Yr 12, Ext 2

Name: Date:

Topic: **COMPLEX NUMBERS - CARTESIAN**

Question 1 [3 marks]

One of the roots of $z^2 - pz + q = 0$, where p and q are real, is $3 - i$, find p and q.

Question 2 [5 marks]

Find the value of z given that: $z(2 - i) + 2\overline{z} = 3 + 4i$

Question 3 [1 + 1 + 1 + 2 + 1 + 2 = 8 marks]

If $z_1 = 1 + 2i$ and $z_2 = 3 - i$, then find:

(a) $z_1 + z_2$

(b) $(z_1)(z_2)$

(c) $\frac{z_1}{z_2}$

(d) $(z_1 + 2i)(\overline{z}_2 - 2i)$

(e) $|z_1|$

(f) $\arg z_2$

Question 4 [4 marks]

Given that $x + 2$ is a factor of $x^3 - 2x^2 - 3x + 10$, find the other factors.

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Question 1

$z^2 - pz + q = [z - (3 - i)][z - (3 + i)]$ [1]

$= z^2 - z(3 + i) - z(3 - i) + 9 - i^2$ [1]

$= z^2 - 6z + 10$

$p = 6$ and $q = 10$ [1]

Question 2

$z(2 - i) + 2\bar{z} = 3 + 4i$

$(x + iy)(2 - i) + 2(x - iy) = 3 + 4i$ [1]

$2x - ix + 2iy - i^2y + 2x - 2iy = 3 + 4i$ [1]

$4x + y - ix = 3 + 4i$

$4x + y = 3$ and $-x = 4$ [1]

$x = -4$ and $y = 19$ [1]

$z = -4 + 19i$ [1]

Question 3

(a) $4 + i$ [1]

(b) $5 + 5i$ [1]

(c) $0.1i + 0.7i$ [1]

(d) $(z_1 + 2i)(\bar{z}_2 - 2i) = (1 + 4i)(3 - i)$ [1]

$= 7 + 11i$ [1]

(e) $\sqrt{5}$ or 2.24 (2dec. pl) [1]

(f) $\arg z_2 = -\tan^{-1}\frac{1}{3} = -0.32$ (2 dec.pl) or -18.43° [2]

Question 4

$$x+2\overline{)\,x^3-2x^2-3x+10\,}\quad x^2-4x+5$$ [2]

$x^3 - 2x^2 - 3x + 10 = (x + 2)(x^2 - 4x + 5)$ [1]

$= (x + 2)[x - (2 + i)][x - (2 - i)]$ [1]

Question 5

$$\frac{a}{1-i} - \frac{b}{1+i} = 3i - 1$$

$$\frac{a(1+i)}{(1-i)(1+i)} - \frac{b(1-i)}{(1+i)(1-i)} = 3i - 1 \quad [1]$$

$$\frac{a+ia}{2} - \frac{b-ib}{2} = 3i - 1 \quad [1]$$

$$\frac{a-b}{2} = -1 \quad \text{and} \quad \frac{a+b}{2} = 3 \quad [1]$$

$$a = 2 \quad \text{and} \quad b = 4 \quad [1]$$

Question 6

(a)

$$|z+i| \geq |z-(2+i)|$$

$$|x+iy+i| \geq |x+iy-(2+i)|$$

$$|x+(y+1)i| \geq |(x-2)+(y-1)i| \quad [1]$$

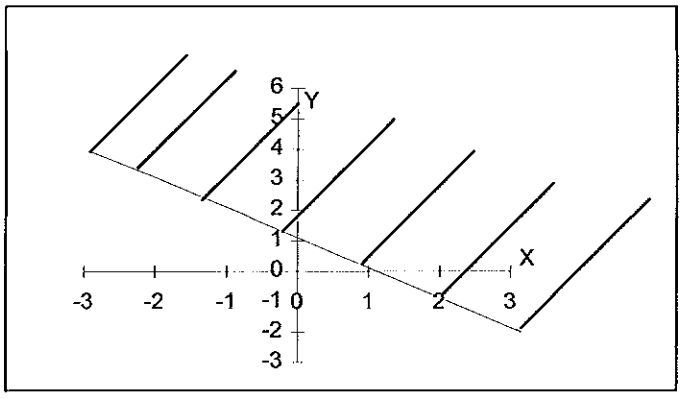
$$x^2+(y+1)^2 \geq (x-2)^2+(y-1)^2 \quad [1]$$

$$x^2+y^2+2y+1 \geq x^2-4x+4+y^2-2y+1 \quad [1,1]$$

$$4x+4y \geq 4$$

$$x+y \geq 1 \quad [1]$$

(b)



[1]

(3 + 5 + 8 + 4 + 4 + 6 = 30 marks)