Yr 12, Ext 2

Name: Date:

Topic: **COMPLEX NUMBERS - POLAR**

Question 1 [2 + 2 + 1 = 5 marks]

Convert the following complex numbers to polar form.

(a) $-1-\sqrt{3}\,i$

(b) $-i\sqrt{2}+\sqrt{2}$

(c) $\sqrt{5}e^{-(\frac{i\pi}{8})}$

Question 2 [2 + 2 + 2 = 6 marks]

Convert each of the following to Cartesian form.

(a) $2\text{cis}(-\frac{3\pi}{2})$

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(b) $e^{i\pi-1}$

(c) $2icis\frac{\pi}{4}$

Question 3 [1 + 3 = 4 marks]

Simplify each of the following, giving the answer in polar form.

(a) $(\sqrt{3}cis\frac{\pi}{2})^4$

(b) $(1-i)^6$

Question 4 [2 + 4 + 2 = 8 marks]

Find z in polar form given,

(a) $z^4 = -81$

(b) $z^3 + 8i = 0$

On the Argand Plane

(c) plot the solutions to 4(a).

Question 5 [1 + 3 + 3 = 7 marks]

Given that $z^k + \frac{1}{z^k} = 2\cos(kx)$,

(a) find $z + \frac{1}{z}$

(b) prove that $\cos^4 x = \frac{1}{16}\cos^4 x + \frac{1}{4}\cos(2x) + \frac{3}{8}$

Hence, find:

(c) $\int_0^{\frac{\pi}{2}} 8\cos^4 x\,dx$

(5 + 6 + 4 + 8 + 7 = 30 marks)

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Question 1

(a) $r = \sqrt{1+3} = 2$ [1]

$\theta = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$ [1]

$-1 - \sqrt{3}\,i = 2\text{cis}(\frac{-2\pi}{3})$

(b) $r = \sqrt{2+2} = 2$ [1]

$\theta = \tan^{-1}(-1) = -\frac{\pi}{4}$ [1]

$-i\sqrt{2} + \sqrt{2} = 2\text{cis}(-\frac{\pi}{4})$

(c) $\sqrt{5}e^{-(\frac{i\pi}{8})} = \sqrt{5}\text{cis}(-\frac{\pi}{8})$ [1]

Question 2

(a) $z = 2\cos(-\frac{3\pi}{2}) + 2i\sin(-\frac{3\pi}{2}) = -2i$ [1,1]

(b) $z = \frac{1}{e}\text{cis}(-\pi) = -\frac{1}{e}$ [1,1]

(c) $z = 2i\cos\frac{\pi}{4} + 2i^2\sin\frac{\pi}{4} = \sqrt{2}\,i - \sqrt{2}$ [1,1]

Question 3

(a) $(\sqrt{3}\text{cis}\frac{\pi}{2})^4 = 9\text{cis}2\pi$ [1]

(b) $1 - i = \sqrt{2}\text{cis}(-\frac{\pi}{4})$ [1]

$(1-i)^6 = (\sqrt{2}\text{cis}(-\frac{\pi}{4}))^6$ [1]

$= 8\text{cis}(-\frac{3\pi}{2})$

$= 8\text{cis}(\frac{\pi}{2})$ [1]

Question 4

(a) $z^4 = (-81)$

$= \{81\text{cis}(-\pi)\}$ [1]

$z = \{81\text{cis}(-\pi), 81\text{cis}\pi, 81\text{cis}3\pi, 81\text{cis}5\pi\}^{1/4}$

$z = 3\text{cis}(-\frac{\pi}{4}), 3\text{cis}\frac{\pi}{4}, 3\text{cis}\frac{3\pi}{4}, 3\text{cis}(-\frac{3\pi}{4})$ [1]

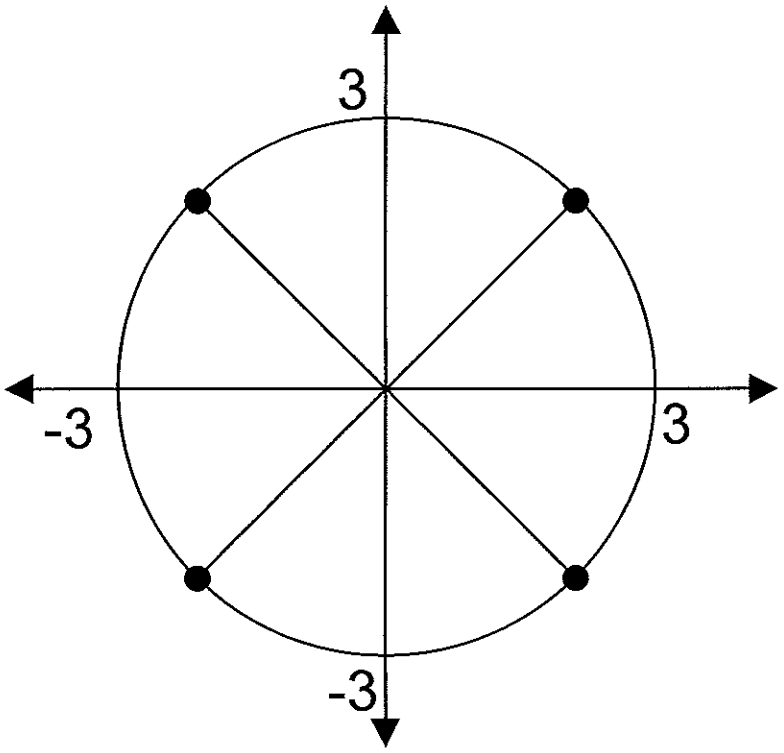
(b) $z^3 = -8i$ [1]

$= 8\text{cis}(-\frac{\pi}{2})$ [1]

$z = \{8\text{cis}(-\frac{\pi}{2}), 8\text{cis}\frac{3\pi}{2}, 8\text{cis}\frac{7\pi}{2}\}^{1/3}$ [1]

$= 2\text{cis}(-\frac{\pi}{6}), 2\text{cis}\frac{\pi}{2}, 2\text{cis}\frac{7\pi}{6}$ [1]

(c)



[3]

Question 5

(a) $2\cos x$ [1]

(b) $(z+\frac{1}{z})^4 = (z^4+\frac{1}{z^4})+4(z^2+\frac{1}{z^2})+6$ [1]

$(2\cos x)^4 = \cos 4x + 4\cos 2x + 6$

$16\cos^4 x = \cos 4x + 4\cos 2x + 6$ [1]

$\cos^4 x = \frac{1}{16}\cos^4 x + \frac{1}{4}\cos 2x + \frac{3}{8}$ [1]

(c)

$$\int_0^{\frac{\pi}{2}} 8\cos^4 x\,dx = \int_0^{\frac{\pi}{2}} (\cos 4x + 4\cos 2x + 3)\,dx$$ [1]

$$= \frac{1}{4}\sin 4x + 2\sin 2x + 3x \Big]_0^{\frac{\pi}{2}}$$ [1]

$$= \frac{3}{2}\pi$$ [1]

(5 + 6 + 4 + 8 + 7 = 30 marks)