

Preliminary Course Task 2 Half Yearly

Mathematics

2016

General Instructions

- Working time – 90 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown on every question

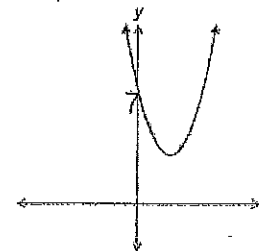
Multiple Choice

1. Which is not the equation of a function?

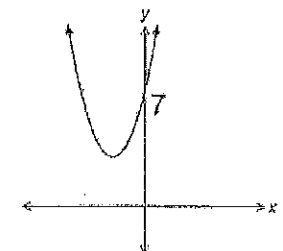
- (A) $y = x^2 + 5$
- (B) $y = -\sqrt{25 - x^2}$
- (C) $x = y^2 + 3$
- (D) $xy = 3$

2. Which is the graph of $f(x) = -(x+2)^2 - 3$?

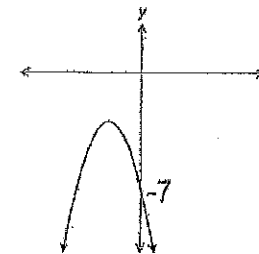
(A)



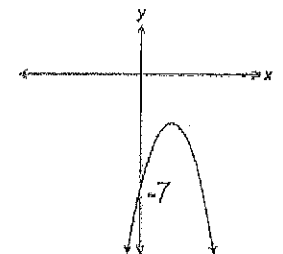
(B)



(C)



(D)

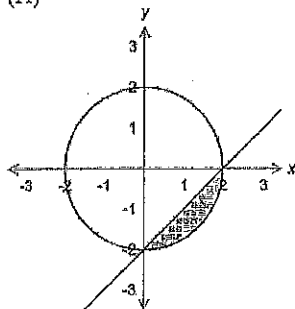


3. If $\tan \theta = \frac{5}{12}$ and $\sin \theta < 0$, then $\sec \theta =$

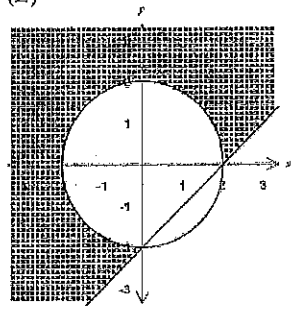
- (A) $\frac{12}{13}$ (B) $-\frac{12}{13}$ (C) $\frac{13}{12}$ (D) $-\frac{13}{12}$

4. Which of the following diagrams show where $x^2 + y^2 \geq 4$ and $y \leq x - 2$ hold simultaneously?

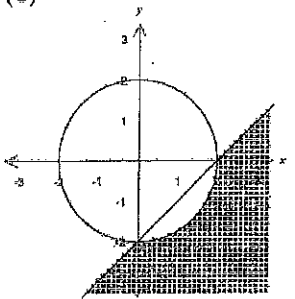
(A)



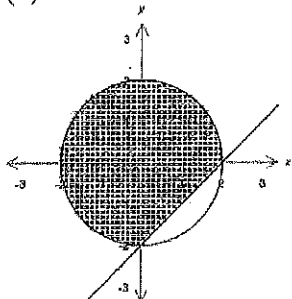
(B)



(C)



(D)



5. $\frac{\sin \theta \cos \theta + \sin^2 \theta}{\sin \theta \cos \theta + \cos^2 \theta} =$

- (A) $\tan \theta$ (B) $\cot \theta$ (C) $\sec^2 \theta$ (D) $\operatorname{cosec}^2 \theta$

Question 6 (Start a new page – 15 marks)

a. Evaluate $\sqrt[3]{\frac{651}{4\pi}}$ correct to 4 significant figures 2

b. Solve $2^{2x+1} = 8$ 2

c. Show that $0.9 = 1$ 2

d. Solve for x if $\left| \frac{4x + 2}{5} \right| \leq 2$ 3

e. Completely factorise the following

i. $6x^2 - 28x + 30$ 2

ii. $x^4 - 16$ 2

iii. $(x - y)^2 - 3x + 3y$ 2

Question 7 (Start a new page - 14 marks)

- a. Expand and simplify $(4\sqrt{3}-\sqrt{2})^2$ 2
- b. Show that $\frac{2\sqrt{3}}{\sqrt{3}+\sqrt{2}}$ can be expressed in the form $a + b\sqrt{6}$. 2
- c. Simplify $\frac{1}{x^2+x} - \frac{1}{x^2-x}$ 3
- d. Solve for x :
- i. $\frac{x-5}{3} - \frac{x+1}{4} = 2$ 2
- ii. $x^2 > x$ 2
- e. Solve these equations simultaneously:
 $x^2 + y^2 - 25 = 0$ and $y + x = 1$ 3

Question 8 (Start a new page - 12 marks)

- a. Sketch these graphs on separate number planes showing all important features:
- i. $f(x) = \sqrt{36-x^2}$ 1
- ii. $f(x) = \sqrt{36-x}$ 2
- iii. $f(x) = -x^2 + 4x - 3$ 3
- b. A function is given by the equation $g(x) = x^2 - 6x + 8$, find :
- i. $g(-3)$ 1
- ii. $g(a-3)$ 2
- iii. all the values of x for which $g(x) < 0$ 2
- c. The function $f(x) = x(x-3)(x+k)$ is an odd function. Find the value of k . 1

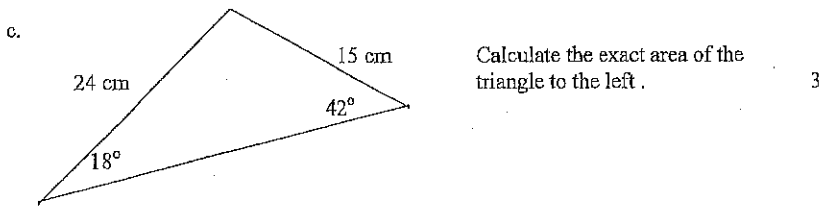
Question 9 (Start a new page – 24 marks)

a. Find the value of x if $\sin(x + 20)^\circ = \cos 30^\circ$. 1

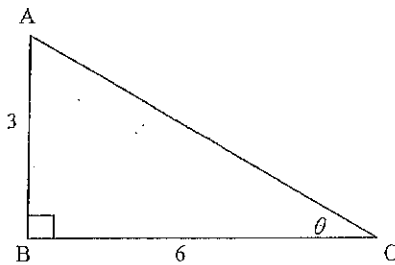
b. Write the following as a trigonometric ratio of θ (if θ is acute), with the correct sign attached.

i. $\sin(180^\circ + \theta)$ 1

ii. $\cos(-\theta)$ 1



d. Consider the triangle ABC.



i. Clearly show that $AC = 3\sqrt{5}$. 1

ii. Hence find the exact and simplified value of $\operatorname{cosec}(90 - \theta)^\circ$ 2

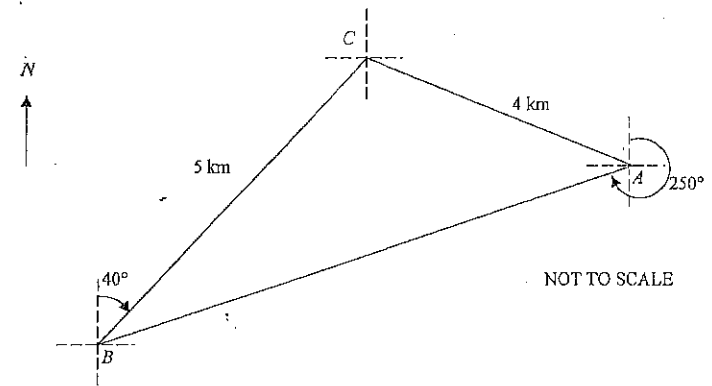
e. Solve for θ over the domain $0^\circ \leq \theta \leq 360^\circ$:

i. $\cos \theta = -\frac{\sqrt{3}}{2}$ 2

ii. $\sin \theta = \sin \theta \tan \theta$ 3

iii. $\tan 2\theta = 0$ 2

f.



$A, B,$ and C are markers in an orienteering course. $AC = 4$ km and $BC = 5$ km. The bearing of C from B is 040°T . The bearing of B from A is 250°T .

i. Copy or trace this diagram into your writing booklet. 3

ii. Find the size of $\angle CAB$ correct to the nearest degree. 1

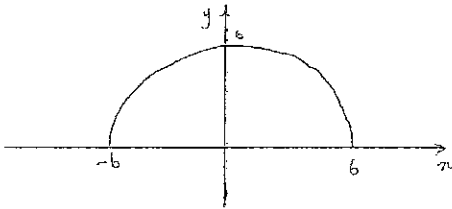
iii. Hence find the bearing of A from C . 1

g. i. Show that $\sin \theta \cos \theta + \frac{\cos^3 \theta}{\sin \theta} = \cot \theta$ 2

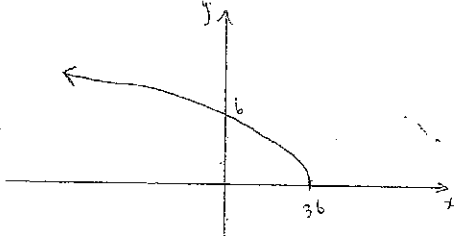
ii. Hence solve $\sin \theta \cos \theta + \frac{\cos^3 \theta}{\sin \theta} = \sqrt{3}$, for $-180^\circ \leq \theta \leq 180^\circ$ 2

END OF EXAM

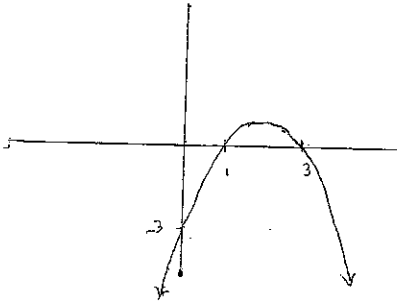
8. a) i)



ii)



iii)



b) i) $g(-3) = (-3)^2 - 6(-3) + 8$

$$= 9 + 18 + 8$$

$$= 35$$

ii) $g(a-3) = (a-3)^2 - 6(a-3) + 8$

$$= a^2 - 6a + 9 - 6a + 18 + 8$$

$$= a^2 - 12a + 35$$

iii) $x^2 - 6x + 8 < 0$

$$(x-2)(x-4) < 0$$

$$2 < x < 4$$

c) $f(x) = -f(-x)$

$$= -(-x)(-x-3)(-x-4)$$

$$= -x(x+3)(x+4)$$

$$k=3$$

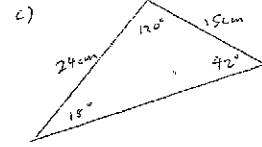
9. a) $\sin(x+20)^\circ = \frac{\sqrt{3}}{2}$

$$x+20 = 60, 240$$

$$x = 40, 220$$

b) i) $-\sin \theta$

ii) $\cos \theta$



c) $Area = \frac{1}{2} \times 24 \times 15 \sin 120^\circ$

$$= 180 \times \frac{\sqrt{3}}{2}$$

$$= 90\sqrt{3} \text{ cm}^2$$

d) i) $AC^2 = 3^2 + 6^2$

$$= 3^2(1+2^2)$$

$$AC = 3\sqrt{5}$$

ii) $\operatorname{cosec}(90-\theta) = \frac{AC}{BC}$

$$= \frac{3\sqrt{5}}{6}$$

$$= \frac{\sqrt{5}}{2}$$

SOLUTIONS

L.C 2.C 3.C 4.C 5.A

6. $\sqrt{3} \cdot 728$

b) $2x+1=3$

$$x=1$$

c) Let $x=0.9$

$$10x=9.9$$

$$x=0.99$$

$$9x=9$$

$$x=1$$

d) If $x \geq -2$

$$\frac{4x+2}{5} \leq 2$$

$$4x+2 \leq 10$$

$$4x \leq 8$$

$$x \leq 2 \Rightarrow -2 \leq x \leq 2$$

If $x < -2$

$$-\frac{4x+2}{5} \leq 2$$

$$4x+2 \geq -10$$

$$4x \geq -12$$

$$x \geq -3 \Rightarrow -3 \leq x < -2$$

$$\Rightarrow -3 \leq x \leq 2$$

e) i) $6x^2 - 28x + 30$

$$= 2(3x^2 - 14x + 15)$$

$$= 2(3x-5)(x-3)$$

ii) $x^4 - 16$

$$= (x^2+4)(x^2-4)$$

$$= (x^2+4)(x+2)(x-2)$$

iii) $(x-y)^2 - 3(x-y)$

$$= (x-y)(x-y-3)$$

7. b) $\frac{2\sqrt{3}}{\sqrt{3}+\sqrt{2}} = \frac{2\sqrt{3}(\sqrt{3}+\sqrt{2})}{3-2}$

$$= 6 + 2\sqrt{6}$$

c) $\frac{1}{x(x+1)} - \frac{1}{x(x-1)}$

$$= \frac{1}{x} \frac{x-1-(x+1)}{(x+1)(x-1)}$$

$$= \frac{-2}{x(x+1)(x-1)}$$

a) $(4\sqrt{3}-\sqrt{2})^2 = 48 - 8\sqrt{6} + 2$

$$= 50 - 8\sqrt{6}$$

d) i) $\frac{x-5}{3} - \frac{x+1}{4} = 2$

$$4(x-5) - 3(x+1) = 24$$

$$x-23=24$$

$$x=47$$

ii) $x^2 - x > 0$

$$x(x-1) > 0$$

$$x > 1, x < 0$$

e) $x^2 + y^2 - 25 = 0$

$$y+x=1 \Rightarrow y=1-x$$

$$x^2 + (1-x)^2 - 25 = 0$$

$$1 - 2x + 2x^2 - 25 = 0$$

$$x^2 - x - 12 = 0$$

$$(x-4)(x+3) = 0$$

$$x=4, -3.$$

If $x=4, y=-3$

$$x=-3, y=4$$

$= 105^\circ$

Bearing of A from C is $19 + 90$

$= 109^\circ$

iii) $x = 39^\circ - 20^\circ$

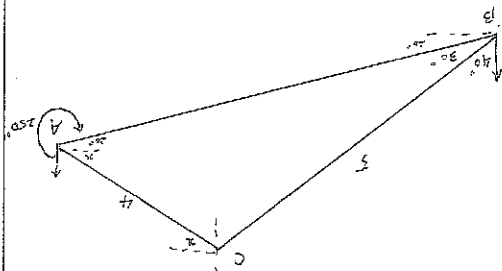
$= 39^\circ$

$\angle CAB = \sin^{-1} \frac{8}{5}$

$\frac{\sin \angle CAB}{5} = \frac{\sin 30^\circ}{4}$

$= 30^\circ$

ii) $\angle CBA = 250^\circ - 180^\circ - 40^\circ$



f) ii)

$\theta = 0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ$
 $2\theta = 0^\circ, 180^\circ, 360^\circ, 540^\circ, 720^\circ$

iii) $\tan 2\theta = 0$

So $\theta = 0^\circ, 45^\circ, 135^\circ, 215^\circ, 360^\circ$

$\theta = 45^\circ, 215^\circ$

if $\tan \theta - 1 = 0$

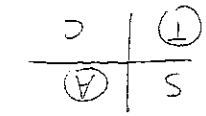
$\theta = 0^\circ, 180^\circ, 360^\circ$

if $\sin \theta = 0$

iii) $\sin \theta (\tan \theta - 1) = 0$

$\theta = 150^\circ, 210^\circ$

e) i) $\alpha = 30^\circ$



$\theta = 30^\circ, -150^\circ$

$\tan \theta = \frac{1}{\sqrt{3}}$

ii) $\cot \theta = \sqrt{3}$

$\cot \theta = \text{RHS}$

$\frac{\cos \theta}{\sin \theta}$

$= \frac{\sin \theta}{(\sin^2 \theta + \cos^2 \theta) \cos \theta}$

$= \frac{\sin \theta}{\sin^2 \theta + \cos^2 \theta}$

g) ii) LHS = $\frac{\sin \theta \cos \theta}{\sin^2 \theta + \cos^2 \theta}$