

J.M.J Ch
MARCELLIN COLLEGE RANDWICK



YEAR 11

MATHEMATICS

HALF YEARLY EXAMINATION

2017

STUDENT NAME: _____

TEACHER: _____

General Instructions

- Reading time - 5 minutes
- Working time – 1.5 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A reference sheet is provided
- Show all necessary working in Questions 6-9

Total marks - 60

Section I

5 marks

- Attempt Questions 1-5
- Allow about 8 minutes for this section

Section II

55 marks

- Attempt Questions 6-9
- Allow about 1 hour 22 minutes for this section

Section I

5 marks

Attempt Questions 1 – 5

Allow about 8 minutes for this section

Use the multiple-choice answer sheet for Questions 1-5

1. What is the value of $\frac{\pi^2}{6}$ correct to three significant figures?

- (A) 1.64
- (B) 1.65
- (C) 1.644
- (D) 1.645

2. Which expression is a factorisation of $8x^3 + 27$?

- (A) $(2x - 3)(4x^2 + 12x - 9)$
- (B) $(2x + 3)(4x^2 - 12x + 9)$
- (C) $(2x - 3)(4x^2 + 6x - 9)$
- (D) $(2x + 3)(4x^2 - 6x + 9)$

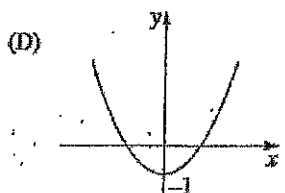
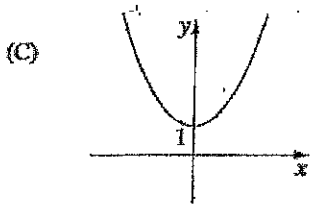
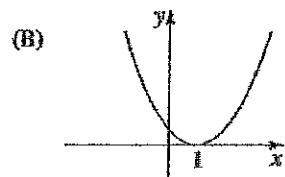
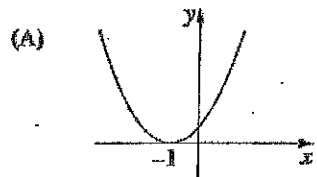
3. Which inequality defines the domain of the function $f(x) \doteq \frac{1}{\sqrt{x+3}}$?

- (A) $x > -3$
- (B) $x \geq -3$
- (C) $x < -3$
- (D) $x \leq -3$

4. The angle of depression from the top of a communications tower measuring 13 m high to a point across the road is 18° . The horizontal distance between the tower and the point on the street is closest to:

- (A) 4 m
- (B) 14 m
- (C) 40 m
- (D) 42 m

5. Which graph best represents $y = (x - 1)^2$?



END OF SECTION I

Section II

55 marks

Attempt Questions 6 – 9

Allow about 1 hour and 22 minutes for this section

Answer each question in the appropriate writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (12 marks)

Marks

- | | |
|---|---|
| (a) Calculate to two decimal places: $5.3 \times 10^9 \div (8.6 \times 10^8)$ | 1 |
| (b) Express $1.2\bar{4}$ as a simplified fraction. | 2 |
| (c) Simplify $(6x)^{-2} \times (4x^2)^{\frac{1}{2}}$ | 2 |
| (d) Write in simplest form: $\sqrt{18} + \sqrt{8}$ | 2 |
| (e) Rationalise the denominator:
$\frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$ | 2 |
| (f) Simplify:
$\frac{\frac{1}{a} - \frac{1}{c}}{\frac{1}{a^2} - \frac{1}{c^2}}$ | 3 |

Question 7 (12 marks)

START A NEW PAGE

Marks

(a) Expand and simplify:

$$-4x^2(x+3) - 2x^2(x-1)$$

1

(b) Solve:

$$\frac{x}{4} - \frac{3x-4}{2} = 6$$

2

(c) The distance to the visible horizon is found using $D = 5\sqrt{\frac{h}{2}}$ Find h correct to two decimal places when $D = 26$.

2

(d) Solve $|4x - 1| = 2x + 7$

3

(e) Solve by completing the square. Leave your answer in surd form.

$$x^2 + 10x = 6$$

2

(f) Solve $x^2 + 6x + 8 > 0$

2

Question 8 (16 marks)

START A NEW PAGE

Marks

(a) Consider the parabola: $y = x^2 + 2x - 24$.(i) Find the x and y intercepts.

2

(ii) Find the coordinates of the vertex and state if this is a maximum or minimum.

2

(iii) What is the range for this function?

1

(iv) Sketch the parabola.

1

(b) For the circle $x^2 + (y - 6)^2 = 64$. Determine its centre and state the radius.

2

(c) Find $\lim_{x \rightarrow -4} \frac{x^2 + 2x - 8}{x + 4}$

2

(d) Sketch the region $y - 2x + 3 < 0$

2

(e) For the curve $y = \frac{1}{x} - 2$

i) State the domain and range

2

ii) Sketch the curve

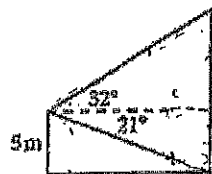
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Question 9 (15 marks)

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Marks

- (a) Find, correct to three significant figures, the value of $\operatorname{cosec} 25^\circ$ 1
- (b) If $\tan \theta = \frac{4}{3}$, find the value of $\sin \theta$, $\cos \theta$ and $\operatorname{cosec} \theta$ 3
- (c) From a point 5 m above the ground, the angle of elevation of the top of a wall is 32° and the angle of depression of the bottom of the wall is 21° . Find

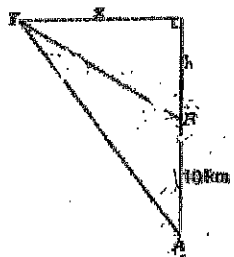


- (i) The horizontal distance from the point of observation to the wall 1
- (ii) The height of the wall, correct to the nearest metre 2

- (d) Find the exact value of the following expression. Write your answer as a single fraction in its simplest form. 2

$$\sin 60^\circ + \cot 45^\circ$$

- (e) From A , the bearing of a tower T is 330° . From B , which is 10km north of A , the bearing of the tower is 290°



- i) Show that
- A) $x = h \tan 70^\circ$ 1
- B) $x = (h + 10) \tan 30^\circ$ 1
- ii) Use the equations in i) to solve for x (answer to 2 decimal places) 3
- iii) Find the distance of the tower from B (answer to 2 decimal places) 1

END OF EXAMINATION

SOLUTIONS.

MC

1. A 2. D 3. A 4. C 5. B

6. a) 6.16

b) Let $x = 1.2\dot{4}$

$$100x = 124.\dot{4}$$

$$10x = 12.\dot{4}$$

$$90x = 112$$

$$x = \frac{112}{90}$$

$$= \frac{56}{45}$$

$$\begin{aligned} c) (6x)^{-2} \times (4x^2)^{\frac{1}{2}} &= \frac{1}{(6x)^2} \times \sqrt{4x^2} \\ &= \frac{1}{36x^2} \times 2x \\ &= \frac{1}{18x} \end{aligned}$$

$$\begin{aligned} d) \sqrt{18} + \sqrt{8} &= \sqrt{9 \times 2} + \sqrt{4 \times 2} \\ &= \sqrt{9}\sqrt{2} + \sqrt{4}\sqrt{2} \\ &= 3\sqrt{2} + 2\sqrt{2} \\ &= 5\sqrt{2} \end{aligned}$$

$$\begin{aligned} e) \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} &= \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} \times \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}} \\ &= \frac{3 + 2\sqrt{3}\sqrt{2} + 2}{3 - 2} \\ &= \frac{5 + 2\sqrt{6}}{1} = 5 + 2\sqrt{6} \end{aligned}$$

$$\begin{aligned} \frac{a}{\frac{1}{a^2} - \frac{1}{c^2}} &= \frac{\frac{ac}{ac}}{\frac{c^2 - a^2}{a^2c^2}} \\ &= \frac{c-a}{ac} \times \frac{a^2c^2}{c^2 - a^2} \\ &= \frac{c-a}{ac} \times \frac{a^2c^2}{(c+a)(c-a)} \\ &= \frac{ac}{c+a} \end{aligned}$$

$$\begin{aligned} 7. a) -4x^2(x+3) - 2x^2(x-1) \\ &= -4x^3 - 12x^2 - 2x^3 + 2x^2 \\ &= -6x^3 - 10x^2 \end{aligned}$$

$$b) \frac{x}{4} - \frac{3x-4}{2} = 6$$

Multiply both sides by 4

$$x - 2(3x-4) = 24$$

$$x - 6x + 8 = 24$$

$$-5x + 8 = 24$$

$$-5x = 16$$

$$x = -\frac{16}{5}$$

11. MARK h the subject:

$$D = 5\sqrt{\frac{h}{2}}$$

$$\frac{D}{5} = \sqrt{\frac{h}{2}}$$

$$\frac{D^2}{25} = \frac{h}{2}$$

$$\frac{2D^2}{25} = h$$

Sub in $D = 26$

$$h = \frac{2(26)^2}{25}$$

$$= 54.08$$

$$d) |4x-11| = 2x+7$$

$$\begin{aligned} \text{If } 4x-1 \geq 0, 4x \geq 1 \\ x \geq \frac{1}{4} \end{aligned}$$

$$\text{and } 4x-1 = 2x+7$$

$$2x = 8$$

$$x = 4 \quad (\text{which is } \geq \frac{1}{4})$$

$$\begin{aligned} \text{If } 4x-1 < 0, 4x \leq 1 \\ x \leq \frac{1}{4} \end{aligned}$$

and

$$-(4x-1) = 2x+7$$

$$-4x+1 = 2x+7$$

$$6x = -6$$

$$x = -1 \quad (\text{which is } < \frac{1}{4})$$

So $x = -1$ or 4

$$c) x^2 + 10x = 0$$

$$x^2 + 10x + 25 = 6 + 25$$

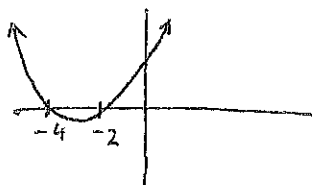
$$(x+5)^2 = 31$$

$$x+5 = \pm\sqrt{31}$$

$$x = -5 \pm \sqrt{31}$$

$$f) x^2 + 6x + 8 > 0$$

$$(x+2)(x+4) > 0$$



$$-4 < x < -2$$

$$8. a) i) x\text{-intercepts: } y = 0$$

$$0 = x^2 + 2x - 24$$

$$0 = (x+6)(x-4)$$

$$x = -6, 4$$

$$y\text{-intercepts } x = 0$$

$$y = 0^2 + 0 - 24$$

$$y = -24$$

$$= x^2 + 2x + 1 - 25$$

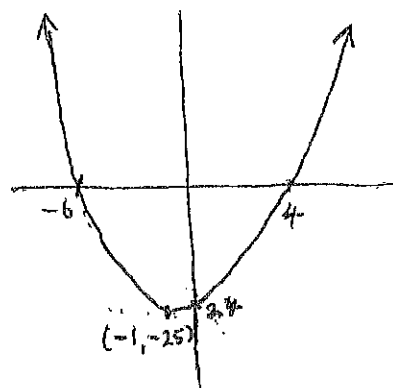
$$= (x+1)^2 - 25$$

Vertex is $(-1, -25)$

It is a minimum because the parabola is concave up: (coefficient of x^2 is positive).

$$iii) y \geq -25$$

iv)



b) Centre is $(0, 6)$

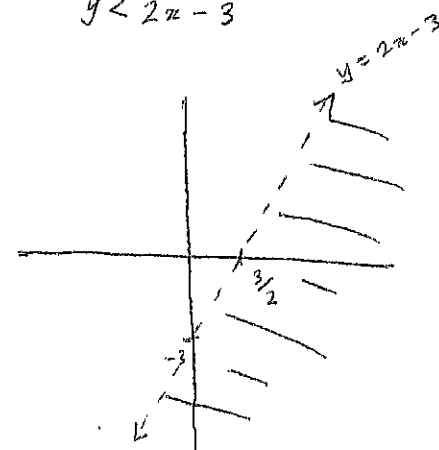
Radius is 8

$$c) \lim_{x \rightarrow -4} \frac{x^2 + 2x - 8}{x + 4} = \lim_{x \rightarrow -4} \frac{(x+4)(x-2)}{x+4}$$

$$= \lim_{x \rightarrow -4} x - 2 = -4 - 2 = -6$$

$$a) y - 2x + 3 < 0$$

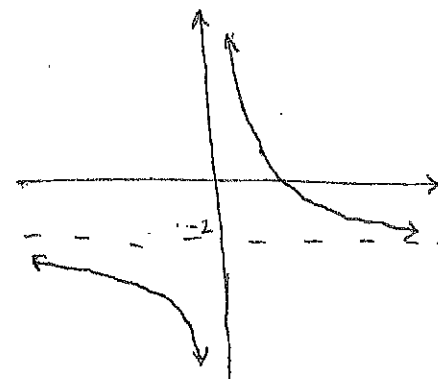
$$y < 2x - 3$$



$$e) i) \text{Domain: } x \neq 0, x \in \mathbb{R}$$

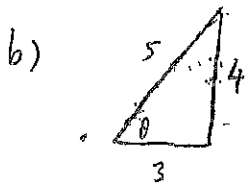
$$\text{Range: } y \neq -2, y \in \mathbb{R}$$

ii)



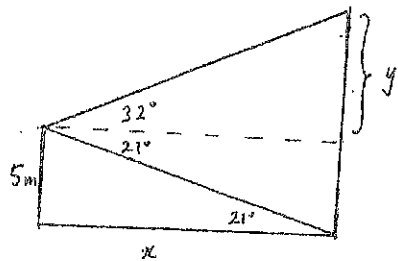
$$i) \operatorname{cosec} 25^\circ = \frac{1}{\sin 25^\circ}$$

$$= 2.37$$



$$\sin \theta = \frac{4}{5}, \cos \theta = \frac{3}{5}, \operatorname{cosec} \theta = \frac{5}{4}$$

c) i)



$$x = \frac{5}{\tan 21^\circ}$$

$$= 13.0 \text{ m}$$

$$ii) y = 13.0 \tan 32^\circ$$

$$= 8.1 \text{ m}$$

$$h = 8.1 + 5$$

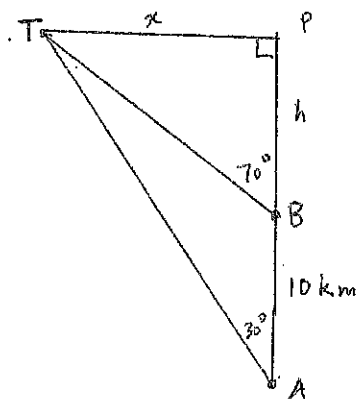
$$= 13 \text{ m (nearest metre)}$$

$$ii) \sin 60^\circ + \cot 45^\circ = \sin 60^\circ + \frac{1}{\tan 45^\circ}$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{1}$$

$$= \frac{\sqrt{3} + 2}{2}$$

e)



i) A) Consider ΔPBT (right angle triangle)

$$\tan 70^\circ = \frac{x}{h}$$

$$x = h \tan 70^\circ$$

B) Consider ΔPAT (right angle triangle)

$$\tan 30^\circ = \frac{x}{h+10}$$

$$x = (h+10) \tan 30^\circ$$

$$ii) h = \frac{x}{\tan 70^\circ} \quad \text{from A)}$$

$$x = \left(\frac{x}{\tan 70^\circ} + 10 \right) \tan 30^\circ$$

$$x \tan 70^\circ = (x + 10 \tan 70^\circ) \tan 30^\circ$$

$$x \tan 70^\circ = x \tan 30^\circ + 10 \tan 70^\circ \tan 30^\circ$$

$$x (\tan 70^\circ - \tan 30^\circ) = 10 \tan 70^\circ \tan 30^\circ$$

$$x = \frac{10 \tan 70^\circ \tan 30^\circ}{\tan 70^\circ - \tan 30^\circ}$$

$$= 7.38 \text{ km}$$

iii) Let $TB = d$.

$$d = \frac{x}{\sin 70^\circ}$$

$$= \frac{7.31}{\sin 70^\circ}$$

$$= 7.78 \text{ km}$$