NAME MASTER

SYDNEY GRAMMAR SCHOOL

2016 Annual Examination

FORM V
MATHEMATICS 2 UNIT

Wednesday 31st August 2016

General Instructions
- Writing time — 2 hours
- Write using black pen.
- Board-approved calculators and templates may be used.

Total — 100 Marks
- All questions may be attempted.

Section I – 9 Marks
- Questions 1–9 are of equal value.
- Record your answers to the multiple choice on the sheet provided.

Section II – 91 Marks
- Questions 10–16 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection
- Write your name, class and Master on each answer booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Write your name, class and Master on this question paper and hand it in with your answers.
- Place everything inside the answer booklet for Question Ten.

5A: DNW	5B: PKH	5C: LRP	5D: FMW
5E: WJM	5F: GMC	5G: NL	5H: SO
5P: TCW	5Q: SDP	5R: RCF	

Checklist
- SGS booklets — 7 per boy
- Multiple choice answer sheet
- Reference sheet
- Candidature — 172 boys

Examiner
NL

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

Which of the following is the gradient of the line $y = 2x + 5$?

(A) -5

(B) $\frac{2}{5}$

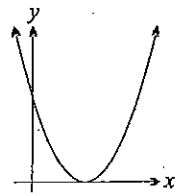
(C) 2

(D) 5

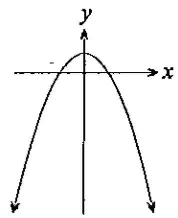
QUESTION TWO

Which of the following is the graph of a negative definite quadratic function?

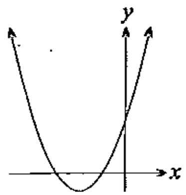
(A)



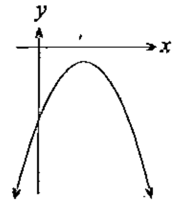
(B)



(C)



(D)



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QUESTION THREE

The angle of elevation from point A to the top of a lighthouse is 40°. What is the angle of depression from the top of the lighthouse to point A?

(A) 40°

(B) 50°

(C) 130°

(D) 140°

QUESTION FOUR

State the number of solutions to the equation $\sin x = 0$, for $0^\circ \le x \le 720^\circ$.

(A) 2

(B) 3

(C) 4

(D) 5

QUESTION FIVE

Which of the following is a geometric sequence?

(A) $5, 9, 13, 17, \ldots$

(B) $3, 9, 18, 36, \ldots$

(C) $81, 27, 9, 3, \ldots$

(D) $2, 4, 8, 24, \ldots$

QUESTION SIX

Which of the following statements is NOT true?

(A) $\log_3 15 - \log_3 5 = 1$

(B) $\log_5 \dfrac{1}{5} = -1$

(C) $\log_5 (3+2) = \log_5 3 \times \log_5 2$

(D) $\dfrac{\log_2 8}{\log_2 4} = \dfrac{3}{2}$

Examination continues overleaf ...

QUESTION SEVEN

If α and β are the solutions to $2x^2 - 6x + 1 = 0$, the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ is:

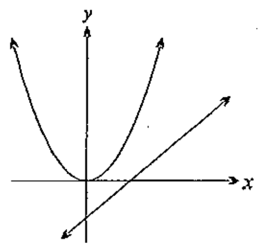
(A) $-\dfrac{1}{3}$

(B) $1{\cdot}5$

(C) $\dfrac{1}{3}$

(D) 6

QUESTION EIGHT



The diagram above shows the parabola $y = ax^2$ and the line $y = bx + c$. Which of the following statements is true?

(A) $b^2 + 4ac < 0$

(B) $b^2 - 4ac < 0$

(C) $b^2 + 4ac > 0$

(D) $b^2 - 4ac = 0$

QUESTION NINE

The expression $\dfrac{\tan^2\theta}{\sec\theta - 1}$ can be simplified to:

(A) $\dfrac{1}{\cos\theta}$

(B) $\sec\theta + 1$

(C) $\dfrac{\sin\theta}{\cos\theta - 1}$

(D) $\tan\theta$

End of Section I

Examination continues next page ...

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION TEN (13 marks) Use a separate writing booklet. **Marks**

(a) Evaluate $|7-12|$. $\boxed{1}$

(b) Expand and simplify $(x+4)(x+5)$. $\boxed{1}$

(c) Simplify:

(i) $5x^2y \times xy$ $\boxed{1}$

(ii) $8x^2y^5 \div 4x^5y^3$ $\boxed{1}$

(d) Evaluate $\log_2 8$. $\boxed{1}$

(e) Simplify $\sqrt{32}+3\sqrt{2}$. $\boxed{1}$

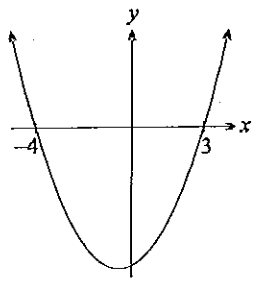
(f) Solve $64^x = 8$. $\boxed{1}$

(g) Find the exact value of $\sin 300°$. $\boxed{1}$

(h) Use the quadratic formula to find the exact solutions to $2x^2+5x+1=0$. $\boxed{2}$

(i) Find the exact distance between the points $A(3,1)$ and $B(-1,7)$. $\boxed{2}$

(j)



The diagram above shows the graph of $y=(x-3)(x+4)$.
Using the graph, or otherwise, state the solution to $(x-3)(x+4) \geq 0$. $\boxed{1}$

Examination continues overleaf ...

QUESTION ELEVEN (13 marks) Use a separate writing booklet. **Marks**

(a) Consider the arithmetic sequence $37, 42, 47, 52, \ldots$

(i) Find the common difference d. $\boxed{1}$

(ii) Find the sum of the first sixty terms. $\boxed{2}$

(b) Sketch the line $y=-2x+1$, showing any intercepts with the coordinate axes. $\boxed{2}$

(c) Consider the graph of the quadratic equation $y=x^2+2x-3$.

(i) State the y-intercept. $\boxed{1}$

(ii) Find the x-intercepts. $\boxed{1}$

(iii) Find the equation of the axis of symmetry. $\boxed{1}$

(iv) Find the coordinates of the vertex. $\boxed{1}$

(v) Hence sketch the graph of $y=x^2+2x-3$, showing all of the above features. $\boxed{2}$

(d) Express $\log_{10}\dfrac{2}{25}$ in terms of $\log_{10} 2$ and $\log_{10} 5$. $\boxed{2}$

Examination continues next page ...

QUESTION TWELVE (13 marks) Use a separate writing booklet. Marks

(a) Differentiate the following:

(i) $y = x^4 - 2x - 1$ $\boxed{1}$

(ii) $y = \dfrac{5}{x^2}$ $\boxed{1}$

(iii) $y = (3x+2)^4$ $\boxed{2}$

(iv) $y = \dfrac{x^2}{x-1}$ $\boxed{2}$

(b) Express $y = x^2 + 6x + 13$ in the form $y = (x+h)^2 + k$. $\boxed{1}$

(c) The fourth term of an arithmetic progression is 51 and the tenth term is 93. Find the value of the thirtieth term. $\boxed{3}$

(d) Find the equation of the tangent to $y = x^2 + 3x$ at $x = 2$. $\boxed{3}$

QUESTION THIRTEEN (13 marks) Use a separate writing booklet. Marks

(a) Evaluate $\displaystyle\sum_{n=2}^{5}(3n+1)$. $\boxed{1}$

(b) Solve $3\tan^2\theta = 1$, for $0° \le \theta \le 360°$. $\boxed{2}$

(c) Solve $3^x = 100$. Express your answer correct to three decimal places. $\boxed{2}$

(d) Consider the sequence $5, 10{\cdot}5, 22{\cdot}05, 46{\cdot}305, \ldots$

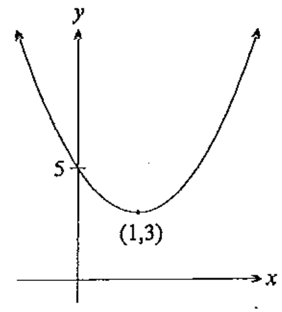
(i) Show that this sequence is geometric. $\boxed{1}$

(ii) Find a formula for the n^{th} term of the sequence, T_n. $\boxed{1}$

(iii) Find the smallest integer value of n for which $T_n > 500\,000$. $\boxed{2}$

(e) Solve $|2x+1| < 5$. $\boxed{2}$

(f)



The parabola with vertex $(1,3)$ and y-intercept $(0,5)$ is shown above. Determine the equation of the parabola. $\boxed{2}$

QUESTION FOURTEEN (13 marks) Use a separate writing booklet. Marks

(a) Quadrilateral $ABCD$ has vertices $A(-2,7)$, $B(-5,3)$, $C(-1,0)$ and $D(2,4)$. It is known that if the diagonals bisect each other at right angles, then the quadrilateral is a rhombus.

(i) Find the midpoints of the diagonals of shape $ABCD$. 2

(ii) Show that $ABCD$ is a rhombus. 2

(b) Sketch the following graphs on separate axes. Show any asymptotes and any intercepts with the coordinate axes.

(i) $y = 3^{-x}$ 2

(ii) $y = \log_2(x-3)$ 2

(iii) $y = -\sqrt{25-x^2}$ 2

(c) Differentiate $y = 3x(5x^2+1)^7$. Give your answer in fully factored form. 3

Examination continues overleaf ...

QUESTION FIFTEEN (13 marks) Use a separate writing booklet. Marks

(a) Find the limiting sum of the geometric progression $8 + 2 + \frac{1}{2} + \cdots$ 2

(b) A mother kangaroo and her joey are initially 50 m apart when they begin hopping directly towards each other. The joey takes one hop every time the mother takes one hop. The mother's first hop is 3·6 m long and the distance she hops in each subsequent hop is 92% of the hop before. The mother and joey are reunited after 30 hops.

(i) How far is the mother from her starting position when she is reunited with her joey? Give your answer correct to the nearest centimetre. 2

(ii) The joey moves the same distance every hop. Find the distance it moves in one hop. Give your answer correct to the nearest centimetre. 1

(c) Differentiate $f(x) = 4x^2 - 3$ from first principles. 2

(d) Solve $\cos^2\theta + \cos\theta = \sin^2\theta$, for $0° \le \theta \le 360°$. 3

(e) Solve $3 \times 9^x - 28 \times 3^x + 9 = 0$. 3

Examination continues next page ...

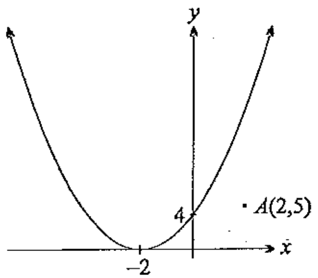
QUESTION SIXTEEN (13 marks) Use a separate writing booklet. **Marks**

(a) Consider the equation $kx^2 - (1+k)x + (3k+2) = 0$. The sum of the roots of the equation is twice the product of the roots.

(i) Find k. **2**

(ii) Hence find the roots. **2**

(b)



The diagram above shows the graph of $y = (x+2)^2$ and the point $A(2,5)$.

(i) Show that a line through A with gradient m has equation $y = mx - 2m + 5$. **1**

(ii) Show that if the line through A is a tangent to $y = (x+2)^2$ then $m = 8 \pm 2\sqrt{11}$. **3**

(iii) Find the equations of these tangents. **1**

(c) Arthur sells gourmet burgers. At a price of \$20 each he sells 7500 a month. For every 25 cent decrease in the price of a burger he sells 150 more each month.

(i) Find an expression for the price of a burger in terms of x, where x stands for the number of times the price of a burger is reduced by 25 cents. **1**

(ii) Arthur's revenue is equal to the product of the price of a burger and the number of burgers sold. Find Arthur's maximum monthly revenue and the number of burgers he must sell to achieve it. **3**

End of Section II

END OF EXAMINATION

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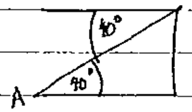
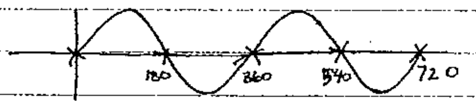
MC

1) $m = 2$ (C)

2) (D)

3) 

(A)

4.) 

5, (D)

5.) (C)

6.) $\log_5(3+2) \neq \log_5 3 \times \log_5 2$ (C)

7.) $\alpha+\beta = \frac{-b}{a}$ $\qquad$ $\alpha\beta = \frac{c}{a}$

$= \frac{6}{2}$ $\qquad$ $= \frac{1}{2}$

$= 3$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta}{\alpha\beta} + \frac{\alpha}{\alpha\beta} = \frac{\alpha+\beta}{\alpha\beta}$$

$$= \frac{3}{1/2}$$

$$= 6$$ (D)

8) $y = ax^2 \qquad y = bx + c$

$ax^2 = bx + c$

$ax^2 - bx - c = 0$

$\Delta = (-b)^2 - 4a(-c)$

$\Delta < 0$

$\therefore \ b^2 + 4ac < 0$ (A)

9) $\dfrac{\tan^2\theta}{\sec\theta - 1}$ $\qquad$ $\tan^2\theta + 1 = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$= \dfrac{\sec^2\theta - 1}{\sec\theta - 1}$

$= \dfrac{(\sec\theta - 1)(\sec\theta + 1)}{\sec\theta - 1}$

$= \sec\theta + 1$ (B)

Q10

a) $|7-12|$

$= |-5|$

$= 5$ ✓

b) $(x+4)(x+5)$

$= x^2 + 5x + 4x + 20$

$= x^2 + 9x + 20$ ✓

c) i) $5xy^{\frac{3}{2}}$ ✓

ii) $\dfrac{2y^2}{x^3}$ ✓

$(\text{or } 2y^2x^{-3})$

d) $\log_2 8 = 3$

e) $\sqrt{32} + 3\sqrt{2}$

$= \sqrt{16 \times 2} + 3\sqrt{2}$

$= 4\sqrt{2} + 3\sqrt{2}$

$= 7\sqrt{2}$ ✓

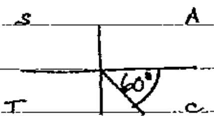
f) $64^x = 8$

$8^{2x} = 8^1$

$2x = 1$

$x = 1/2$ ✓

g) $\sin 300^\circ$



$= -\sin 60^\circ$

$= -\dfrac{\sqrt{3}}{2}$ ✓

h) $2x^2 + 5x + 1 = 0$

$a = 2 \quad b = 5 \quad c = 1$

$x = \dfrac{-5 \pm \sqrt{5^2 - 4 \times 2 \times 1}}{2 \times 2}$ ✓

$x = \dfrac{-5 \pm \sqrt{17}}{4}$ ✓

i) $A(3,1) \quad B(-1,7)$

$d = \sqrt{(7-1)^2 + (-1-3)^2}$ ✓

$d = \sqrt{36 + 16}$

$d = \sqrt{52}$

$d = 2\sqrt{13}$ ✓

j) $x \le -4$ or $x \ge 3$

Q11

a) 37, 42, 47, 52, ...

i) $42-37 = 47-42 = 52-47 = 5$

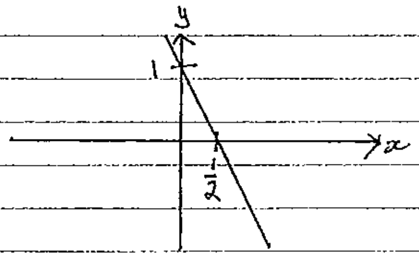
$d = 5$ ✓

ii) $S_n = \frac{1}{2}n(2a+(n-1)d)$

$S_{60} = \frac{1}{2} \times 60(2 \times 37 + 59 \times 5)$ ✓

$S_{60} = 11070$ ✓

b) $y = -2x + 1$



General shape/slope ✓
Intercepts + axes ✓

c) $y = x^2 + 2x - 3$

i) $(0, -3)$ ✓

ii) $0 = (x-1)(x+3)$

$x = 1$ or $x = -3$

$(1, 0)$ or $(-3, 0)$ ✓

iii) $x = \frac{-b}{2a}$

$x = \frac{-2}{2 \times 1}$

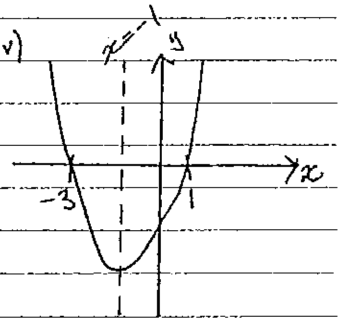
$x = -1$ ✓

iv) $x = -1$, $y = (-1)^2 + 2(-1) - 3$

$y = -4$

$(-1, -4)$ ✓

v)



Concavity ✓
intercepts ✓

d) $\log_{10}\frac{2}{25} = \log_{10}2 - \log_{10}25$ ✓

$= \log_{10}2 - \log_{10}5^2$

$= \log_{10}2 - 2\log_{10}5$ ✓

Q12

a) i) $y = x^4 - 2x - 1$

$y' = 4x^3 - 2$

ii) $y = 5x^{-2}$

$y' = -10x^{-3}$ ✓

$y' = -\frac{10}{x^3}$

iii) $y = (3x+2)^4$

$y' = 4(3x+2)^3 \times 3$ ✓ (Even if 3 is missing).

$y' = 12(3x+2)^3$ ✓

iv) $y = \frac{x^3}{x-1}$

$u = x^2 \qquad v = x-1$

$u' = 2x \qquad v' = 1$

$y' = \frac{(x-1)\times 2x - x^2}{(x-1)^2}$ ✓

$y' = \frac{2x(x-1) - x^2}{(x-1)^2}$

$y' = \frac{2x^2 - 2x - x^2}{(x-1)^2}$

$y' = \frac{x^2 - 2x}{(x-1)^2}$ ✓

$y' = \frac{x(x-2)}{(x-1)^2}$

(b) $y = x^2 + 6x + 13$

$y = (x+3)^2 - 9 + 13$ (or equivalent)

$y = (x+3)^2 + 4$ ✓

(c) $T_4 = 51 \quad T_{10} = 93$

$51 = a + (4-1)d$. $\qquad 93 = a + (10-1)d$

$51 = a + 3d$ (1) $\qquad 93 = a + 9d$ (2) ✓ (both)

(2)-(1) $\quad 42 = 6d$

$d = 7$ ✓

$a = 30$.

$T_{30} = 30 + 29 \times 7$

$T_{30} = 233$ ✓

(d) $y = x^2 + 3x$.

$y' = 2x + 3$

at $x = 2 \quad m = 2\times 2 + 3$

$m = 7$ ✓

at $x = 2 \quad y = 2^2 + 3\times 2$

$y = 10$ ✓ (or valid coordinate)

$y - y_1 = m(x - x_1)$

$y - 10 = 7(x-2)$

$y = 7x - 14 + 10$

$y = 7x - 4$ ✓ (or equivalent)

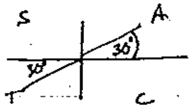
Q13

a) $$\sum_{n=2}^{5}(3n+1) = (3\times2+1)+(3\times3+1)+(3\times4+1)+(3\times5+1)$$
$$= 46 \checkmark$$

b) $3\tan^2\theta = 1$
$\tan^2\theta = \frac{1}{3}$

$\tan\theta = \pm\frac{1}{\sqrt{3}}$ or $\pm\frac{\sqrt{3}}{3}$ ✓

$\theta_r = 30^\circ$



$\theta = 30^\circ, (180+30)^\circ, (180-30)^\circ, (360-30)^\circ$

$\theta = 30^\circ, 210^\circ, 150^\circ, 330^\circ$ ✓

c) $3^x = 100$
$x = \log_3 100$

$x = \dfrac{\log_{10}100}{\log_{10}3}$ ✓

$x = 4.192$ (to 3 dp) ✓

d) i) $\dfrac{10.5}{5} = \dfrac{22.05}{10.5} = \dfrac{46.305}{22.05} = 2.1$ ✓ (Any 2)

ii) $r = 2.1 \quad a = 5$
$T_n = ar^{n-1}$
$T_n = 5\times 2.1^{n-1}$ ✓

iii) $5\times2.1^{n-1} > 500000$
$2.1^{n-1} > 100000$
$n-1 > \dfrac{\log_{10}100000}{\log_{10}2.1}$ ✓

$n-1 > 15.517\ldots$
$n > 16.517$

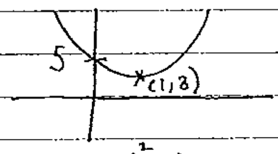
$\therefore n = 17$ ✓

e) $|2x+1| < 5$

$2x+1 < 5$ $\qquad$ $2x+1 > -5$ ✓
$2x < 4$ $\qquad$ $2x > -6$
$x < 2$ $\qquad$ $x > -3$

$\therefore -3 < x < 2$ ✓

f)



$y = a(x+h)^2 + k$
$y = a(x-1)^2+3$ ✓
Sub $(0,5)$
$5 = a(-1)^2+3$
$\underline{\underline{a=2}}$

$y = 2(x-1)^2+3$ ✓
or $y = 2x^2-4x+5$

Q14

a) i) midpoint of AC $= \left(\frac{-2+-1}{2}, \frac{7+0}{2}\right)$

$= \left(-\frac{3}{2}, \frac{7}{2}\right)$

midpoint of BD $= \left(\frac{-5+2}{2}, \frac{3+4}{2}\right)$

$= \left(-\frac{3}{2}, \frac{7}{2}\right)$

ii) Midpoints have the same coordinates $\therefore$ diagonals bisect each other.

$m_{AC} = \frac{7-0}{-2--1}$ $m_{BD} = \frac{3-4}{-5-2}$

$= \frac{7}{-1}$ $= \frac{-1}{-7}$

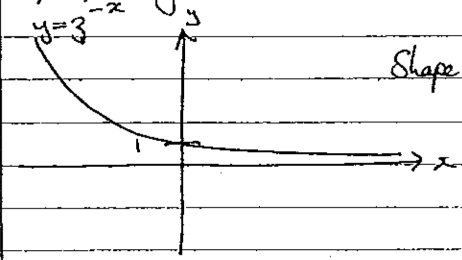
$= -7$ $= \frac{1}{7}$

$m_{AC} \times m_{BD} = -7 \times \frac{1}{7} = -1$

$\therefore$ Diagonals are perpendicular.

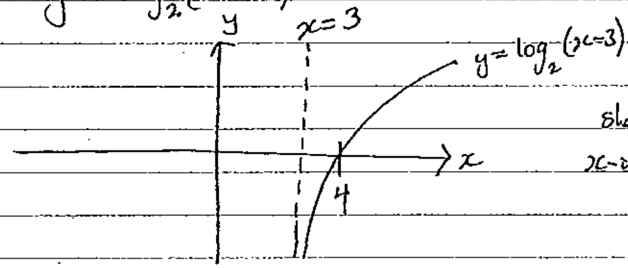
$\therefore$ ABCD is a rhombus.

b) i) $y = 3^{-x}$



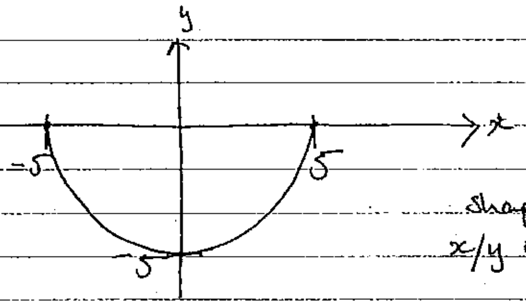
Shape / asymptopic at x-axis ✓
~~y~~ y intercept ✓

ii) $y = \log_2(x-3)$



shape/asymptote ✓
x-intercept ✓

iii) $y = -\sqrt{25-x^2}$



shape ✓
x/y intercepts ✓

c) $y = 3x(5x^2+1)^7$

$u = 3x \qquad v = (5x^2+1)^7$

$u' = 3 \qquad v' = 7(5x^2+1)^6 \times 10x = 70x(5x^2+1)^6$ ✓

$y' = 3(5x^2+1)^7 + 210x^2(5x^2+1)^6$ ✓

$y' = 3(5x^2+1)^6(5x^2+1+70x^2)$

$y' = 3(5x^2+1)^6(75x^2+1)$ ✓

Q15

a) $r = 1/4$ ✓

$S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{8}{1-1/4}$

$S_\infty = \frac{32}{3}$

$= 10\frac{2}{3}$ ✓

b) i) $r = 0.92$

$S_n = \frac{a(1-r^n)}{1-r}$

$S_{30} = \frac{3.6(1-0.92^{30})}{1-0.92}$

$S_{30} = 41.31$ m (to 2 dp)

$= 41.31$ m (to nearest m)

ii) Joey moves $50 - 41.31 = 8.688...$ m.

$\frac{8.688}{30} = 0.2896$ m

$= 29$ cm

c) $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{4(x+h)^2-3-(4x^2-3)}{h}$ ✓

$= \lim_{h\to 0} \frac{4(x^2+2xh+h^2)-3-4x^2+3}{h}$

$= \lim_{h\to 0} \frac{4x^2+8xh+4h^2-4x^2}{h}$

$= \lim_{h\to 0} \frac{8xh+4h^2}{h}$

$\left.\begin{aligned} &= \lim_{h\to 0} (8x+4h) \\ &= 8x \end{aligned}\right\}$ ✓ (both).

d) $\cos^2\theta + \cos\theta = \sin^2\theta$ for $0^\circ \leq \theta \leq 360^\circ$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

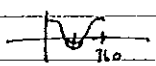
$\cos^2\theta + \cos\theta = 1 - \cos^2\theta$

$2\cos^2\theta + \cos\theta - 1 = 0$

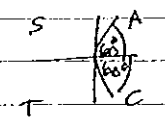
$(2\cos\theta - 1)(\cos\theta + 1)$

$2\cos\theta = 1$

$\cos\theta = 1/2 \qquad \cos\theta = -1$



$\theta_r = 60^\circ \qquad \theta_r = 180^\circ$



$\theta = 60^\circ, 180^\circ, 300^\circ$

e) $3\times 9^x - 28\times 3^x + 9 = 0$

$\left.\begin{aligned} &3\times 3^{2x} - 28\times 3^x + 9 = 0 \\ &\text{let } u = 3^x \\ &3u^2 - 28u + 9 = 0 \end{aligned}\right\}$ either ✓

$(3u-1)(u-9) = 0$ ✓

$3u = 1 \qquad u = 9$

$u = 1/3$

$3^x = 3^{-1} \qquad 3^x = 3^2$

$\underline{\underline{x = -1}} \qquad \underline{\underline{x = 2}}$ ✓

Q16

a) i) $kx^2 - (1+k)x + (3k+2) = 0$

$\alpha + \beta = 2\alpha\beta$ ✓

$\alpha + \beta = \frac{1+k}{k}$

$\alpha\beta = \frac{3k+2}{k}$

$\frac{1+k}{k} = 2\left(\frac{3k+2}{k}\right)$

$1 + k = 6k + 4$

$-3 = 5k$

$k = -\frac{3}{5}$ ✓

ii) $\alpha + \beta = \frac{1 - 3/5}{-3/5}$

$\left.\begin{array}{l} \alpha + \beta = -2/3 \\ \alpha\beta = \frac{3(-3/5)+2}{-3/5} \\ \alpha\beta = -1/3 \end{array}\right\}$ Any valid working ✓

$\beta = -\frac{1}{3\alpha}$

$\alpha + \beta = \alpha - \frac{1}{3\alpha} = -2/3$

$3\alpha^2 - 1 = -2\alpha$

$3\alpha^2 + 2\alpha - 1 = 0$

$(3\alpha - 1)(\alpha + 1) = 0$

$3\alpha = 1$

$\alpha = 1/3 \quad \therefore \beta = -1$ ✓ (both).

b) i) $A(2,5)$

$y - y_1 = m(x - x_1)$

$y - 5 = m(x - 2)$ ✓

$y = mx - 2m + 5$

ii) $(x+2)^2 = mx - 2m + 5$

$x^2 + 4x + 4 - mx + 2m - 5 = 0$

$x^2 + 4x - mx + 4 + 2m - 5 = 0$

$x^2 + (4-m)x + 2m - 1 = 0$ ✓

if tangent $\Delta = 0$

$b^2 - 4ac = 0$

$(4-m)^2 - 4 \times 1 \times (2m-1) = 0$ ✓

$16 - 8m + m^2 - 8m + 4 = 0$

$m^2 - 16m + 20 = 0$

$m = \frac{16 \pm \sqrt{16^2 - 4 \times 1 \times 20}}{2 \times 1}$

$m = \frac{16 \pm \sqrt{176}}{2}$

$\left.\begin{array}{l} m = \frac{16 \pm 4\sqrt{11}}{2} \\ m = 8 \pm 2\sqrt{11} \end{array}\right\}$ ✓

iii) $y = mx - 2m + 5 \qquad m = 8 + 2\sqrt{11}$

$\left.\begin{array}{l} y = (8+2\sqrt{11})x - 2(8+2\sqrt{11}) + 5 \\ y = (8+2\sqrt{11})x - 11 - 4\sqrt{11} \\ \\ m = 8 - 2\sqrt{11} \\ y = (8-2\sqrt{11})x - 2(8-2\sqrt{11}) + 5 \\ y = (8-2\sqrt{11})x - 11 + 4\sqrt{11} \end{array}\right\}$ (both) ✓

c) \$20 sells 7500 burgers.
Every decrease of \$0.25 sells 150 more/month.

i) Price $= 20 - 0.25x$ ✓

ii) Revenue = Price × Number
$= (20-0.25x)(7500+150x)$ ✓

Vertex

$0.25x = 20$ $\qquad$ $150x = -7500$

$x = 80$ $\qquad$ $x = -50$

$\frac{80-50}{2} = 15$

Number of burgers $= 7500 + 150 \times 15$
$= 9750$ burgers. ✓

Price $= 20 - 0.25 \times 15$
$= \$16.25$

Revenue = \$158,437.50. ✓