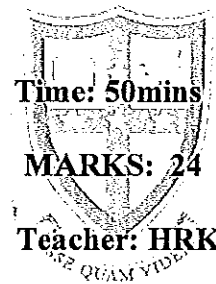


Year 12 2 Unit Sequences and Series – Alternative Task 1

Term 1, 2015

Name: \_\_\_\_\_



Question 1

Start a new Booklet

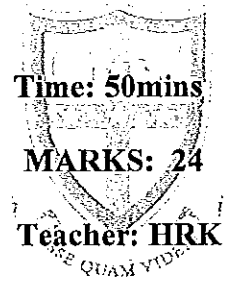
Total Marks 8

- a.  $\sum_{n=7}^{15} 3n + 2$  1
- b. The 10<sup>th</sup> term of an arithmetic sequence is 93 and the 3<sup>rd</sup> term is 30. Find an expression for  $T_n$ , the  $n$ th term in the sequence. 2
- c. The 5<sup>th</sup> term of a geometric series is 2 and the 8<sup>th</sup> is 54. Find the sum of the first 8 terms of the series. 2
- d. Rick earns \$68,000 in his first year working for a bank. His salary increases by 6% per annum (each year).
- i. Write an expression for  $T_n$ , the amount Rick will earn in his  $n$ th year at the bank. 1
- ii. How much will Rick earn in his 10<sup>th</sup> year at the bank? 1
- iii. What will the total of his earnings be after 10 years? 1

Year 12 2 Unit Sequences and Series – Alternative Task 1

Term 1, 2015

Name: \_\_\_\_\_



Question 2

Start a new Booklet

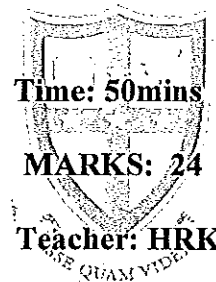
Total Marks 8

- a. Consider the series  $x + x(2 - x) + x(2 - x)^2 + \dots$
- i. Find all possible values  $x$  if the series is to have a limiting sum. 2
  - ii. It is known that the limiting sum is  $\frac{7}{4}$ . Find the value of  $x$  2
- b. The Snowy River is 30 metres deep. A new dam is built that will remove 6 metres of water in the first year and then  $\frac{2}{3}$  of that amount each year thereafter. Ultimately, what will the level of water be? 2
- c. A Cranbrook student, looking to increase his fitness, takes up running. He runs 10 kilometres in the first week and each week after that runs an additional 2 kilometres.
- i. How far will he run in the 24<sup>th</sup> week? 1
  - ii. At the end of the 24<sup>th</sup> week, how far will he have run in total? 1

Year 12 2 Unit Sequences and Series – Alternative Task 1

Term 1, 2015

Name: \_\_\_\_\_



Question 3

Start a new Booklet

Total Marks 8

a. Jake begins a new savings plan at First National Bank by investing \$3000 at the end of each year beginning in 2012. His investment earns 7% per annum.

- i. Show that at the end of 2014, the amount he has in his account,  $A_3$ , can be expressed by:

$$A_3 = 3000(1.07)^2 + 3000(1.07) + 3000 \quad 1$$

- ii. How much money will he have saved when he retires at the end of 2030? 2

b. Kate borrows 800,000 to buy her dream home at 9% per annum, reducible monthly interest. The loan is to be paid back in monthly instalments. The bank is offering the first 3 months interest free and the loan is to be repaid over 20 years.

If  $A_n$  is the amount owing after  $n$  months, and  $M$  is the monthly instalment:

- i. Find an expression for  $A_1, A_2, A_3, A_4$  in terms of  $M$  2
- ii. Find an expression for  $A_{240}$  1
- iii. Find  $M$ , the monthly instalment. 2

Start here.

Q1

$$a) \sum_{n=1}^{15} 3n+2$$

$$= S_{15} - S_0 \times$$

$$3 \times (1+2) = 5$$

$$S_{15} - S_1$$

$$= \frac{15}{2} (5+5 + (15-1) \times 3) - \frac{2}{2} (5+5 + (2-1) \times 3)$$

$$= 292$$

x

$$b) T_{10} = 93, T_3 = 30$$

$$10d + a = 93 \quad (1)$$

$$3d + a = 30 \quad (2)$$

$$10d + a = 93 \quad (1) - (2)$$

$$- 3d + a = 30$$

$$7d = 63$$

$$d = 9$$

sub d into (2)

$$3 \times 9 + a = 30$$

$$a = 3$$

$$T_n = 9n + 3$$

✓ 2

$$c) T_5 = 2, T_8 = 54$$

$$r = \sqrt[3]{\frac{54}{2}}$$

$n = 3$  ✓

$T_1 = \frac{T_2}{r^4}$

$= \frac{T_2}{81}$

$= \frac{2}{81}$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_8 = \frac{\frac{2}{81}(3^8 - 1)}{3 - 1}$

$= 80 \frac{80}{81}$  ✓ 2

d)

i)  $T_n = 68000 \times (1.06)^{n-1}$  . |

ii)  $T_{10} = 68000 \times (1.06)^{10-1}$

$= 68000 \times (1.06)^9$

$= \$114884.57$  ✓ |

iii)  $S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{68000((1.06)^{10} - 1)}{1.06 - 1}$

$= \$896294.06$  ✓ |

Start here.

Q2

a)

i)  $|r| < 1$

$$|2-x| < 1$$

✓ |

⊕

⊖

$$2-x < 1$$

$$-(2-x) < 1$$

$$-x < -1$$

$$x-2 < 1$$

$$x > 1$$

$$x < 3$$

$$1 < x < 3$$

✓ |

~~ii)  $\frac{9}{1-r} = \frac{7}{4}$~~

~~$\frac{x}{1-(2-x)} = \frac{7}{4}$~~

~~$7(2-x) = 4x$~~

~~$14 - 7x = 4x$~~

~~$14 - 11x = 0$~~

~~$-11x = -14$~~

~~$x = \frac{14}{11}$~~

$$\frac{9}{1-r} = \frac{7}{4}$$

$$\frac{x}{1-(2-x)} = \frac{7}{4}$$

~~$\frac{x}{x-1} = \frac{7}{4}$~~

$$4x = 7(x-1)$$

$$4x = 7x - 7$$

$$-3x = -7$$

$$x = \frac{7}{3}$$

✓ 2

b)  $6 + 6 \times \frac{2}{3} + 6 \times \left(\frac{2}{3}\right)^2 \dots$

$$\begin{aligned} S_{\infty} &= \frac{a}{1-r} \\ &= \frac{6}{1-\frac{2}{3}} \\ &= \frac{6}{\frac{1}{3}} \\ &= 18m \end{aligned}$$

Level of water =  $30 - 18$   
 $= 12m$  ✓ Good. ✓

c)  $10 + 12 + 14 + 16 \dots$

$$d = 2$$

$$a = 10$$

$$T_n = 2n + 8$$

i)  $T_{24} = 24 \times 2 + 8$   
 $= 56 \text{ km}$  ✓

ii)  $S_n = \frac{n}{2} (a + l)$

$$S_{24} = \frac{24}{2} (10 + 56)$$

$$= 12 \times 66$$

$$= 792 \text{ km}$$
 ✓

Start here.

Q3

a)

i)  $A_1 = 3000$  ✓

$A_2 = 3000 \times (1.07) + 3000$  ✓

$A_3 = 3000 \times (1.07)^2 + 3000(1.07) + 3000$  ✓

ii) End of 2030 =  $A_{19}$

$A_{19} = 3000 \times (1.07)^{18} + 3000 \times (1.07)^{17} \dots + 3000$

$= 3000 (1 + 1.07 + 1.07^2 + \dots + 1.07^{18})$

$= 3000 \times \frac{1(1.07^{19} - 1)}{1.07 - 1}$

$= \$112136.89$  ✓

3

b)  $\frac{0.09}{12} = 0.0075$  ✓

i)  $A_1 = 800000 - M$

$A_2 = 800000 - 2M$  ✓

$A_3 = 800000 - 3M$  ✓

$A_4 = 800000 \times (1.0075) - 3M(1.0075) - M$  ✓

ii)  $A_{240} = 800000 \times (1.0075)^{247} - M(3 \times 1.0075^{297} + 1.0075^{296} + 1.0075^{295} \dots + 1)$  X

iii)  $M(3 \times 1.0075^{297} + 1.0075^{296} + 1.0075^{295} \dots + 1) = 800000 \times (1.0075)^{297}$

~~$2 \times 1.0075^{296} \times M + M \times \frac{1(1.0075^{298} - 1)}{1.0075 - 1}$~~  X

$2 \times 1.0075^{297} \times M + M \times \frac{1(1.0075^{298} - 1)}{1.0075 - 1} = 800000 \times (1.0075)^{297}$

$1120.914 \dots M$

$= 7359889.031$

$M = \$6565.97$  X