Year 12 2 Unit Sequences and Series – Alternative Task 1 **Time: 50mins**

Term 1, 2015 **MARKS: 24**

Name: ____________________ **Teacher: HRK**

Question 1 **Start a new Booklet** **Total Marks 8**

a. $\sum_{n=7}^{15} 3n+2$ **1**

b. The 10^{th} term of an arithmetic sequence is 93 and the 3^{rd} term is 30. Find an expression for T_n, the nth term in the sequence. **2**

c. The 5^{th} term of a geometric series is 2 and the 8^{th} is 54. Find the sum of the first 8 terms of the series. **2**

d. Rick earns $68,000 in his first year working for a bank. His salary increases by 6% per annum (each year).

   i. Write an expression for T_n, the amount Rick will earn in his nth year at the bank. **1**

   ii. How much will Rick earn in his 10^{th} year at the bank? **1**

   iii. What will the total of his earnings be after 10 years? **1**

Year 12 2 Unit Sequences and Series – Alternative Task 1 **Time: 50mins**

Term 1, 2015 **MARKS: 24**

Name: ______________________ **Teacher: HRK**

Question 2 **Start a new Booklet** **Total Marks 8**

a. Consider the series $x+x(2-x)+x(2-x)^2+\ldots$

i. Find all possible values x if the series is to have a limiting sum. **2**

ii. It is known that the limiting sum is $\frac{7}{4}$. Find the value of x **2**

b. The Snowy River is 30 metres deep. A new dam is built that will remove 6 metres of water in the first year and then $\frac{2}{3}$ of that amount each year thereafter. Ultimately, what will the level of water be? **2**

c. A Cranbrook student, looking to increase his fitness, takes up running. He runs 10 kilometres in the first week and each week after that runs an additional 2 kilometres.

i. How far will he run in the 24^{th} week? **1**

ii. At the end of the 24^{th} week, how far will he have run in total? **1**

Year 12 2 Unit Sequences and Series – Alternative Task 1 **Time: 50mins**

Term 1, 2015 **MARKS: 24**

Name: ____________________ **Teacher: HRK**

Question 3 **Start a new Booklet** **Total Marks 8**

a. Jake begins a new savings plan at First National Bank by investing \$3000 at the end of each year beginning in 2012. His investment earns 7% per annum.

i. Show that at the end of 2014, the amount he has in his account, A_3, can be expressed by:

$A_3 = 3000(1.07)^2 + 3000(1.07) + 3000$ **1**

ii. How much money will he have saved when he retires at the end of 2030? **2**

b. Kate borrows 800,000 to buy her dream home at 9% per annum, reducible monthly interest. The loan is to be paid back in monthly instalments. The bank is offering the first 3 months interest free and the loan is to be repaid over 20 years.

If A_n is the amount owing after n months, and M is the monthly instalment:

i. Find an expression for A_1, A_2, A_3, A_4 in terms of M **2**

ii. Find an expression for A_{240} **1**

iii. Find M, the monthly instalment. **2**

Start here.

Q1

a) $\sum_{n=7}^{15} 3n+2$

$= S_{15} - S_7$ x

$3\times 1 + 2 = 5$

$S_{15} - S_7$

$= \frac{15}{2}(5+5+(15-1)\times 3) - \frac{7}{2}(5+5+(7-1)\times 3)$

$= \underline{292}$

x

b) $T_{10} = 93$, $T_3 = 30$

$10d + a = 93$ (1)

$3d + a = 30$ (2)

$10d + a = 93$ (1) − (2)

$- \quad 3d + a = 30$

$7d = 63$

$d = 9$

sub d into (2)

$3\times 9 + a = 30$

$a = 3$

$T_n = 9n + 3$ ✓ 2

c) $T_5 = 2$, $T_8 = 54$

$r = \sqrt[3]{\frac{54}{2}}$

$= 3$ ✓

$T_1 = \frac{T_5}{r^4}$

$= \frac{T_5}{81}$

$= \frac{2}{81}$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_8 = \frac{\frac{2}{81}(3^8 - 1)}{3 - 1}$

$= 80\frac{80}{81}$ ✓ 2

d)

i) $T_n = 68000 \times (1.06)^{n-1}$

ii) $T_{10} = 68000 \times (1.06)^{10-1}$

$= 68000 \times (1.06)^9$

$= \$114\,884.57$ ✓

iii) $S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{68000((1.06)^{10} - 1)}{1.06 - 1}$

$= \$896294.06$ ✓

Additional writing space on back page

Start here.

Q2

a)

i) $|r|<1$

$|2-x|<1$ 1

$\oplus$ $\ominus$

$2-x<1$ $-(2-x)<1$

$-x<-1$ $x-2<1$

$x>1$ $x<3$

$\underline{1<x<3}$

ii) $\cancel{\frac{a}{1-r}=\frac{7}{4}}$

$\cancel{\frac{x}{1-2-x}=\frac{7}{4}}$

$\cancel{7(2-x)=4x}$

$\cancel{14-7x=4x}$

$\cancel{14-11x=0}$

$\cancel{-11x=-14}$

$\cancel{x=1\frac{3}{11}}$

$\frac{a}{1-r}=\frac{7}{4}$

$\frac{x}{1-(2-x)}=\frac{7}{4}$

$\frac{x}{x-1}=\frac{7}{4}$

$4x=7(x-1)$

$4x=7x-7$

$-3x=-7$ 2

$\underline{x=\frac{7}{3}}$

b) $6+6\times\frac{2}{3}+6\times\left(\frac{2}{3}\right)^2\ldots$

$S_\infty = \frac{a}{1-r}$

$= \frac{6}{1-\frac{2}{3}}$

$= \frac{6}{\frac{1}{3}}$

$= 18m$

Level of water $= 30-18$

$= 12m$ ✓ Good. 2

c) $10+12+14+16..$

$d=2$

$a=10$

$T_n = 2n+8$

i) $T_{24} = 24 \times 2 + 8$

$= 56km$ ✓ 1

ii) $S_n = \frac{n}{2}(a+l)$

$S_{24} = \frac{24}{2}(10+56)$ 1

$= 12 \times 66$

$= 792km$ ✓

Additional writing space on back page

Start here.

Q3

a)

i) $A_1 = 3000$

$A_2 = 3000 \times (1.07) + 3000$

$A_3 = 3000 \times (1.07)^2 + 3000(1.07) + 3000$

ii) End of 2030 : A_{19}

$A_{19} = 3000 \times (1.07)^{18} + 3000 \times (1.07)^{17} \ldots + 3000$

$= 3000(1 + 1.07 + 1.07^2 + \ldots + 1.07^{18})$

$= 3000 \times \frac{1(1.07^{19} - 1)}{1.07 - 1}$

$= \$112136.89$

3

b) $\frac{0.09}{12} = 0.0075$

i) $A_1 = 800000 - M$

$A_2 = 800000 - 2M$

$A_3 = 800000 - 3M$

$A_4 = 800000 \times (1.0075) - 3M(1.0075) - M$

2

..

?

ii) $A_{240} = 800000 \times (1.0075)^{297} - M(3 \times 1.0075^{297} + 1.0075^{296} + 1.0075^{295} \ldots + 1)$ ✗

iii) $M(3 \times 1.0075^{297} + 1.0075^{296} + 1.0075^{295} \ldots + 1) = 800000 \times (1.0075)^{297}$

$\cancel{2 \times 1.0075^{296} \times M + M \times \frac{1(1.0075^{298} - 1)}{1.0075 - 1}}$ ✗

$2 \times 1.0075^{298} \times M + M \times \frac{1(1.0075^{298} - 1)}{1.0075 - 1} = 800000 \times (1.0075)^{297}$

$1120.914\ldots M = 7359889.034$

$M = \$6565.97$ ✗ ✗

0