ $y = -5, 4$ d) $(2x+4)(2x+3) = 0$ $x = -2, -1.5$ e) $m+5 = \pm\sqrt{7}$ $m = -5 \pm \sqrt{7}$ f) $3x^2 + 16x - 8 = 0$ $x = \frac{-16 \pm \sqrt{256 - 4(-24)}}{6} = \frac{-16 \pm \sqrt{400}}{6} = \frac{-16 \pm 20}{6}$ $x = \frac{4}{6} = \frac{2}{3}, x = \frac{-36}{6} = -6$	1 2 2 2 2 2 2 2
Question 2 1. $x^2 = 2x + 2x + 1$ $x^2 - 4x - 1 = 0$ $x = \frac{4 \pm \sqrt{16 + 4}}{2} = \frac{4 \pm \sqrt{20}}{2} = 2 \pm \sqrt{5}$ 2. Hypotenuse = $2\sqrt{2} \times (2 + \sqrt{2})$ $= 2\sqrt{2} \times 2(1 + \sqrt{2})$ $= 4\sqrt{2} \times (1 + \sqrt{2})$ $= 4\sqrt{2} + 8$ 3. $x = \frac{-1 \pm \sqrt{1 + 4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$ 4. $x = \frac{-2 \pm \sqrt{4 + 20}}{2} = \frac{-2 \pm \sqrt{24}}{2} = -1 \pm \sqrt{6}$ 5. $x = 1.12, y = 7.12$	3 3 3 3 3 3 3 3
Question 3 1. $x^2 - 4x - 1 = 0$ $x = \frac{4 \pm \sqrt{16 + 4}}{2} = 2 \pm \sqrt{5}$ 2. $x = \frac{-1 \pm \sqrt{1 + 4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$ 3. $x = \frac{-2 \pm \sqrt{4 + 20}}{2} = -1 \pm \sqrt{6}$ 4. $x = \frac{-1 \pm \sqrt{1 + 4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$ 5. $x = \frac{-2 \pm \sqrt{4 + 20}}{2} = -1 \pm \sqrt{6}$	3 3 3 3 3 3 3 3
Question 4 1. $x^2 - 14x + 4 = 0$ $x = \frac{14 \pm \sqrt{196 - 16}}{2} = \frac{14 \pm \sqrt{180}}{2} = 7 \pm 3\sqrt{5}$ 2. $x = \frac{-4 \pm \sqrt{16 - 4(1)(-5)}}{2} = \frac{-4 \pm \sqrt{36}}{2} = -2 \pm 3$ 3. $x = \frac{-5 \pm \sqrt{25 - 4(1)(16)}}{2} = \frac{-5 \pm \sqrt{9}}{2} = \frac{-5 \pm 3}{2} = -1, -4$ 4. $x = \frac{-2 \pm \sqrt{4 - 4(1)(-16)}}{2} = \frac{-2 \pm \sqrt{68}}{2} = -1 \pm \sqrt{17}$ 5. $x = \frac{-4 \pm \sqrt{16 - 4(1)(-20)}}{2} = \frac{-4 \pm \sqrt{96}}{2} = -2 \pm 2\sqrt{6}$	3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3

Question 2 continued 5. $A = P(1+r)^n$ $1900 = 1500(1+0.25)^n$ $1.25^n = \frac{1900}{1500}$ $n \log 1.25 = \log \frac{1900}{1500}$ $n = \frac{\log 1.2733}{\log 1.25} = 1.56$ 6. $A = P(1+r)^n$ $120000 = P(1+0.08)^5$ $P = \frac{120000}{1.4693} = 81707.16$	4 3 3 3 3 3
Question 3 1. Disregard the information but not in rectangles OR Disregard the information which they pass in rectangles but not in rectangles OR Disregard one side in rectangle but not in rectangles	3 3 3 3 3 3
2. $x + x + 50 = 180$ $2x = 130$ $x = 65$ 3. $\angle CAD = \angle BCA$ (Angles in same seg) $65 + y + 30 = 180$ $y = 85$	3 3 3 3 3 3
4. $\angle DCB = 130$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 50$ 5. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$	3 3 3 3 3 3 3 3 3 3
6. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$ 7. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$	3 3 3 3 3 3 3 3 3 3
8. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$ 9. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$	3 3 3 3 3 3 3 3 3 3
10. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$ 11. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$	3 3 3 3 3 3 3 3 3 3
12. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$ 13. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$	3 3 3 3 3 3 3 3 3 3
14. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$ 15. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$	3 3 3 3 3 3 3 3 3 3
16. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$ 17. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$	3 3 3 3 3 3 3 3 3 3
18. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$ 19. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$	3 3 3 3 3 3 3 3 3 3
20. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$ 21. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$	3 3 3 3 3 3 3 3 3 3
22. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$ 23. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$	3 3 3 3 3 3 3 3 3 3
24. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$ 25. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$	3 3 3 3 3 3 3 3 3 3
26. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$ 27. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$	3 3 3 3 3 3 3 3 3 3
28. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$ 29. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$	3 3 3 3 3 3 3 3 3 3
30. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$ 31. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$	3 3 3 3 3 3 3 3 3 3
32. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$ 33. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$	3 3 3 3 3 3 3 3 3 3
34. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$ 35. $\angle DCB = 50$ $\angle DCB + \angle CDB = 180$ $\angle CDB = 130$	3 3 3 3 3 3 3 3 3 3

$$3. (a) P(\langle 125) = \frac{2}{19} \quad (1)$$

$$(b) P(\frac{45}{30-135}) = \frac{8}{19} \quad (1)$$

4.

(a)		E	80
	C	S	
	24	11	9
			36

(3)

$$(b) P(C \text{ but not } S) = \frac{24}{80}$$

$$= \frac{3}{10} \quad (1)$$

$$(c) P(\text{not } C \text{ or } S) = \frac{36}{80}$$

$$= \frac{9}{20} \quad (1)$$

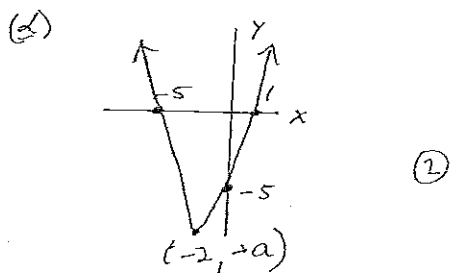
Question 5

$$1 \quad y = x^2 + 4x - 5$$

(a) let $x = 0$
 $y \text{ intercept} = -5 \quad (1)$

(b) Let $y = 0$
 $0 = (x+5)(x-1) \quad (2)$
 $x = -5 \text{ and } 1 \text{ (x intercepts)}$

(c) Axis of sym $x = -\frac{b}{2a}$
 $x = -\frac{4}{2} = -2$
 Sub into $y = x^2 + 4x - 5$
 $y = 4 - 8 - 5 = -9$
 $\therefore \text{Vertex} = (-2, -9) \quad (3)$



$$2 \quad y = k(x-x_1)(x-x_2)$$

$$y = k(x+2)(x-4)$$

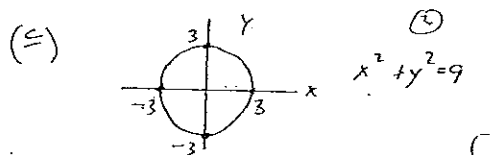
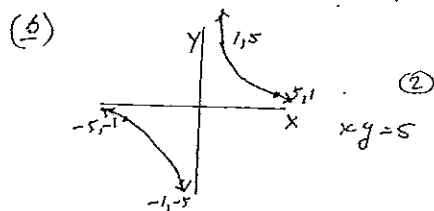
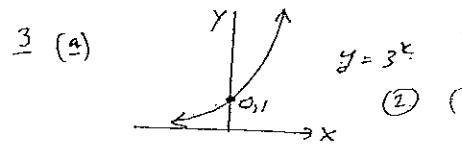
Substitute $(0, -16)$

$$-16 = k(2)(-4)$$

$$\therefore k = 2$$

$$y = 2(x+2)(x-4)$$

$$\therefore y = 2x^2 - 4x - 16 \quad (3)$$



$$4 \quad y = -\frac{1}{20}x^2 + 3x$$

Axis of sym $x = -\frac{b}{2a}$

$$x = \frac{-3}{(2 \times \frac{1}{20})} = 30$$

$$y = -\frac{1}{20} \times 30^2 + 3 \times 30$$

$$= 45 \quad (3)$$

$\therefore \text{Max. height} = 45 \text{ km}$