St Catherine's School
Waverley

Student Number: ____________________

YEAR 11 MATHEMATICS EXTENSION 1

PRELIMINARY TASK 1

1st April 2016

General Instructions

- Reading Time – 3 minutes
- Working Time – 55 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Marks may be deducted for careless or badly arranged work
- Task Weighting – 30%
- Total Marks – 38

SECTION I **13 marks**

- Attempt Questions 1 – 4 in one booklet
- Show all necessary working

SECTION II **25 marks**

- Attempt Questions 5 – 6
- Answer each question in a separate booklet
- Show all necessary working

Question 1 – 3	/3
Question 4	/10
Question 5	/12
Question 6	/13
TOTAL	/38

SECTION I **START A NEW BOOKLET**
Total Marks 13
Attempt Questions 1 – 4

Questions 1 to 3, answer either A, B, C or D. **3 Marks**

Question 1

What is the exact value of $\cos 135°$?

(A) $\frac{\sqrt{2}}{2}$

(B) $\sqrt{3}$

(C) $-\frac{1}{\sqrt{2}}$

(D) $-\frac{1}{\sqrt{3}}$

Question 2

What is the acute angle between the lines $y = 2x - 3$ and $3x + 5y - 1 = 0$?
Answer to the nearest degree.

(A) 32°

(B) 50°

(C) 82°

(D) 86°

Question 3

What are the coordinates of the point that divides the interval joining the points $A(7,1)$ and $B(0,-6)$ internally in the ratio 4:3 ?

(A) $(3,-3)$

(B) $(3,-2)$

(C) $(4,-2)$

(D) $(4,-3)$

Question 4 (10 marks) **Marks**

(a) Suppose that α is an obtuse angle and $\sin\alpha = \frac{\sqrt{7}}{3}$, find the exact value of $\cos\alpha$. **2**

(b) Solve $\frac{x-3}{x} > 0$ **2**

(c) Point $C(5,-2)$ divides the interval AB where $A(3,0), B(h,1)$, externally in the ratio 2:3. Find the value of h. **2**

(d) Solve for $0° \leq \alpha \leq 360°$ to the nearest minute **2**

$4\cos\alpha - 1 = 0$

(e) Prove: **2**

$$\frac{1}{1-\cos\alpha} + \frac{1}{1+\cos\alpha} = 2cosec^2\alpha$$

End of Section I

SECTION II

Total Marks 25

Question 5 (12 marks) **START A NEW BOOKLET** **Marks**

(a) Show that: $\tan(90° - x)\sec(180° + x)\cos(90° - x) = -1$ **2**

(b) Solve: **3**

$$\frac{2x+5}{x+1} \leq 3$$

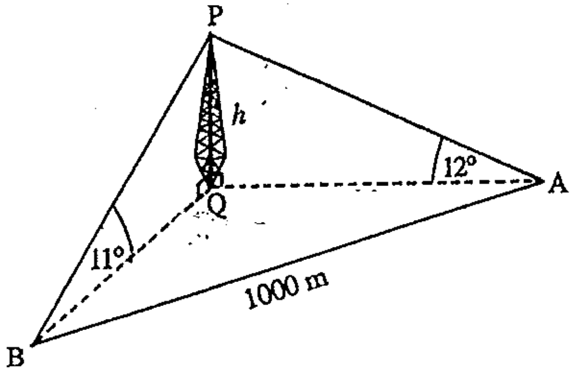
(c) By using the difference of two squares or otherwise, prove: **3**

$$\left(1 + tanx + \frac{1}{cosx}\right)\left(1 + tanx - \frac{1}{cosx}\right) = 2tanx$$

(d) Find the exact value of b if the lines $2x - y + 1 = 0$ and $bx + 4y - 10 = 0$ intersect at 60°. **4**

Question 6 (13 marks) START A NEW BOOKLET **Marks**

(a)



The angle of elevation of a tower PQ of height h metres at a point A due east is 12°. From another point B, the bearing of the tower is 051° and the angle of elevation is 11°. The points A and B are 1000 metres apart and on the same level as the base Q of the tower.

(i) Show that $< AQB = 141°$. **1**

(ii) Consider the triangle APQ and show that AQ= $h\tan 78°$. **1**

(iii) Find a similar expression for BQ **1**

(iv) Use the cosine rule in the triangle AQB to calculate h to the nearest metre. **2**

(b) A point C divides the interval joining $A(-4,2)$ and $B(6,8)$ internally in the ratio m:n.

(i) Write the coordinates of point C in terms of m and n. **1**

(ii) A line l: $3x + y - 17 = 0$ passes through the point C. Using part (i) show that: **1**

$$3(-4n + 6m) + 2n + 8m - 17(m + n) = 0$$

(iii) Hence, using the expression from (ii) find the ratio m:n. **2**

Question 6 continues

(c)

(i) Show that **2**

$$4\cos^2 x + \cos x \sin x + 2\cos x + 4\sin^2 x = 4$$

can be written as $\cos x\,(\sin x + 2) = 0$

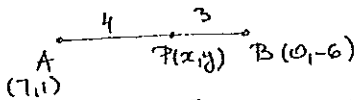
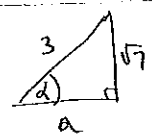
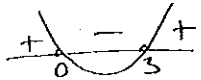
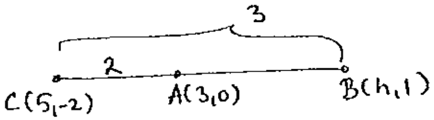
(ii) Hence, solve **2**

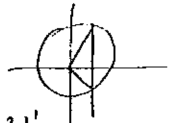
$4\cos^2 x + \cos x \sin x + 2\cos x + 4\sin^2 x = 4$ for $-180° \leq x \leq 180°$.

End of Section II

END OF TASK

Yr 11 Ext 1 Task 1 - 2016

Qn	Solutions	Marks	Comments: Criteria
1.	$\cos 135° = \cos(180° - 45°)$ $= -\cos 45°$ $= -\frac{1}{\sqrt{2}}$ C	1	
2.	$y = 2x - 3$ $m_1 = 2$ $m_2 = -\frac{3}{5}$ $3x + 5y - 1 = 0$ $5y = -3x + 1$ $y = -\frac{3}{5}x + \frac{1}{5}$ $\tan\theta = \left\lvert \frac{2 + \frac{3}{5}}{1 - \frac{6}{5}} \right\rvert = 13 \therefore \theta \approx 86°$ D	1	
3.	 $x = \frac{4\times 0 + 3\times 7}{7}$ $= 3$ $y = \frac{4\times(-6) + 3\times 1}{7}$ $= -3$ $P(3,-3)$ A	1	
4. a)	$\sin\alpha = \frac{\sqrt{7}}{3}$  $a = \sqrt{9-7}$ $a = \sqrt{2}$ $\cos\alpha = -\frac{\sqrt{2}}{3}$	2	
b)	$\frac{x-3}{x} > 0 \quad x \neq 0$  $x < 0$ or $x > 3$	2	
c)	 $AC : CB = -2 : 3$ $5 = \frac{3\times 3 - 2\times h}{1}$ $9 - 2h = 5$ $2h = 4 \quad \therefore h = 2$	2	

Qn	Solutions	Marks	Comments: Criteria
d)	$4\cos\alpha - 1 = 0 \quad , \quad 0° \leq \alpha \leq 360°$ $4\cos\alpha = 1$ $\cos\alpha = \frac{1}{4}$  $\alpha = 75°31' \, , \, 360° - 75°31'$ $\alpha = 75°31' \, , \, 284°29'$	2	
e)	$\frac{1}{1-\cos\alpha} + \frac{1}{1+\cos\alpha} = 2\operatorname{cosec}^2\alpha$ LHS: $\frac{1 + \cos\alpha + 1 - \cos\alpha}{1 - \cos^2\alpha} = \frac{2}{\sin^2\alpha}$ $= 2\operatorname{cosec}^2\alpha$ $=$ RHS.	2	
5. a)	$\tan(90° - x)\sec(180° + x)\cos(90° - x) = -1$ LHS: $\cot x \; \frac{1}{\cos(180° + x)} \times \sin x$ $= \frac{\cos x}{\sin x} \times \frac{\sin x}{-\cos x}$ $= -1$ $=$ RHS.	2	
b)	$\frac{2x+5}{x+1} \leq 3 \quad , \quad x \neq -1$ $(2x+5)(x+1) \leq 3(x+1)^2$ $(2x+5)(x+1) - 3(x+1)^2 \leq 0$ $(x+1)(2x+5 - 3(x+1)) \leq 0$ $(x+1)(2x+5 - 3x - 3) \leq 0$ $(x+1)(-x+2) \leq 0$  $x < -1$ or $x \geqslant 2$	3	

Qn	Solutions	Marks	Comments: Criteria

c) $\left(1+\tan x+\frac{1}{\cos x}\right)\left(1+\tan x-\frac{1}{\cos x}\right)=2\tan x$ — **3**

LHS: $(1+\tan x)^2-\frac{1}{\cos^2 x}$

$=1+2\tan x+\tan^2 x-\frac{1}{\cos^2 x}$

$=2\tan x+1+\frac{\sin^2 x}{\cos^2 x}-\frac{1}{\cos^2 x}$

$=2\tan x+\frac{\cos^2 x+\sin^2 x-1}{\cos^2 x}$

$=2\tan x+\frac{1-1}{\cos^2 x}$

$=2\tan x$

$=$ RHS.

d) — **4**

$2x-y+1=0$

$y=2x+1$

$bx+4y-10=0$

$4y=-bx+10$

$y=-\frac{b}{4}x+\frac{10}{4}$

$m_1=2 \quad m_2=-\frac{b}{4}$

$\tan\theta=\left|\frac{2+\frac{b}{4}}{1-\frac{2b}{4}}\right|$

$\sqrt{3}=\left|\frac{8+b}{4-2b}\right|$

(I)

$\frac{8+b}{4-2b}=\sqrt{3}$

$8+b=4\sqrt{3}-2\sqrt{3}b$

$b+2\sqrt{3}b=4\sqrt{3}-8$

$b=\frac{4\sqrt{3}-8}{1+2\sqrt{3}}$

(II)

$\frac{8+b}{4-2b}=-\sqrt{3}$

$8+b=-4\sqrt{3}+2\sqrt{3}b$

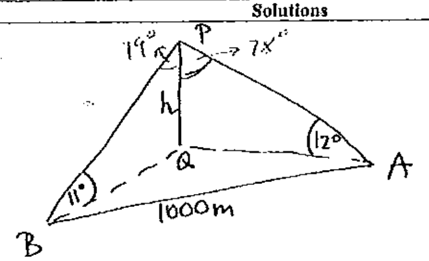
$b-2\sqrt{3}b=-4\sqrt{3}-8$

$b(1-2\sqrt{3})=-4\sqrt{3}-8$

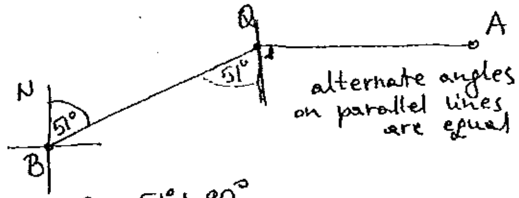
$b=\frac{-4\sqrt{3}-8}{1-2\sqrt{3}}$

Qn	Solutions	Marks	Comments: Criteria

6. a)



i) — **1**



alternate angles on parallel lines are equal

$\angle AQB = 51°+90°$
$= 141°$

ii) $\triangle APQ$ — **1**

$\angle APQ = 90°-12$
$= 78°$

$\tan 78° = \frac{AQ}{h}$

$\therefore AQ = h\tan 78°$

(iii) $\triangle PBQ$ — **1**

$\angle QPB = 90°-11°$
$= 79°$

$\tan 79° = \frac{BQ}{h}$

$\therefore BQ = h\tan 79°$

(iv) $\triangle AQB$ (cosine rule) — **2**

$1000^2 = AQ^2+BQ^2-2AQ\times BQ\cos 141°$

$1000^2 = h^2\tan^2 78°+h^2\tan^2 79°-2h^2\tan 78°\tan 79°\cos 141°$

$h=\frac{1000}{\sqrt{\tan^2 78°+\tan^2 79°-2\tan 78°\tan 79°\cos 141°}}$

$h \approx 108\,m$

Qn	Solutions	Marks	Comments: Criteria
b)	i) m n		
	A$(-4,2)$ C(x,y) B$(6,8)$		
	$AC : CB = m : n$		
	$x = \frac{6m-4n}{m+n}$ $y = \frac{8m+2n}{m+n}$	1	
	ii) $l: 3x+y-17=0$		
	$3 \times \frac{6m-4n}{m+n} + \frac{8m+2n}{m+n} - 17 = 0$	1	
	$3(6m-4n) + 8m+2n - 17(m+n) = 0$		
	(iii) $18m - 12n + 8m + 2n - 17m - 17n = 0$	2	
	$9m - 27n = 0$		
	$9m = 27n$		
	$m = \frac{27n}{9}$		
	$\frac{m}{n} = 3$ $m:n = 3:1$		
c)	i) $\underbrace{4\cos^2 x + 4\sin^2 x}_{4} + \cos x \sin x + 2\cos x = 4$	2	
	$\cancel{4} + \cos x \sin x + 2\cos x - \cancel{4} = 0$		
	$\cos x(\sin x + 2) = 0$		
	ii) $\cos x(\sin x + 2) = 0$, $-180° \le x \le 180°$		
	$\cos x = 0$ or $\sin x + 2 = 0$ no solutions	2	
	$x = 90°, -90°$		