Student Number: ____________________

St Catherine's School

Waverley

# YEAR 11 MATHEMATICS EXTENSION 1

## PRELIMINARY TASK 1

## 24th March 2015

**General Instructions**

- Reading Time – 3 minutes
- Working Time – 55 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Marks may be deducted for careless or badly arranged work
- Task Weighting – 25%
- Total Marks – 51

**SECTION I** **18 marks**

- Attempt Questions 1 – 4 in one booklet
- Show all necessary working

**SECTION II** **33 marks**

- Attempt Questions 5 – 6
- Answer each question in a separate booklet
- Show all necessary working

| | |
|---|---|
| Question 1 – 3 | /3 |
| Question 4 | /15 |
| Question 5 | /16 |
| Question 6 | /17 |
| **TOTAL** | **/51** |



**SECTION I** **START A NEW BOOKLET**

**Total Marks 18**

**Attempt Questions 1 – 4**

**Questions 1 to 3, answer either A, B, C or D.** **3 Marks**

**Question 1**

What is the exact value of $\tan 60° \sin 30°$ ?

(A) $\frac{1}{2\sqrt{3}}$ (B) $\frac{\sqrt{3}}{4}$

(C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$

**Question 2**

Determine the centre and radius of $x^2 + y^2 - 2y - 3 = 0$.

(A) centre $(0, 2)$ and radius = $\sqrt{3}$ units

(B) centre $(0, 1)$ and radius = $\sqrt{3}$ units

(C) centre $(1, 0)$ and radius = 2 units

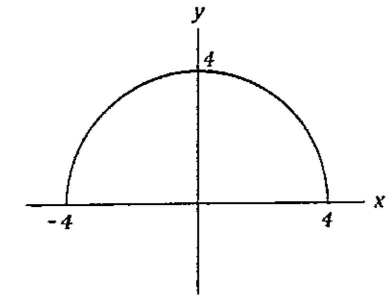
(D) centre $(0, 1)$ and radius = 2 units

**Question 3**

Which one of the following graphs best represents the curve $f(x) = \sqrt{16 - x^2}$ ?

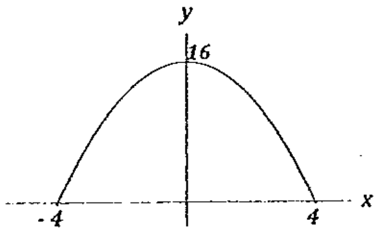
(A)

Diagram not to scale



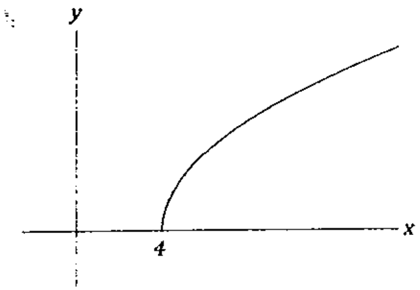

(B)

Diagram not to scale



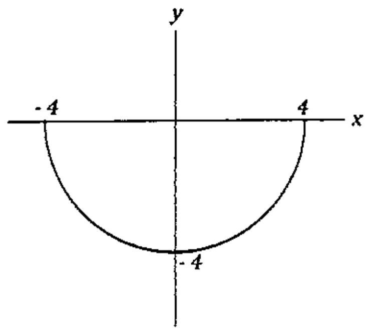

(C)

Diagram not to scale




(D)

Diagram not to scale






**Question 4** **(15 marks)** **Marks**

(a) Solve $\frac{2}{x} \geq 5$ 2

(b) Solve $\frac{2x+1}{3x-2} < 2$ 3

(c) Determine whether $f(x) = \frac{3x}{x^4-2}$ is even, odd or neither. 2

(d) Evaluate $\frac{\cos 60°}{\sin 45°} + \cot 30°$. Leave your answer as an exact value. 2

(e) For the parabola $f(x) = x^2 + 2x - 8$,

(i) Sketch $y = f(x)$ for $-5 \leq x \leq 4$, clearly labelling the $x$-intercept(s), $y$-intercept(s) and the endpoints. 3

(ii) What is the minimum value of $f(x)$. 1

(iii) Find the range of $f(x)$ for $-5 \leq x \leq 4$. 1

(iv) For what values of $x$ is the curve decreasing? 1

**End of Section I**

**SECTION II**
**Total Marks 33**

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**Question 5** **(16 marks)** **START A NEW BOOKLET** **Marks**

(a) For $f(x) = |2x + 1|$

(i) Sketch the curve, clearly labelling all essential features, including any intercepts. 2

(ii) State the natural domain and range. 1

(b) For $f(x) = 3^x + 1$

(i) Sketch the curve, clearly labelling all essential features, including any intercepts and asymptotes. 2

(ii) State the natural domain and range. 1

(c) For $f(x) = \dfrac{3}{x+2} - 1$

(i) Sketch the curve, clearly labelling all essential features, including any intercepts and asymptotes. 3

(ii) State the natural domain and range. 1

(d) A function is defined as below:

$$f(x) = \begin{cases} x^2 + 2 & \text{for } x \geq 0 \\ x + 1 & \text{for } x < 0 \end{cases}$$

(i) Sketch $f(x)$, clearly labelling $x$-intercept(s) and $y$-intecept(s). 2

(ii) Evaluate $f(2) - 3f(-1) + f(0)$. 2

(e) Shade the region defined by $(x-1)^2 + (y-1)^2 \leq 9$, $x \leq 2$ and $y \geq 1$. 2



**Question 6** **(17 marks)** **START A NEW BOOKLET** **Marks**

(a) Ray drives from town P to his home on a bearing of $300°T$ for 30 $km$.
Tina drives from town P to her home on a bearing of $230°T$ for 80 $km$.

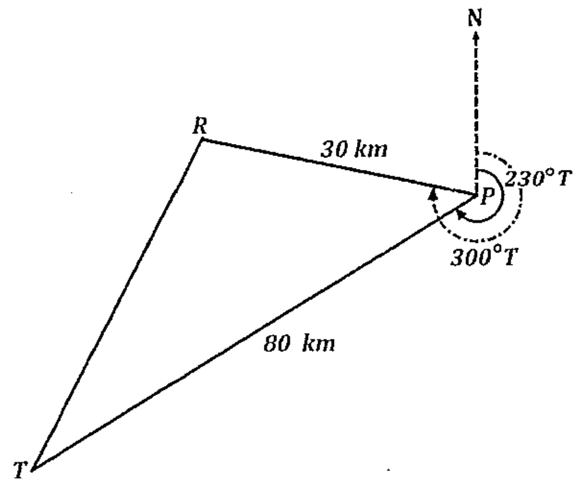



Diagram not to scale

(i) What is the distance between Ray and Tina? 3

Leave your answer to the nearest kilometre.

(ii) Find the bearing of Tina from Ray. 3

Leave your answer to the nearest degree.

(b) State the largest possible domain for the function $y = \sqrt{x^2 - 4x}$ . 2

**Question 6 continues**

**Question 6 continued**

(c) From an observation tower $PQ$ of height $h$ metres, two points $A$ and $B$ at ground level have bearings of $172°T$ and $060°T$ respectively. The angles of elevation to the top of this tower from $A$ is $48°$ and from $B$ is $32°$. The distance between the points $A$ and $B$ is $105\ m$.

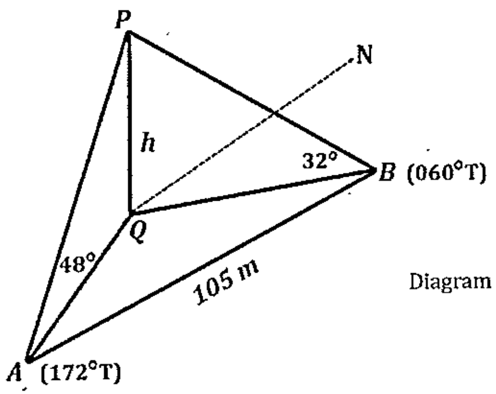



Diagram not to scale

(i) Show that $BQ = h\tan 58°$ . 1

(ii) Similarly, find an expression for $AQ$ . 1

(iii) Hence, find the height of the observation tower $PQ$, to the nearest metre. 3

**Question 6 continues**



**Question 6 continued**

(d)

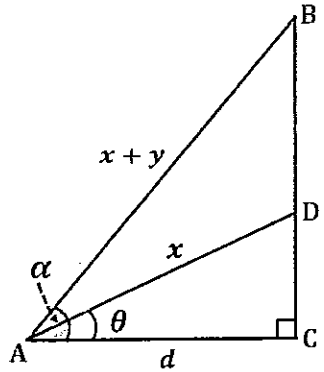



Diagram not to scale

In $\Delta ABC$, $AB = x + y$ , $AD = x$ , $AC = d$ , $\angle BAC = \alpha$ and $\angle DAC = \theta$.

(i) Find an expression for $\cos\theta$. 1

(ii) Prove that $d = \frac{y\cos\theta\cos\alpha}{\cos\theta - \cos\alpha}$ . 3

**End of Section II**

**END OF TASK**

| Qn | Solutions | Marks | Comment: Criteria |
|---|---|---|---|

Year 11 Mathematics Extension 1

Preliminary Task 1

SOLUTIONS

## Section 1

(1) $\tan 60^\circ \sin 30^\circ = \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{2}$ (C)

(2)
$$x^2 + y^2 - 2y - 3 = 0$$
$$x^2 + y^2 - 2y = 3$$
$$x^2 + y^2 - 2y + \left(\frac{-2}{2}\right)^2 = 3 + \left(\frac{-2}{2}\right)^2$$
$$x^2 + y^2 - 2y + 1 = 4$$
$$x^2 + (y-1)^2 = 2^2$$

$\therefore$ centre $(0, 1)$ and radius $= 2$ units (D)

(3) $f(x) = \sqrt{16 - x^2}$

is a positive semicircle with centre $(0, 0)$ and radius 4. (A)

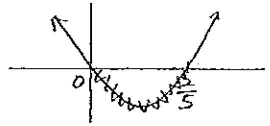


| Qn | Solutions | Marks | Comment: Criteria |
|---|---|---|---|

(4)

(a) $\frac{2}{x} \geq 5$ where $x \neq 0$ — 2

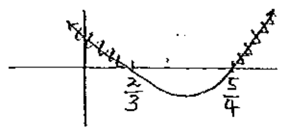
$$x^2 \times \frac{2}{x} \geq 5x^2$$
$$2x \geq 5x^2$$
$$0 \geq 5x^2 - 2x$$
$$0 \geq x(5x - 2)$$




$\therefore \quad 0 < x \leq \frac{2}{5}$

(b) $\frac{2x+1}{3x-2} < 2$ — 3

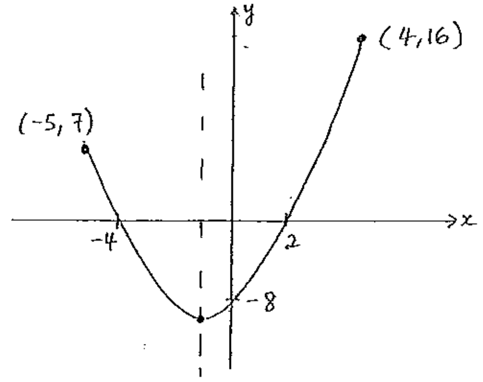
$$(3x-2)^2 \times \frac{2x+1}{3x-2} < 2(3x-2)^2$$
$$(3x-2)(2x+1) < 2(3x-2)^2$$
$$0 < 2(3x-2)^2 - (3x-2)(2x+1)$$
$$0 < (3x-2)(6x-4-2x-1)$$
$$0 < (3x-2)(4x-5)$$

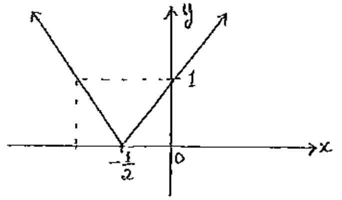
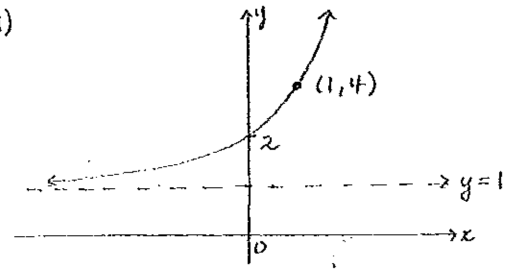



$\therefore \quad x < \frac{2}{3}$ or $x > \frac{5}{4}$

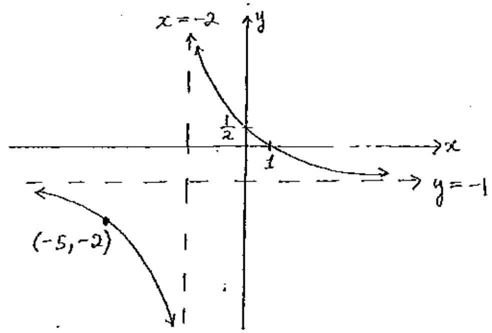
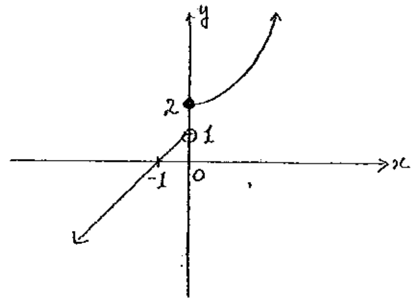
| Qn | Solutions | Marks | Comment: Criteria |
|---|---|---|---|
| (c) | $f(x) = \dfrac{3x}{x^4 - 2}$<br>$f(-x) = \dfrac{3(-x)}{(-x)^4 - 2}$<br>$= -\dfrac{3x}{x^4-2}$<br>$= -f(x)$<br>$\therefore$ it's an odd function | 2 | |
| (d) | $\dfrac{\cos 60^\circ}{\sin 45^\circ} + \cot 30^\circ = \dfrac{\cos 60^\circ}{\sin 45^\circ} + \dfrac{1}{\tan 30^\circ}$<br>$= \dfrac{\frac{1}{2}}{\frac{1}{\sqrt{2}}} + \dfrac{1}{\frac{1}{\sqrt{3}}}$<br>$= \dfrac{\sqrt{2}}{2} + \sqrt{3}$<br>OR<br>$= \dfrac{\sqrt{2} + 2\sqrt{3}}{2}$ | 2 | |
| (e) | $f(x) = x^2 + 2x - 8$<br>(i) $x$-intercept when $y = 0$:<br>$0 = x^2 + 2x - 8$<br>$0 = (x+4)(x-2)$<br>$\therefore x = -4$ or $x = 2$<br>$y$-intercept when $x = 0$:<br>$y = 0^2 + 2(0) - 8$<br>$\therefore y = -8$ | 2 | |

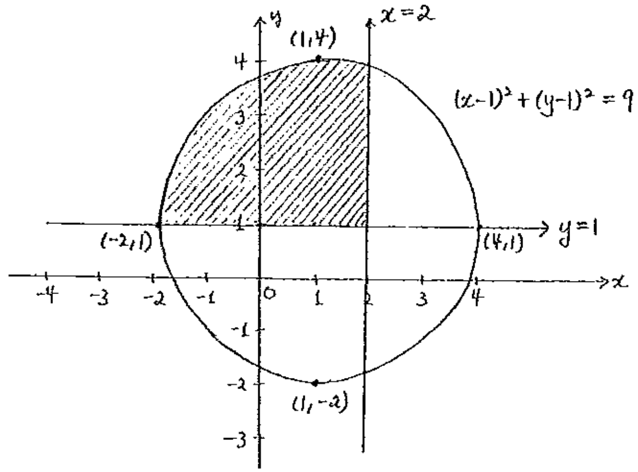


| Qn | Solutions | Marks | Comment: Criteria |
|---|---|---|---|
| | when $x = -5$, $y = (-5)^2 + 2(-5) - 8$<br>$= 7$<br>when $x = 4$, $y = 4^2 + 2(4) - 8$<br>$= 16$<br><br> | | |
| (ii) | axis of symmetry $= \dfrac{-4+2}{2}$<br>$= -1$<br>then $f(-1) = (-1)^2 + 2(-1) - 8$<br>$= -9$<br>$\therefore$ minimum value of $f(x)$ is $-9$. | 1 | |
| (iii) | range of $f(x)$ is $-9 \leq y \leq 16$ | 1 | |
| (iv) | $x < -1$<br>OR<br>$-5 \leq x < -1$ (for restricted domain) | | |

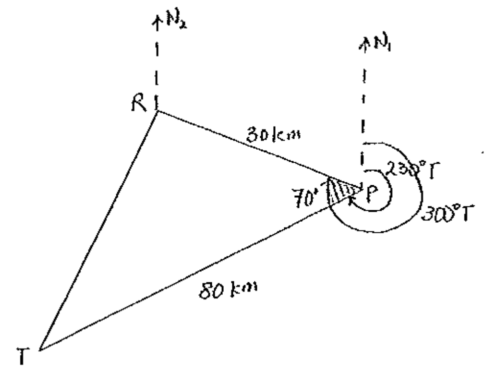
| Qn | Solutions | Marks | Comment: Criteria |
|---|---|---|---|
| | Section 2 | | |
| 5 (a) | $f(x) = \lvert 2x+1 \rvert$ | | |
| | (i) $f(x) = \begin{cases} 2x+1 & \text{for } x \geqslant -\frac{1}{2} \\ -(2x+1) & \text{for } x < -\frac{1}{2} \end{cases}$ | 2 | |
| | for $f(x) = 2x+1$ : $y$-intercept $= 1$, $x$-intercept $= -\frac{1}{2}$ | | |
| |  | | |
| | (ii) Domain : $x \in \mathbb{R}$<br>Range : $y \geqslant 0$ | 1 | |
| (b) | $f(x) = 3^x + 1$ | | |
| | (i)  | 2 | |
| | (ii) Domain : $x \in \mathbb{R}$<br>Range : $y > 1$ | 1 | |



| Qn | Solutions | Marks | Comment: Criteria |
|---|---|---|---|
| (c) | $f(x) = \frac{3}{x+2} - 1$ | | |
| | (i) vertical asymptote : $x+2=0$<br>$x = -2$ | 3 | |
| | horizontal asymptote : $y = -1$ | | |
| |  | | |
| | $x$-intercept : $x = 1$<br>$y$-intercept : $y = \frac{1}{2}$ | | |
| | (ii) Domain : $x \in \mathbb{R}$ but $x \neq -2$<br>Range : $y \in \mathbb{R}$ but $y \neq -1$ | 1 | |
| (d) | $f(x) = \begin{cases} x^2+2 & \text{for } x \geqslant 0 \\ x+1 & \text{for } x < 0 \end{cases}$ | | |
| | (i)  | 2 | |

| Qn | Solutions | Marks | Comment: Criteria |
|---|---|---|---|
| | (ii) $f(2) = 2^2 + 2 = 6$<br>$f(-1) = -1+1 = 0$<br>$f(0) = 0^2 + 2 = 2$<br>$\therefore f(2) - 3f(-1) + f(0) = 6 - 3(0) + 2$<br>$= 8$ | 2 | |
| (e) |   | 2 | |
| | test (0,0) for $(x-1)^2 + (y-1)^2 \leq 9$<br>$(0-1)^2 + (0-1)^2 \leq 9$<br>$1 + 1 \leq 9$<br>$2 \leq 9$<br>$\therefore$ True | | |



| Qn | Solutions | Marks | Comment: Criteria |
|---|---|---|---|
| 6 (a) |   | | |
| | (i) $\angle RPT = 300° - 230°$<br>$= 70°$<br>using Cosine Rule :<br>$RT^2 = PT^2 + RT^2 - 2 \times PT \times RT \times \cos\angle RPT$<br>$= 80^2 + 30^2 - 2 \times 80 \times 30 \times \cos 70°$<br>$RT = 75.2216...$<br>$\therefore RT = 75$ km (nearest km)<br>$\therefore$ the distance between Ray and Tina is 75 km. | 3 | |
| | (ii) $\angle N_1PR = 360° - 300°$ ($\angle$ of revolution)<br>$= 60°$<br>$\angle N_2RP = 180° - \angle N_1PR$ (co-interior $\angle$s of // lines are supplementary)<br>$= 180° - 60°$<br>$= 120°$<br>using Sine Rule :<br>$\frac{\sin\angle TRP}{TP} = \frac{\sin\angle RPT}{RT}$ | 3 | |

| Qn | Solutions | Marks | Comment: Criteria |
|---|---|---|---|

$$\frac{\sin\angle TRP}{80} = \frac{\sin 70^\circ}{75.2216\ldots} \quad \text{using part (i)}$$

$$\angle TRP = \sin^{-1}\left(\frac{80\sin 70^\circ}{75.2216\ldots}\right)$$

$$= 87.9899\ldots$$

$$= 88^\circ \quad \text{(nearest degree)}$$

then $\angle N_2RT = \angle TRP + \angle N_2RP$

$$= 88^\circ + 120^\circ$$

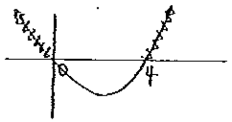
$$= 208^\circ$$

$\therefore$ the bearing of Tina from Ray is $208^\circ T$.

**(b)** $y = \sqrt{x^2 - 4x}$ — Marks: 2

Domain: $x^2 - 4x \geq 0$

$x(x-4) \geq 0$

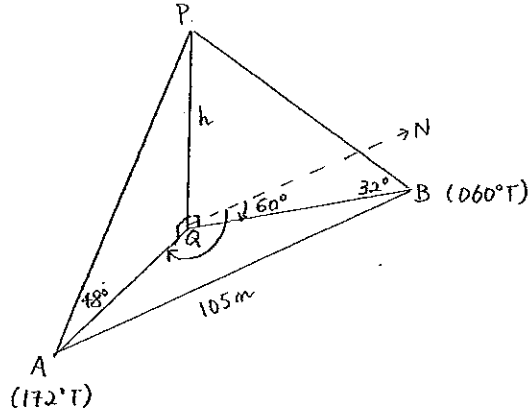



$\therefore \; x \leq 0 \text{ or } x \geq 4$

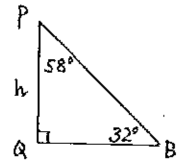


| Qn | Solutions | Marks | Comment: Criteria |
|---|---|---|---|

**(c)**




(i) In $\triangle PQB$: — Marks: 1



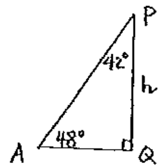

$\angle QPB = 180^\circ - 90^\circ - 32^\circ$ ($\angle$ sum of $\triangle$)

$= 58^\circ$

$$\tan 58^\circ = \frac{BQ}{h}$$

$\therefore BQ = h\tan 58^\circ$ as required

(ii) In $\triangle PQA$: — Marks: 1




$\angle APQ = 180^\circ - 90^\circ - 48^\circ$ ($\angle$ sum of $\triangle$)

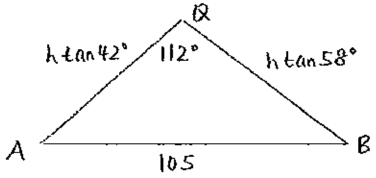
$= 42^\circ$

$$\tan 42^\circ = \frac{AQ}{h}$$

$\therefore AQ = h\tan 42^\circ$

| Qn | Solutions | Marks | Comment: Criteria |
|---|---|---|---|

(iii) In $\triangle AQB$: **[3]**




$$\angle AQB = \angle NQA - \angle NQB$$
$$= 172° - 60°$$
$$= 112°$$

using Cosine Rule:

$$AB^2 = AQ^2 + BQ^2 - 2 \times AQ \times BQ \times \cos \angle AQB$$

$$105^2 = (h\tan 42°)^2 + (h\tan 58°)^2 - 2(h\tan 42°)(h\tan 58°)\cos 112°$$

$$= (\tan^2 42°)h^2 + (\tan^2 58°)h^2 - (2\tan 42° \cdot \tan 58° \cos 112°)h^2$$

$$= (\tan^2 42° + \tan^2 58° - 2\tan 42° \tan 58° \cos 112°)h^2$$

$$h^2 = \frac{105^2}{\tan^2 42° + \tan^2 58° - 2\tan 42° \tan 58° \cos 112°}$$

$$h = 49.767\ldots$$

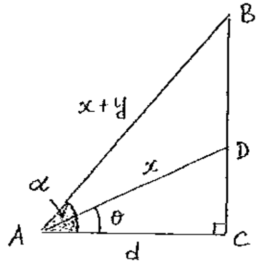
$$h = 50 \text{ m (nearest metre)}$$

$\therefore$ The height of the observation tower PQ is 50m.



| Qn | Solutions | Marks | Comment: Criteria |
|---|---|---|---|

(d)




(i) In $\triangle ACD$: $\cos\theta = \frac{AC}{AD}$ **[1]**

$$\therefore \cos\theta = \frac{d}{x} \quad (1)$$

(ii) In $\triangle ABC$: $\cos\alpha = \frac{AC}{AB}$ **[2]**

$$\cos\alpha = \frac{d}{x+y} \quad (2)$$

rewriting (1): $x = \frac{d}{\cos\theta}$ (3)

rewriting (2): $x+y = \frac{d}{\cos\alpha}$

$$x = \frac{d}{\cos\alpha} - y \quad (4)$$

substitute (3) and (4):

$$\frac{d}{\cos\alpha} - y = \frac{d}{\cos\theta}$$

$$\frac{d}{\cos\alpha} - \frac{d}{\cos\theta} = y$$

$$\frac{d\cos\theta - d\cos\alpha}{\cos\alpha\cos\theta} = y$$

$$d(\cos\theta - \cos\alpha) = y\cos\alpha\cos\theta$$

$$\therefore \quad d = \frac{y\cos\alpha\cos\theta}{\cos\theta - \cos\alpha} \text{ as required}$$

OR

show:

$$d = \frac{y\left(\frac{d}{x}\right)\left(\frac{d}{x+y}\right)}{\frac{d}{x} - \frac{d}{x+y}}$$

$$RHS = \frac{\frac{d^2y}{x(x+y)}}{\frac{d(x+y)-dx}{x(x+y)}}$$

$$= \frac{d^2y}{dx+dy-dx}$$

$$= d$$

$$= LHS$$