



# St. Catherine's School

## Year 10 Mathematics Semester 2 Examination October 2006

Time allowed: 2 hours + 5 minutes reading time

### INSTRUCTIONS

- There are 3 sections in this paper.
- Complete all three sections on the separate paper provided.
- Marks for each part of a question are indicated.
- All questions should be attempted.
- All necessary working should be shown.
- Start each section on a new page
- Approved scientific calculators and drawing templates may be used

### Section 1

27 Marks

1. Simplify each of the following algebraic expressions:

(n)  $x^9 \div x^2 \times x^3$

(b)  $8y^7 \div 2y^0$

(c)  $(-3m^2)^3$

2. Expand and simplify the following algebraic expressions:

(a)  $3(x+4) - 2(x-5)$

(b)  $6 - 3(k-8)$

3. Factorise the algebraic expression:

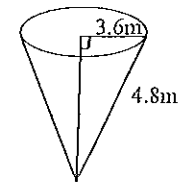
(a)  $4m^2 - 32m$

4. Solve the following quadratic equations:

(a)  $h^2 - 25 = 0$

(b)  $g^2 + 4g + 3 = 0$

5. Calculate the volume of the cone below and leave your answer correct to 3 significant figures.



PAS5.2.1

1

1

1

PAS 5.2.1

2

2

PAS 5.2.1

2

PAS 5.2.2

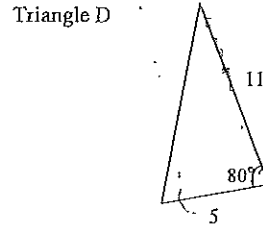
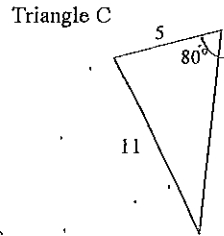
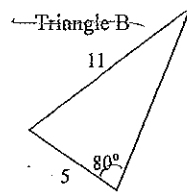
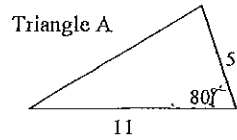
2

2

MAS 5.2.2

3

6. Determine which two of these triangles are congruent and state the congruence test you would use.



7. A die is rolled thirty times to obtain the following results:

|   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|
| 6 | 5 | 1 | 3 | 3 | 4 | 2 | 6 | 1 | 2 |
| 3 | 4 | 4 | 5 | 3 | 5 | 6 | 1 | 1 | 3 |
| 5 | 2 | 2 | 4 | 1 | 6 | 6 | 3 | 3 | 2 |

- (a) Construct a frequency distribution table including a cumulative frequency column and use it to determine the mode, range and mean of the above set of data.
- (i) Draw a cumulative frequency histogram and polygon (ogive) for the above set of data and use it to find the median.

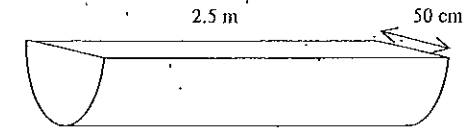
2  
SGS 5.2.2

DS 5.1.1

Section 2

54 Marks

1. On the day Melissa turned 16 years old, her parents decided to invest \$2500 in a bank account where interest is compounded yearly at 4.5% p.a.
- What will the amount be if Melissa decides to withdraw the money at the age of 25?
2. A semicircular water trough is shown in the diagram. It is made of sheet metal with a length of 2.5 metres and a diameter of 50 centimetres.
- (i) Find the volume of water when the trough is filled (in cubic centimetres to the nearest whole number).
- (ii) Find the area of sheet metal required to make the trough (the nearest whole number).



3. Simplify the following leaving your answer in exact form.
- (a)  $\cos 61^\circ + \sin 29^\circ$
- (b)  $\cos \theta - \sin(90^\circ - \theta)$
4. If  $\cos \theta = -\frac{4}{5}$  and  $\tan \theta > 0$ , find the ratio of  $\sin \theta$ .
5. Find the exact value of:
- (a)  $\cos 315^\circ$
- (b)  $\tan 120^\circ$

NS 5.2.2

MS 5.2.2

3

MS 5.3.2

2

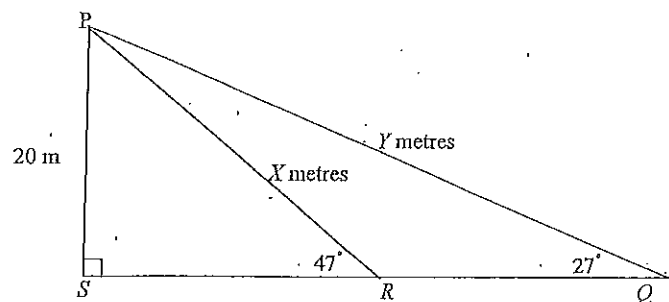
MS 5.3.2

MS 5.3.2

1

6. Find all possible values of  $\theta$ , for  $0 \leq \theta \leq 360^\circ$ , to the nearest degree if  $\tan \theta = -\frac{5}{7}$ .

7.



In the diagram above, a ramp runs from R to P, a vertical height of 20 metres, and the ramp makes an angle of  $47^\circ$  with the horizontal.

- (a) Show that  $X = \frac{20}{\sin 47^\circ}$  and find the value of  $X$  correct to 1 decimal place.
- (b) This ramp was found to be too steep and replaced by the ramp  $QP$  inclined at  $27^\circ$  to the horizontal. Find the value of  $Y$  and the difference in lengths of the ramps.
8. Solve the following linear inequalities:

(a)  $3m + 5 > -10$

(b)  $-2t - 6 \leq -4$

(c)  $\frac{x-6}{7} > 3 - \frac{x}{2}$

9. Solve the following two equations simultaneously

(a)  $y = \frac{x+2}{2}$  and  $4y + x = -20$

(b)  $-3p + 2q = 2$  and  $2p - 10q = -62$

3 MS 5.3.2

MS 5.2.3

2

2

PAS 5.2.2

1

2

3

PAS 5.2.2

3

3

10. Jacinta's average mark for seven Maths class tests is 82%. What minimum mark does Jacinta need to obtain in her eighth test to improve her average to 90%?

11. Joan and John are golfers. Each has played ten rounds of golf on the same course and their scores have been recorded below.

Joan's scores: 73 81 77 85 76 76 84 73 80 75

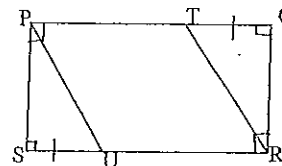
John's scores: 70 84 82 78 83 73 73 74 85 78

- (i) Calculate the mean and standard deviation for each set of scores.
- (ii) Considering your results from part (i), who is the more consistent golfer? Justify your answer.
- (iii) Find the interquartile range for Joan's scores and draw a box-and-whisker plot for her scores.

12. A letter is chosen at random from the word HIPPOPOTOMAS. Find these probabilities:

- (i) P(letter O)
- (ii) P(vowel)
- (iii) P(letter P or I)

13.  $PQRS$  is a rectangle.  $T$  is a point on  $PQ$  and  $U$  is a point on  $SR$  such that  $TQ = SU$ .



- (i) Prove that  $\triangle PSU$  is congruent to  $\triangle RQT$ .
- (ii) Hence, prove that  $PU = TR$ , giving reasons for your answer.

3 DS 5.2.1

DS 5.2.1

4

4

NS 5.1.3

1

1

1

SGS 5.3.1

3

1

1. Expand and simplify the following:

$$(5 - 2\sqrt{7})^2$$

2

NS 5.3.1

2. Rationalise the denominator:

$$\frac{7}{\sqrt{11} - 3}$$

2

NS 5.3.1

3. Three points,  $A$ ,  $B$  and  $C$ , are such that  $A$  is 72 m from  $B$  and 86 m from  $C$ . The bearing of  $B$  from  $A$  is  $105^\circ$  and of  $C$  from  $A$  is  $214^\circ$ .

(i) Show this information on a diagram.

2

MS 5.3.2

(ii) Find the distance from  $B$  to  $C$ , to the nearest metre.

3

4. Solve the following quadratic equations:

(a) By factorising:  $2x^2 + 11x - 6 = 0$

2

PAS 5.3.2

(b) By completing the square:  $4x^2 - 36x + 75 = 0$

2

(c) Using the quadratic formula:  $5x^2 - 4x - 2 = 0$

2

5.

In the diagram below,  $AB$  is parallel to  $CD$ .

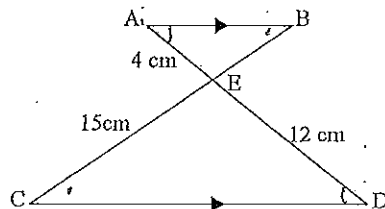
(a) Prove that  $\triangle AEB$  is similar to  $\triangle CED$ , giving reasons for your answers.

3

SGS 5.3.3

(b) Find the length of  $BE$ , giving reasons for your answer.

2



$$\frac{-4 \pm 20}{12400}$$

$$\frac{-403.836}{12344} \quad 7$$

$$2x - 1 = 0$$

$$2x = 1$$

$$x = \frac{1}{2}$$

6. A box contains 6 red marbles and 4 blue marbles. Two marbles are drawn out in succession, without replacement.

(a) Draw a tree diagram that shows the possible outcomes from the above information including the probabilities on the branches.

2

(b) Use the tree diagram from part (a) to find the probability of:

1

(i) two blue marbles being drawn

(ii) a red marble and a blue marble being drawn in any order.

2

7. A baby's shoe box is similar to an adult's one. Their surface areas are in the ratio of 25:49. If the volume of the children's shoe box is  $2400 \text{ cm}^3$ , find the volume of the adult's one.

3

MS 5.3.1 PAS 5.3.3

8. (i) On a number plane, plot the points  $A(2,3)$ ,  $B(5,2)$  and  $C(4,-1)$

1

(ii) Find the midpoint  $E$  of  $AC$ .

1

(iii) If  $E$  is the midpoint of  $BD$ , find the coordinates of  $D$ .

2

(iv) Find the gradient  $m_1$  of  $AB$ .

1

(v) Prove that  $AB$  is parallel to  $DC$ .

2

(vi) Find the equation of the line  $DC$ .

2

End of Examination

$$\frac{25\frac{1}{2}}{27}$$

Section 1.

1.  
a)  $x^1 \cdot x^2 \cdot x^3$   
 $= x^6$

b)  $8y^7 \div 2y^0$   
 $= 8y^7 \div 2$   
 $= 4y^7$

c)  $(-3m^2)^3$   
 $= -27m^6$

2.  
a)  $3(x+4) - 2(x-5)$   
 $3x + 12 - 2x + 10$   
 $= x + 22$

b)  $6 - 3(k-8)$   
 $= 6 - 3k + 24$   
 $= 30 - 3k$

3.  
a)  $4m^2 - 32m$   
 $= 4m(m-8)$

4.  
a)  $h^2 - 25 = 0$   
 $(h+5)(h-5) = 0$   
 $h = -5, 5$

b)  $g^2 + 4g + 3 = 0$   
 $(g+3)(g+1) = 0$   
 $g = -3, -1$

5.  $V = \frac{1}{3}Ab$   
 $= \frac{1}{3}\pi r^2 h$   
 $= \frac{1}{3}\pi \times 3.6^2 \times 10.08$   
 $= 43.08874$   
 $= 43.1 m^3$  (3 sig fig.)

$a^2 = b^2 + c^2$   
 $= 3.6^2 + 4.8^2$   
 $= 36 + 23.04$   
 $= 59.04$   
 $a = \sqrt{59.04}$   
 $a = 7.68$

6. ~~Triangle C is congruent to Triangle D (SAS test)~~  
Triangle A is congruent to Triangle D  
Triangle C is congruent to Triangle B (SAS test)

7.

a)

| x | tally | f               | cf | fx                |
|---|-------|-----------------|----|-------------------|
| 1 |       | 5               | 5  | 5                 |
| 2 |       | 5               | 10 | 10                |
| 3 |       | 7               | 17 | 21                |
| 4 |       | 4               | 21 | 16                |
| 5 |       | 4               | 25 | 20                |
| 6 |       | 5               | 30 | 30                |
|   |       | $\Sigma f = 30$ |    | $\Sigma fx = 102$ |

mode = 3 ✓  
range = 6 - 1 = 5 ✓  
mean =  $\frac{\Sigma fx}{\Sigma f}$   
 $= \frac{102}{30}$   
 $= 3.4$  ✓

$$\frac{5}{3}$$

56/56

FANTASTIC

Section 2

$$1. 2500(1 + \frac{4.3}{100})^9$$

$$= 3715.2378...$$

$$\approx 3715.24$$

3/3

2.

$$(i) V = Ah$$

$$= \frac{1}{2} \pi r^2 \times 2.5h$$

$$= \frac{1}{2} \pi \times 25^2 \times 25$$

$$= \frac{1}{2} \times \pi \times 25^2 \times 250$$

$$= 248436.9261...$$

$$= 248437 \text{ cm}^3$$

2/2

$$iii) SA = \pi r^2 + \frac{1}{2} 2\pi r l$$

$$= \pi \times 25^2 + \frac{1}{2} \times 2 \times \pi \times 25 \times 250$$

$$= 21898.4494... \text{ cm}^2$$

$$= 21898 \text{ cm}^2$$

3/3

3.

$$a) \cos 61 + 3 \sin 29$$

$$\cos \theta = \sin(90 - \theta)$$

$$\cos 61 = \sin(90 - 61)$$

$$= \sin 29$$

2/2

$$\therefore \sin 29 + \sin 29 = 2 \sin 29$$

$$b) \cos \theta - \sin(90 - \theta)$$

$$\cos \theta = \sin(90 - \theta)$$

$$\therefore \cos \theta - \cos \theta = 0$$

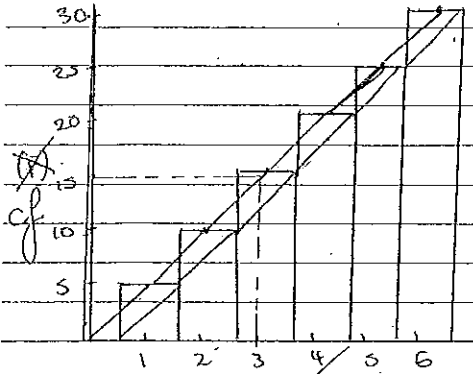
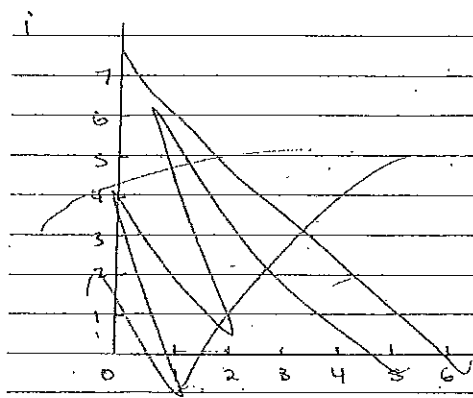
2/2

$$4. \cos \theta = \sin(90 - \theta) \quad a^2 = b^2 + c^2$$

$$\cos \theta = \frac{3}{5} \quad = \frac{3^2 + 4^2}{5^2}$$

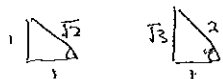
$$\therefore \sin \theta = \frac{3}{5} \quad a = 4$$

but  $\sin \theta$  is in 3rd quadrant  $\therefore \sin \theta = -\frac{3}{5}$



3/4

median = 3



5.

$$\begin{aligned} \text{a) } \cos 315 &= \cos(360-45) \\ &= \cos 45 \\ &= \frac{1}{\sqrt{2}} \end{aligned}$$

$$\left(\frac{1}{\sqrt{2}}\right)$$

$$\begin{aligned} \text{b) } \tan 120 &= \tan(180-60) \\ &= -\tan 60 \\ &= -\sqrt{3} \end{aligned}$$

$$\left(\frac{1}{1}\right)$$

$$\begin{aligned} \text{6. } \tan \theta &= \frac{5}{7} \\ \theta &= \tan^{-1} \frac{5}{7} \\ &= 35.53767\dots \end{aligned}$$

|   |     |
|---|-----|
| ✓ |     |
| S | A   |
| T | C ✓ |

$$\left(\frac{35}{30}\right)$$

$\tan \theta$  is in 2 and 4th quadrant

$$\therefore 180 - 35.53767\dots = 144.4623\dots$$

$$\hat{=} 144^\circ \text{ (nearest degree)}$$

$$360 - 35.53767\dots = 324.46\dots$$

$$\hat{=} 324^\circ \text{ (nearest degree)}$$

$$\begin{aligned} \text{7a) } \sin 47 &= \frac{20}{x} \\ \frac{1}{\sin 47} &= \frac{x}{20} \end{aligned}$$

$$x = \frac{20}{\sin 47}$$

$$= 27.3465\dots$$

$$= 27.3 \text{ (1dp)}$$

$$\left(\frac{2}{2}\right)$$

$$\text{b) } \sin 27 = \frac{20}{x}$$

$$x = \frac{20}{\sin 27}$$

$$= 44.05378\dots$$

$$\hat{=} 44.1 \text{ (1dp)}$$

$$\text{difference} = 44.1 - 27.3 = 16.7$$

$$= 16.7 \text{ (1dp)}$$

$$\hat{=} 16.7 \text{ (1dp)}$$

$$\left(\frac{2}{2}\right)$$

8.

$$\text{a) } 3m + 5 > -10$$

$$3m > -15$$

$$m > -5$$

$$\left(\frac{1}{1}\right)$$

$$\text{b) } -2t - 6 < -4$$

$$-2t < 2$$

$$t > -1$$

$$\left(\frac{2}{2}\right)$$

$$\text{c) } \frac{x-6}{7} > 3 - \frac{x}{2}$$

$$\frac{2(x-6)}{14} > \frac{42}{14} - \frac{7x}{14}$$

$$2x - 12 > 42 - 7x$$

$$2x + 7x > 42 + 12$$

$$9x > 54$$

$$x > 6$$

$$\left(\frac{3}{13}\right)$$

9.

$$\text{a) } y = x + 2 \quad \textcircled{1}$$

$$4y - x = -20 \quad \textcircled{2}$$

Subs  $\textcircled{1}$  into  $\textcircled{2}$

$$4(x+2) - x = -20$$

$$4x + 8 - x = -20$$

$$3x = -24$$

$$x = -8$$

subst  $x = -8$  into  $\textcircled{1}$

$$y = -8 + 2$$

$$= -6$$

$$= -3$$

$$\left(\frac{3}{3}\right)$$

$$b) -3p + 2q = 2 \quad \textcircled{1}$$

$$2p - 10q = -62 \quad \textcircled{2}$$

$$2p = -62 + 10q$$

$$p = -31 + 5q$$

subst equation ② into ① /

$\frac{3}{3}$

$$-3(-31 + 5q) + 2q = 2$$

$$93 - 15q + 2q = 2$$

$$-13q = -91$$

$$q = 7$$

subst  $q = 7$  into ①

$$-3p + 2 \times 7 = 2$$

$$-3p + 14 = 2$$

$$-3p = -12$$

$$p = 4$$

10.  $90 \times 8 = 720$  ✓  $574$  (total marks to get 90% average)

$$7 \times 82 = 574$$
 (total marks)

$$720 - 574 = 146\%$$

$\frac{3}{3}$

11.

i. Joan

$$\text{mean} = 78$$

$$\text{standard deviation} = 4.07430 \dots$$

$$\approx 4.1$$

$\frac{4}{4}$

John

$$\text{mean} = 78$$

$$\text{standard deviation} = 5.059644 \dots$$

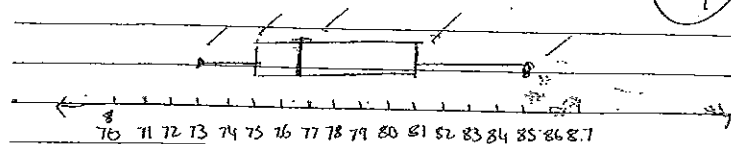
ii. Joan is a more consistent golfer as her standard deviation is lower meaning her scores are less spread out

$\frac{2}{2}$

11. 73, 73, 75, 76, 76, 77, 80, 81, 84, 85

$$81 - 75 = 6$$

$\frac{4}{4}$



12.

$$(i) P(O) = \frac{3}{12} = \frac{1}{4}$$

$\frac{1}{4}$

$$(ii) P(\text{vowel}) = \frac{5}{12}$$

$\frac{1}{4}$

$$(iii) P(P, I) = \frac{4}{12} = \frac{1}{3}$$

$\frac{1}{3}$

13.

$$(i) SU = TQ \text{ (given)}$$

$$\angle PSU = \angle RQT = 90^\circ \text{ (given, } \angle \text{ on a rectangle)}$$

$$PS = QR \text{ (opposite sides on rectangle are equal)}$$

$\frac{3}{3}$

$$\therefore \Delta PSU \cong \Delta RQT \text{ (SAS test)}$$

$$(ii) PV = TR \text{ (corresp sides on congruent } \Delta \text{ are equal)}$$

$\frac{1}{1}$



b)

(i) P(two blue) =  $\frac{4}{10} \times \frac{3}{9}$   
 $= \frac{2}{15}$  ✓  $\left(\frac{1}{15}\right)$

(ii) P(red and blue) =  $\frac{6}{10} \times \frac{4}{9} + \frac{4}{10} \times \frac{6}{9}$   
 $= \frac{4}{15} + \frac{4}{15}$   
 $= \frac{8}{15}$  ✓  $\left(\frac{2}{15}\right)$

7.  $\sqrt{25^2 + 49}$   
 $5 : 7$  ✓  
 $5^2 : 7^2$  ✓

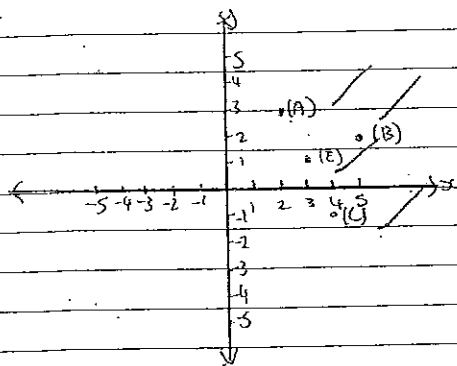
125 : 343 (ratio of volumes)

$\frac{5x}{2800} = \frac{343}{125}$  ✓

$125x = 823200$  ✓

$x = 6585.6 \text{ cm}^3$  ✓  $\left(\frac{3}{15}\right)$

8.



(i)  $E = \left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$   
 $= \left(\frac{2+4}{2}, \frac{3+1}{2}\right)$   
 $= (3, 1)$  ✓  $\left(\frac{1}{1}\right)$

(ii)  $E = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$   
 $(3, 1) = \left(\frac{5+x}{2}, \frac{2+y}{2}\right)$  ✓

$\therefore \frac{5+x}{2} = 3$

$5+x = 6$

$x = 1$

$\frac{2+y}{2} = 1$

$2+y = 2$

$y = 0$  D(1, 0)

(iv)  $m_1 = \frac{y_2-y_1}{x_2-x_1}$   
 $= \frac{3-2}{2-5}$   
 $= -\frac{1}{3}$  ✓  $\left(\frac{1}{1}\right)$

(v)  $m_{BC} = \frac{y_2-y_1}{x_2-x_1}$   
 $= \frac{0+1}{1-4}$   
 $= -\frac{1}{3}$  ✓  $\left(\frac{2}{2}\right)$   
 $\therefore AB \parallel BC$  (same gradient)

(vi)  $y - y_1 = m(x - x_1)$   $y + 1 = \frac{1}{3}(x - 4)$   
 $y - 0 = -\frac{1}{3}(x - 1)$   $3y + 3 = x - 4$   
 $3(y - 0) = -1(x - 1)$   $x + 3y - 1 = 0$  ✓  $\left(\frac{2}{2}\right)$

$\frac{35}{35}$  Excellent

Section 3

1.  $25 - 2 \times 8 \times 2\sqrt{7} + (2\sqrt{7})^2$   
 $= 25 - 20\sqrt{7} + 4 \times 7$   
 $= 25 - 20\sqrt{7} + 28$   
 $= 53 - 20\sqrt{7}$

$\frac{2}{2}$

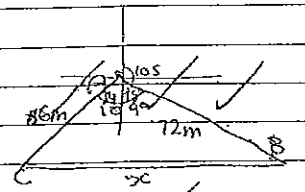
2.  $\frac{7}{\sqrt{11}-3} \times \frac{\sqrt{11}+3}{\sqrt{11}+3}$

$\frac{7(\sqrt{11}+3)}{11-9}$   
 $= \frac{7\sqrt{11}+21}{2}$

$\frac{2}{2}$

$= \frac{7(\sqrt{11}+3)}{2}$

3.



$\frac{2}{2}$

(ii)  $x^2 = 86^2 + 12^2 - 2 \times 86 \times 12 \times \cos 109$   
 $= 16611.836 \dots$   
 $x = \sqrt{16611.836} \dots = 128.88 \dots$   
 $\approx 129.536 \dots = 128.88 \dots$   
 $\approx 130 \text{m (nearest m)} = 129 \text{m (nearest m)}$

$\frac{2}{3}$

4.

a)  $2x^2 + 11x - 6 = 0$        $P = -12$   
 $2x^2 + 12x - 1x - 6 = 0$        $S = 11$   
 $2x(x+6) = 1(x+6) = 0$   
 $(2x-1)(x+6) = 0$   
 $x = \frac{1}{2}, -6$

$\frac{2}{2}$

c)  $5x^2 - 4x - 2 = 0$   
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$   
 $= \frac{4 \pm \sqrt{16 + 40}}{10}$   
 $= \frac{4 \pm 2\sqrt{14}}{10}$   
 $= \frac{2 \pm \sqrt{14}}{5}$

$\frac{2}{2}$

5.

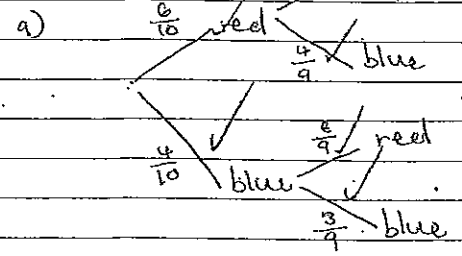
a)  $\angle AEB = \angle CED$  (vert. opp.  $\angle$ s are equal) ✓  
 $\angle ABE = \angle ECD$  (alt.  $\angle$ s on // lines are equal  $AB \parallel CD$ ) ✓  
 $\angle AAE = \angle EDC$  (alt.  $\angle$ s on // lines are equal  $AD \parallel BC$ ) ✓  
 $\therefore \triangle AEB \sim \triangle DEC$  (equiangular) ✓

$\frac{3}{1}$

b) ratios =  $\frac{AE}{DE} = \frac{4}{12} = \frac{1}{3}$  (similar triangles)  
 $\therefore \frac{BE}{CE} = \frac{1}{3}$  (corresp sides of similar  $\Delta$ s are in same ratio)  
 $3BE = CE$   
 $BE = 5 \text{cm}$

$\frac{2}{2}$

6.



$\frac{2}{2}$