Student's Number: _____

St Catherine's School
Waverley

Year 11 Mathematics

Preliminary Task 2

10th May 2016

Time allowed: *55 minutes*

Total marks: *55 marks*

Weighting: *25%*

INSTRUCTIONS

- Start each question in a new booklet.
- Marks for each question are indicated.
- All necessary working should be shown.
- Diagrams should be drawn using pencil and ruler.
- Approved scientific calculators may be used.
- Marks may be deducted for careless or badly arranged work.

Section I	Multiple Choice	/5
Section II	Real Functions	/22
Section III	Trigonometric Ratios	/25
	TOTAL	/52

Section I **5 Marks**

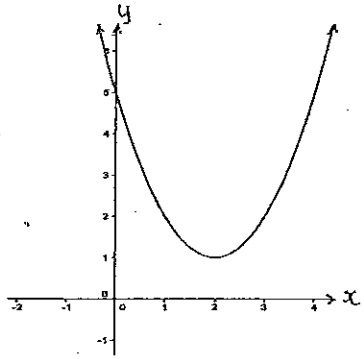
Multiple Choice

Choose the correct answer. Answer on the multiple choice answer sheet.

1 If $g(x) = 2^x + |x+1|$, then
$g(-2) =$

(A) -3 (B) $\frac{5}{4}$ (C) $-\frac{3}{4}$ (D) 3

2



The equation of the parabola shown could be

(A) $y = (x-1)^2 + 2$ (B) $y = (x+2)^2 + 1$

(C) $y = (x-2)^2 + 1$ (D) $y = 1 - (x-2)^2$

3 If $\cos\theta = \frac{5}{13}$ and θ is acute, the exact value of $\tan\theta$ is

(A) $\frac{12}{5}$ (B) $\frac{12}{13}$ (C) $\frac{5}{12}$ (D) $\frac{4}{5}$

4 $\cos(180-\theta)\tan(180+\theta) =$

(A) $-\sin\theta$ (B) $\cot\theta$ (C) $-\sec\theta$ (D) $\sin\theta$

5 If $f(x) = \begin{cases} 2x & x < -2 \\ 5 & -2 \le x \le 2 \\ x^2 + 1 & x > 2 \end{cases}$

Then $f(-3) + f(2) + f(3) =$

(A) 2 (B) 14 (C) 10 (D) 9

END OF SECTION I

Section II (Functions) 25 marks START A NEW BOOKLET

Marks

1) If $f(x) = x^2 + 5x$, find the value(s) of x for which $f(x) = 14$. 2

2) Sketch the following graphs on *separate number planes*, showing x and y intercepts and other important features.
Under each sketch clearly state the **domain** and **range** of the curve.

a) $y = |x - 3|$ 3

b) $y = -\frac{12}{x+2}$ 3

c) $y = \sqrt{9 - x^2}$ 3

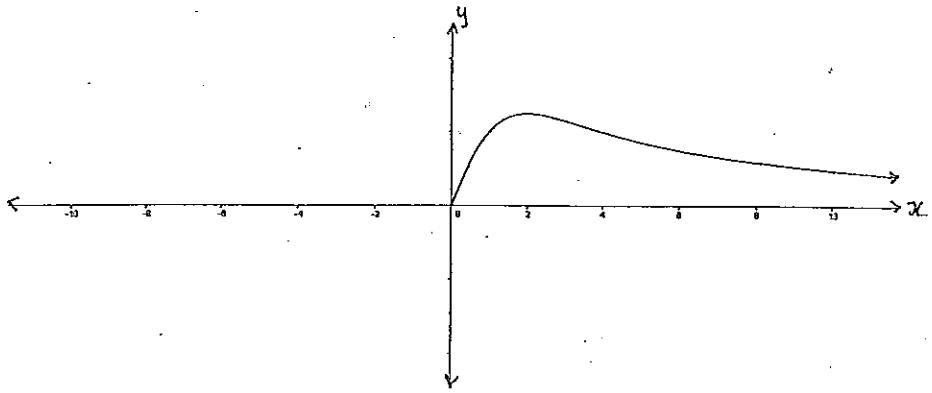
3) a) Sketch $y = x^2 - 2x$ for the domain $-1 \le x \le 4$. 2
Show the x and y intercepts and the vertex if they are in the given domain.

b) State the range of $y = x^2 - 2x$ for this domain. 1

4) For the function $f(x) = \frac{x}{x^2+4}$

a) Prove that $f(x)$ is an odd function. 2

b) One half of the function $f(x) = \frac{x}{x^2+4}$ has been sketched below.



Copy this sketch into your answer booklet and complete the other half of the function, given that it is odd. 1

5) Sketch the region satisfying the inequalities $x^2 + y^2 \leq 1$ and $y \leq |x|$. 3

6) State the domain of $f(x) = \sqrt{x-2} + \sqrt{6-x}$ 2

END SECTION II

Section III (Trigonometry) 25 marks START A NEW BOOKLET

1) Find the exact value of

a) $\cos 150°$ 1

b) $\sin 60° \cos 45° \tan 60°$ 2

c) $\sec 495°$ 1

2) Solve for $0° \leq \theta \leq 360°$

a) $\sin\theta = -\frac{1}{\sqrt{2}}$ 2

b) $\tan 2\theta = \sqrt{3}$ 2

3) The angle of depression from the top of a cliff to a ship at sea is 12°. The cliff is $120m$ high.

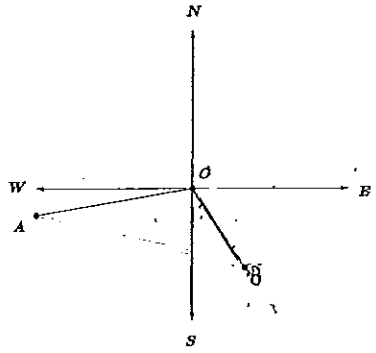
a) Draw a diagram for this information, marking on it the angle of depression and the height of the cliff. 1

b) Find the distance from the ship to the base of the cliff (nearest metre). 2

4) Prove that $\tan\theta + \cot\theta = \sec\theta \operatorname{cosec}\theta$ 2

Question 5 is on the next page.

5) Two planes leave an airport. Plane A flies 300 km on a bearing of 245°, while plane B flies 160 km on a bearing of 140°.



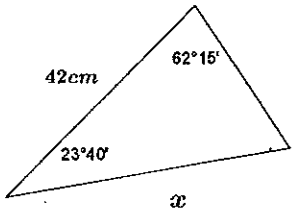
a) Copy the diagram above and mark on it the information given. 1

b) Explain why $\angle AOB = 105°$. 1

c) Find the distance between the two planes, AB, correct to the nearest kilometre. 2

d) Find the bearing of A from B (nearest degree). 2

6)



Not to scale

a) Use the Sine Rule to find the value of x correct to one decimal place. 2

b) Find the area of ΔABC. 2

7) Simplify $\frac{1+\cos\theta}{1-\sin\theta} \div \frac{1+\sin\theta}{1-\cos\theta}$ 2

Qn	Solutions	Marks	Comments: Criteria

MULTIPLE CHOICE

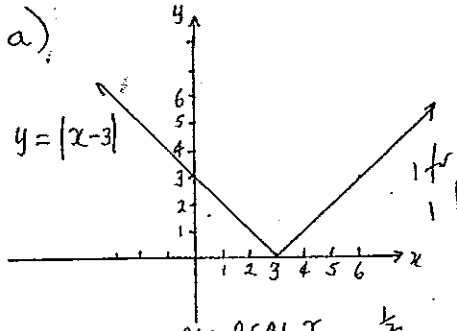
(1) B
(2) C
(3) A
(4) A
(5) D

$f(-3) = -6$
$f(2) = 5$
$f(3)$ no

SECTION 2

(1) $x^2+5x=14$ (1M)
$x^2+5x-14=0$
$(x+7)(x-2)=0$
$x=-7, 2$ (1M)

(2) a)

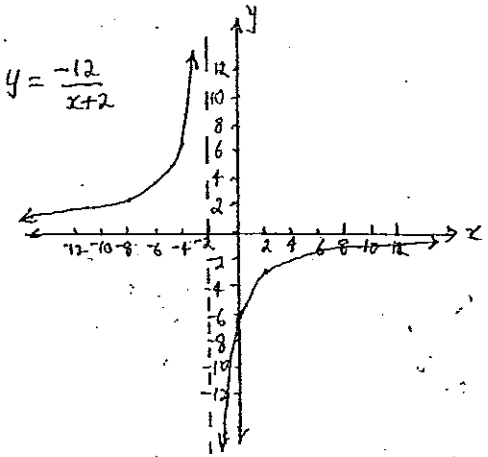


1 for shape
1 for showing sufficient pts

DOMAIN : ALL REAL x $\frac{1}{2}$
RANGE : $y \geq 0$ $\frac{1}{2}$

Qn	Solutions	Marks	Comments: Criteria

b)

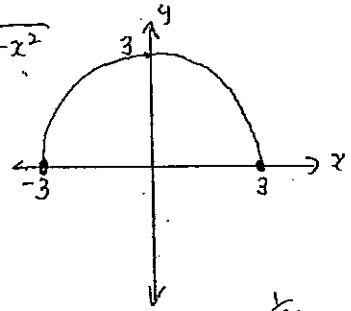


1 for shape
1 for indicating position including (0, -6)

y-INT
$x=0$
$y=-6$

DOMAIN; ALL REAL x EXCEPT $x=-2$ ½
RANGE: ALL REAL y EXCEPT $y=0$ ½

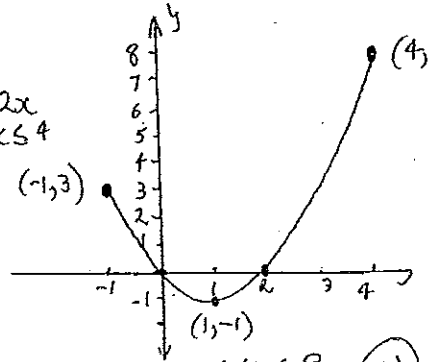
c) $y=\sqrt{9-x^2}$



1 for shape
1 for position

DOMAIN: $-3 \le x \le 3$ ½
RANGE: $0 \le y \le 3$ ½

3 a) $y=x^2-2x$ FOR $-1 \le x \le 4$



x-INT $y=0$
$x^2-2x=0$
$x(x-2)=0$
$x=0, 2$
VERTEX $(1,-1)$

1½ if fixed end points not shown.
-½ if rate, not shown.

b) THE RANGE IS $-1 \le y \le 8$ (1M)

Qn	Solutions	Marks	Comments: Criteria

4 a) $f(x)=\frac{x}{x^2+4}$

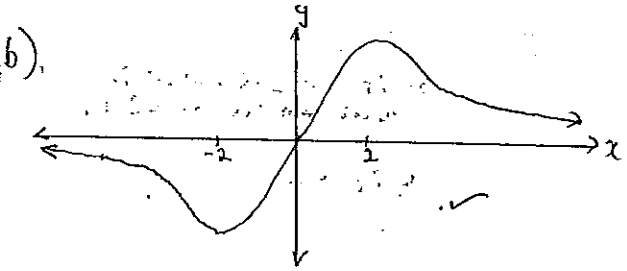
$f(-x)=\frac{-x}{(-x)^2+4}$ ✓

$=\frac{-x}{x^2+4}$

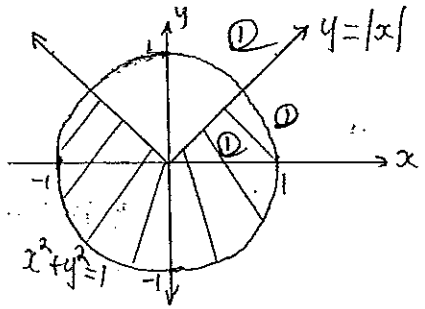
$=-f(x)$ ✓

SINCE $f(-x)=-f(x)$, $\therefore f(x)$ IS ODD.

b)



5



6 $x-2 \ge 0$ $\quad 6-x \ge 0$

$x \ge 2$ $\quad x \le 6$

Dom: $2 \le x \le 6$.

Qn	Solutions	Marks	Comments: Criteria

SECTION 3

(1) a) $\cos 150^\circ = -\cos 30^\circ$ ½ if for $\cos 30^\circ$

$= -\frac{\sqrt{3}}{2}$

b) $\sin 60^\circ \cos 45^\circ \tan 60^\circ = \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} \cdot \sqrt{3}$ ½ ½ ½

$= \frac{3}{2\sqrt{2}}$ ½

$= \frac{3\sqrt{2}}{4}$ or

c) $\sec 480^\circ = \frac{1}{\cos 135^\circ}$ 0 if the answer is wrong and no working.

$= \frac{1}{-\cos 45^\circ}$ ½ if $\sqrt{2}$

$= -\sqrt{2}$

(2) a) $\sin\theta = -\frac{1}{\sqrt{2}}$

$\theta = 180^\circ + 45^\circ, 360^\circ - 45^\circ$

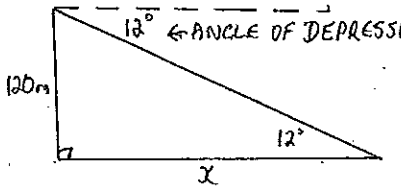
$= 225^\circ, 315^\circ$ (1) (1)

b) $\tan 2\theta = \sqrt{3}$

$2\theta = 60^\circ, 240^\circ, 420^\circ, 600^\circ$ ½ for each angle

$\theta = 30^\circ, 120^\circ, 210^\circ, 300^\circ$

(3)



½ for showing the angle of depression

½ for h.

$\tan 12^\circ = \frac{120}{x}$

$x = \frac{120}{\tan 12^\circ}$

$\doteq 565$ m $-\frac{1}{2}$ if not rounded properly

Qn	Solutions	Marks	Comments: Criteria

(4) $\tan\theta + \cot\theta = \sec\theta \operatorname{cosec}\theta$

LHS $= \tan\theta + \cot\theta$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$ (1)

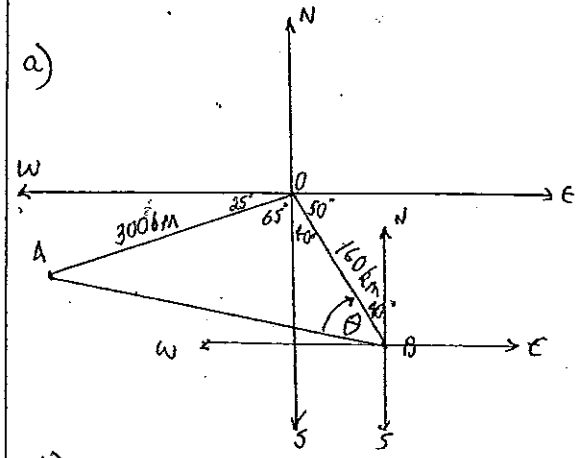
$= \frac{1}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta} \times \frac{1}{\cos\theta}$ (1)

$= \operatorname{cosec}\theta \sec\theta$

$=$ RHS

(5) a)



½ distance

½ angles

b) $\angle AOB = 245^\circ - 140^\circ$

$= 105^\circ$ (1)

c) $c^2 = a^2 + b^2 - 2ab\cos C$

$AB^2 = 160^2 + 300^2 - 2(160)(300)\cos 105^\circ$

$AB \doteq 375$ km

−1 Sub Calc. mistake

Qn	Solutions	Marks	Comments: Criteria
	d) BEARING $= 360° - 40° - \theta$ $(\theta = \angle OBA)$ $\frac{\sin\theta}{300} = \frac{\sin 105°}{375}$ → (STUDENTS CAN USE THIS VALUE OR EXACT VALUE) $\sin\theta = \frac{300\sin 105°}{375°}$ $\theta \doteqdot 51°$ $\therefore$ BEARING $\doteqdot 360° - 40° - 51°$ $\doteqdot 269°$	(1) (1)	
(6)	a) $\frac{a}{\sin A} = \frac{b}{\sin B}$ $\frac{x}{\sin 62°15'} = \frac{42}{\sin 94°5'}$ $x = \frac{42\sin 62°15'}{\sin 94°5'}$ $\doteqdot 37.3$ cm	(1) (1)	
	b) $A \doteqdot \frac{1}{2}ab\sin C$ $= \frac{1}{2}(42)(37.3)\sin 23°40'$ $\doteqdot 314.4\ cm^2$ (1 DEC. PL.)	(1) (1)	
(7)	$\frac{1+\cos\theta}{1-\sin\theta} \div \frac{1+\sin\theta}{1-\cos\theta} = \frac{1+\cos\theta}{1-\sin\theta} \times \frac{1-\cos\theta}{1+\sin\theta}$ $= \frac{1-\cos^2\theta}{1-\sin^2\theta}$ $= \frac{\sin^2\theta}{\cos^2\theta}$ $= \tan^2\theta$	(1) (1)	