St Catherine's School

Waverley

Year 11 Mathematics

Preliminary Task 1

10th March 2016

Time allowed: **55 minutes plus 3 minutes reading time**

Total marks: **60 marks**

Weighting: **10%**

INSTRUCTIONS

- There are 2 sections – section are NOT equal value.
- Start each section in a new booklet:
  - Section 1 consists of 3 multiple choice questions and questions 4 and 5.
  - Section 2 consists of questions 6 and 7.
- Start each question on a new page.
- Marks for each question are indicated.
- All necessary working should be shown.
- Approved scientific calculators may be used.
- Students will be provided with the HSC Mathematics Reference Sheet
- Marks may be deducted for careless or badly arranged work.

SECTION 1 START A NEW BOOKLET

Questions 1-3 are multiple choice

Choose the correct answer: Answer in your writing booklet.

QUESTION 1 **(1 mark)**

The value of $\frac{24(0.5)^{\frac{5}{2}}}{\sqrt{7}}$, correct to 3 significant figures is:

(A) 1.603 (B) 1.604 (C) 1.60 (D) 1.61

QUESTION 2 **(1 mark)**

The algebraic expression $27a^3 + 8b^3$ when fully factorised is:

(A) $(9a + 2b)(8a^2 + 16b + 2b^2)$ (B) $(3a + 2b)(9a^2 - 6ab + 4b^2)$

(C) $(3a + 2b)(9a^2 + 6ab + 4b^2)$ (D) $(3a + 2b)(9a^2 - 12ab + 4b^2)$

QUESTION 3 **(1 mark)**

After rationalising the denominator, the expression $\frac{1}{3\sqrt{2}-\sqrt{5}}$ is equivalent to:

(A) $\frac{3\sqrt{2}+\sqrt{5}}{23}$ (B) $\frac{3\sqrt{2}-\sqrt{5}}{23}$ (C) $\frac{3\sqrt{2}+\sqrt{5}}{13}$ (D) $\frac{3\sqrt{2}+\sqrt{5}}{11}$

QUESTION 4 **START A NEW PAGE** **(13 marks)**

(a) Expand and simplify:

(i) $5x - 3(x + 4)$ 1

(ii) $(3m - 2n)^2$ 1

(iii) $(x - 2)(x^2 + 3x + 5)$ 2

(iv) $(y + 4)^2 - (y + 4)(y - 4)$ 2

(b) Factorise fully:

(i) $p^3 - 8q^3$ 1

(ii) $(b - 2)^2 - 7(b - 2)$ 1

(iii) $3x^2 + 7x - 6$ 1

(iv) $t^2 + 2 + \frac{1}{t^2}$ 1

(c) Find integers a and b, such that: 3

$\frac{9}{\sqrt{7}-2} = a + b\sqrt{7}$

QUESTION 5 **START A NEW PAGE** **(15 marks)**

(a) Solve:

(i) $|x + 4| = 5$ 2

(ii) $|5b - 2| = 2b + 4$ 3

(iii) $2^{5x+2} = 8$ 2

(iv) $m^{-\frac{3}{2}} = \frac{1}{8}$ 2

(v) $4x^2 - 11x + 6 = 0$ 2

(vi) $2m^2 + 3m - 4 = 0$ *(Leave your answer in exact form)* 2

(vii) $\frac{x+1}{2} + \frac{x+3}{6} = \frac{2}{3}$ 2

END OF SECTION I

SECTION 2 START A NEW BOOKLET

QUESTION 6 (20 marks)

(a) Solve and sketch the solution on a number line:

(i) $|3a-2| \geq 1$ **3**

(ii) $x^2 - 3x \leq 10$ **3**

(b) Solve simultaneously:

(i) $\begin{cases} 2p+3q=-4 \\ 3p+2q=-6 \end{cases}$ **3**

(ii) $\begin{cases} x+y=5 \\ x^2+y^2=25 \end{cases}$ **3**

(c) Simplify, leaving your answer as a single fraction:

(i) $\frac{12ab-6b^2}{9ab}$ **1**

(ii) $\frac{m^2-9}{m^2-m-12} \div \frac{m^2-3m}{m^2-9m+20}$ **3**

(iii) $\frac{5}{2p+6} + \frac{p}{p^2-9}$ **3**

(iv) $(4x^3)^{\frac{1}{2}}$ **1**

QUESTION 7 START A NEW PAGE (9 marks)

(a) Write down $0.2\dot{8}\dot{3}$ as a fraction in simplest form. **3**
You should include all necessary working.

(b) Expand and simplify, leaving your answer in simplest surd form: **2**

$$(\sqrt{5}+2\sqrt{2})(\sqrt{6}-1)$$

(c) Factorise fully: $a^3+3a^2b+ab^2+3b^3$ **2**

(d) Factorise fully: $4x^4+8x^2-12$ **2**

END OF SECTION 2

END OF TASK

Mathematics

Factorisation

$$a^2-b^2=(a+b)(a-b)$$

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

Angle sum of a polygon

$$S=(n-2)\times 180^\circ$$

Equation of a circle

$$(x-h)^2+(y-k)^2=r^2$$

Trigonometric ratios and identities

$$\sin\theta=\frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta=\frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta=\frac{\text{opposite side}}{\text{adjacent side}}$$

$$\operatorname{cosec}\theta=\frac{1}{\sin\theta}$$

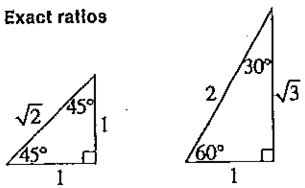
$$\sec\theta=\frac{1}{\cos\theta}$$

$$\tan\theta=\frac{\sin\theta}{\cos\theta}$$

$$\cot\theta=\frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta+\cos^2\theta=1$$

Exact ratios



Sine rule

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

Cosine rule

$$c^2=a^2+b^2-2ab\cos C$$

Area of a triangle

$$\text{Area}=\frac{1}{2}ab\sin C$$

Distance between two points

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Perpendicular distance of a point from a line

$$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

Slope (gradient) of a line

$$m=\frac{y_2-y_1}{x_2-x_1}$$

Point-gradient form of the equation of a line

$$y-y_1=m(x-x_1)$$

***n*th term of an arithmetic series**

$$T_n=a+(n-1)d$$

Sum to *n* terms of an arithmetic series

$$S_n=\frac{n}{2}[2a+(n-1)d] \quad \text{or} \quad S_n=\frac{n}{2}(a+l)$$

***n*th term of a geometric series**

$$T_n=ar^{n-1}$$

Sum to *n* terms of a geometric series

$$S_n=\frac{a(r^n-1)}{r-1} \quad \text{or} \quad S_n=\frac{a(1-r^n)}{1-r}$$

Limiting sum of a geometric series

$$S=\frac{a}{1-r}$$

Compound interest

$$A_n=P\left(1+\frac{r}{100}\right)^n$$

Mathematics (continued)

Differentiation from first principles

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Derivatives

If $y=x^n$, then $\dfrac{dy}{dx}=nx^{n-1}$

If $y=uv$, then $\dfrac{dy}{dx}=u\dfrac{dv}{dx}+v\dfrac{du}{dx}$

If $y=\dfrac{u}{v}$, then $\dfrac{dy}{dx}=\dfrac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}$

If $y=F(u)$, then $\dfrac{dy}{dx}=F'(u)\dfrac{du}{dx}$

If $y=e^{f(x)}$, then $\dfrac{dy}{dx}=f'(x)e^{f(x)}$

If $y=\log_e f(x)=\ln f(x)$, then $\dfrac{dy}{dx}=\dfrac{f'(x)}{f(x)}$

If $y=\sin f(x)$, then $\dfrac{dy}{dx}=f'(x)\cos f(x)$

If $y=\cos f(x)$, then $\dfrac{dy}{dx}=-f'(x)\sin f(x)$

If $y=\tan f(x)$, then $\dfrac{dy}{dx}=f'(x)\sec^2 f(x)$

Solution of a quadratic equation

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Sum and product of roots of a quadratic equation

$$\alpha+\beta=-\frac{b}{a} \qquad \alpha\beta=\frac{c}{a}$$

Equation of a parabola

$$(x-h)^2=\pm 4a(y-k)$$

Integrals

$$\int (ax+b)^n\,dx=\frac{(ax+b)^{n+1}}{a(n+1)}+C$$

$$\int e^{ax+b}\,dx=\frac{1}{a}e^{ax+b}+C$$

$$\int \frac{f'(x)}{f(x)}\,dx=\ln|f(x)|+C$$

$$\int \sin(ax+b)\,dx=-\frac{1}{a}\cos(ax+b)+C$$

$$\int \cos(ax+b)\,dx=\frac{1}{a}\sin(ax+b)+C$$

$$\int \sec^2(ax+b)\,dx=\frac{1}{a}\tan(ax+b)+C$$

Trapezoidal rule (one application)

$$\int_a^b f(x)\,dx\approx\frac{b-a}{2}[f(a)+f(b)]$$

Simpson's rule (one application)

$$\int_a^b f(x)\,dx\approx\frac{b-a}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]$$

Logarithms – change of base

$$\log_a x=\frac{\log_b x}{\log_b a}$$

Angle measure

$$180^\circ=\pi \text{ radians}$$

Length of an arc

$$l=r\theta$$

Area of a sector

$$\text{Area}=\frac{1}{2}r^2\theta$$

2016 Mathematics (2U) Assessment 1 – SOLUTIONS

① $1.603567\ldots \doteq 1.60$ (3 sig fig)

so (C) —— ①

② $(3a+2b)(9a^2-6ab+4b^2)$

so (B) —— ①

③ $\frac{1}{3\sqrt{2}-\sqrt{5}} \times \frac{3\sqrt{2}+\sqrt{5}}{3\sqrt{2}+\sqrt{5}} = \frac{3\sqrt{2}+\sqrt{5}}{18-5}$

$= \frac{3\sqrt{2}+\sqrt{5}}{13}$

So (C) —— ①

④ (a) (i) $5x-3(x+4)$

$= 5x - 3x - 12$

$= 2x - 12$ —— ①

ii) $(3m-2n)^2 = 9m^2 - 12mn + 4n^2$ —— ①

iii) $(x-2)(x^2+3x+5) = x^3+3x^2+5x-2x^2-6x-10$ —— ①

$= x^3 + x^2 - x - 10$ —— ①

iv) $(y+4)^2 - (y+4)(y-4) = y^2+8y+16-(y^2-16)$ —— ①

$= y^2+8y+16-y^2+16$

$= 8y+32$ —— ①

(b) i) $p^3-8q^3 = (p-2q)(p^2+2pq+4q^2)$ —— ①

ii) $(b-2)^2 - 7(b-2) = (b-2)[b-2-7]$

$= (b-2)(b-9)$ —— ①

iii) $3x^2+7x-6 = (3x-2)(x+3)$ —— ①

iv) $t^2+2+\frac{1}{t^2} = (t+\frac{1}{t})^2$ —— ①

⑤ (c) $\frac{9}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2} = \frac{9(\sqrt{7}+2)}{7-4}$ —— ①

$= \frac{{}^{3}\cancel{9}(\sqrt{7}+2)}{\cancel{3}_{1}}$

$= 3(\sqrt{7}+2)$ —— ①

$= 3\sqrt{7}+6$

so $a=6,\ b=3$ —— ①

⑤ (a) i) $|x+4| = 5$

$x+4 = \pm 5$

$x = -4 \pm 5$

$= -9, 1$ —— ① for each (②, total)

ii) $|5b-2| = 2b+4$

Case 1: $5b-2 \geq 0 \Rightarrow b \geq \frac{2}{5}$

Then $5b-2 = 2b+4$

$3b = 6$

$b = 2$ —— ①

Check: $|8| = 8$ ✓

Case 2: $5b-2 < 0 \Rightarrow b < \frac{2}{5}$

Then $-(5b-2) = 2b+4$

$-5b+2 = 2b+4$

$-7b = 2$

$b = -\frac{2}{7}$ —— ①

Check: $\left|-\frac{24}{7}\right| = \frac{24}{7}$ ✓ —— (+1) for correct check or correctly identify restricted domain

iii) $2^{5x+2} = 2^3$ —— ①

$5x+2 = 3$

$5x = 1$

$x = \frac{1}{5}$ —— ①

(5) (a) iv) $m^{-\frac{3}{2}} = \frac{1}{8}$ } take reciprocals

$m^{\frac{3}{2}} = 8$ —— (1)

$m^3 = 8^2$

$= 64$

$m = \sqrt[3]{64}$

$= 4$ —— (1)

v) $4x^2 - 11x + 6 = 0$

$(4x-3)(x-2) = 0$ —— (1)

$\therefore 4x - 3 = 0$ or $x - 2 = 0$

$4x = 3$ $\quad x = 2$

$x = \frac{3}{4}$

$\therefore x = \frac{3}{4}, 2$ —— (1)

vi) $2m^2 + 3m - 4 = 0$

$m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-3 \pm \sqrt{9 + 32}}{4}$ —— (1)

$= \dfrac{-3 \pm \sqrt{41}}{4}$ —— (1)

vii) $\dfrac{x+1}{2} + \dfrac{x+3}{6} = \dfrac{2}{3}$

$\dfrac{3(x+1)}{6} + \dfrac{x+3}{6} = \dfrac{4}{6}$ —— (1)

$3(x+1) + (x+3) = 4$

$3x + 3 + x + 3 = 4$

$4x = -2$

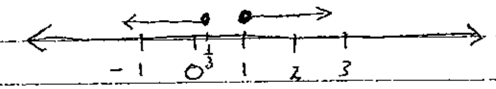
$x = -\frac{1}{2}$ —— (1)

Q 6 (a) i) $|3a - 2| \geq 1$

$3a - 2 \geq 1$ or $3a - 2 \leq -1$

$3a \geq 3$ $\quad 3a \leq 1$

$a \geq 1$ —— (1) $\quad a \leq \frac{1}{3}$ —— (1)



—— (1)

ii) $x^2 - 3x \leq 10$

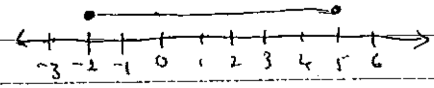
Solve: $x^2 - 3x - 10 \leq 0$

$(x-5)(x+2) \leq 0$

$x = -2, 5$ —— (1)

Test: when $x = 0$, $0 - 0 \leq 10$ ✓

So $x = 0$ is included



—— (1)

$\therefore -2 \leq x \leq 5$ —— (1)

(6) (b)(i) $2p + 3q = -4$ —— (1)

$3p + 2q = -6$ —— (2)

(1)×3 $\quad 6p + 9q = -12$ —— (3)

(2)×2 $\quad 6p + 4q = -12$ —— (4)

} correct variation — (1m)

(3)−(4) $\quad 5q = 0$

$q = 0$ —— first pronumeral found (1m)

Sub $q = 0$ into (1) $\quad 2p + 0 = -4$

$p = -2$

So $p = -2, q = 0$ —— (1 mark)

6 (b) (ii) $x + y = 5$ ……(1)

$x^2 + y^2 = 25$ ……(2)

From (1) $x = 5 - y$ ……(3) — (1)

Sub (3) into (2) $(5-y)^2 + y^2 = 25$

$25 - 10y + y^2 + y^2 = 25$

$2y^2 - 10y = 25 - 25$

$2y(y-5) = 0$

$y = 0, 5$

} correct solution of quadratic — (1)

Sub into (3) when $y = 0$, $x = 5 - 0$

$= 5$

when $y = 5$, $x = 5 - 5$

$= 0$

So solutions are $(0,5)$ and $(5,0)$ — (1)

(c) i) $\frac{12ab - 6b^2}{9ab} = \frac{2\cancel{6}\cancel{b}(2a-b)}{3\cancel{9}a\cancel{b}}$

$= \frac{2(2a-b)}{3a}$ — (1)

ii) $\frac{m^2 - 9}{m^2 - m - 12} \times \frac{m^2 - 9m + 20}{m^2 - 3m}$

$= \frac{(m-3)(m+3)}{(m-4)(m+3)} \times \frac{(m-4)(m-5)}{m(m-3)}$ correct factorising (2) (−1 for each error)

$= \frac{m-5}{m}$ cancelling (1)

iii) $\frac{5}{2p+6} + \frac{p}{p^2-9} = \frac{5}{2(p+3)} + \frac{p}{(p+3)(p-3)}$ factorising (1)

$= \frac{5(p-3) + 2p}{2(p+3)(p-3)}$ correct common denominator (1)

$= \frac{5p - 15 + 2p}{2(p+3)(p-3)}$

$= \frac{7p - 15}{2(p+3)(p-3)}$ — (1)

6 (c) (iv) $(4x^3)^{\frac{1}{2}} = \sqrt{4} \times \sqrt{x^3}$

$= 2\sqrt{x^3}$ } either — (1)

OR $2x^{\frac{3}{2}}$

7 (a) Let $x = 0.2\dot{8}\dot{3}$ — (1)

$100x = 28.2\dot{8}\dot{3}$

$99x = 28.2\dot{8}\dot{3} - 0.2\dot{8}\dot{3}$ — (1)

$= 28.1$

$\therefore x = \frac{28.1}{99}$

$= \frac{281}{990}$ — (1)

(b) $(\sqrt{5} + 2\sqrt{2})(\sqrt{6} - 1) = \sqrt{30} - \sqrt{5} + 2\sqrt{12} - 2\sqrt{2}$ — (1)

$= \sqrt{30} - \sqrt{5} + 4\sqrt{3} - 2\sqrt{2}$ — (1)

(c) $a^3 + 3a^2b + ab^2 + 3b^3 = a^2(a+3b) + b^2(a+3b)$ — (1)

$= (a+3b)(a^2 + b^2)$ — (1)

(d) $4x^4 + 8x^2 - 12 = (4x^2 - 4)(x^2 + 3)$ — (1)

$= 4(x^2 - 1)(x^2 + 3)$

$= 4(x-1)(x+1)(x^2+3)$ — (1)