NSW EDUCATION STANDARDS AUTHORITY

2017 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions 11–16, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2–5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6–16)

- Attempt Questions 11–16
- Allow about 2 hours and 45 minutes for this section

1250

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

1 What is the gradient of the line $2x + 3y + 4 = 0$?

A. $-\frac{2}{3}$

B. $\frac{2}{3}$

C. $-\frac{3}{2}$

D. $\frac{3}{2}$

2 Which expression is equal to $3x^2 - x - 2$?

A. $(3x-1)(x+2)$

B. $(3x+1)(x-2)$

C. $(3x-2)(x+1)$

D. $(3x+2)(x-1)$

3 What is the derivative of e^{x^2}?

A. $x^2 e^{x^2-1}$

B. $2xe^{2x}$

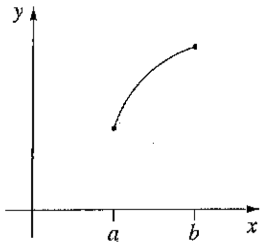
C. $2xe^{x^2}$

D. $2e^{x^2}$

4 The function $f(x)$ is defined for $a \le x \le b$.

On this interval, $f'(x) > 0$ and $f''(x) < 0$.

Which graph best represents $y = f(x)$?

A.


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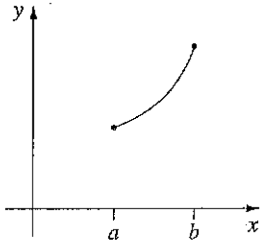

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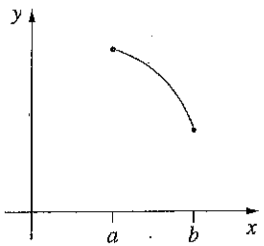

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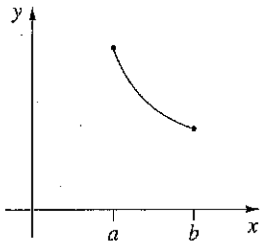


5 It is given that $\ln a = \ln b - \ln c$, where $a, b, c > 0$.

Which statement is true?

A. $a = b - c$

B. $a = \dfrac{b}{c}$

C. $\ln a = \dfrac{b}{c}$

D. $\ln a = \dfrac{\ln b}{\ln c}$

6 The point P moves so that it is always equidistant from two fixed points, A and B.

What best describes the locus of P?

A. A point

B. A circle

C. A parabola

D. A straight line

7 Which expression is equivalent to $\tan\theta + \cot\theta$?

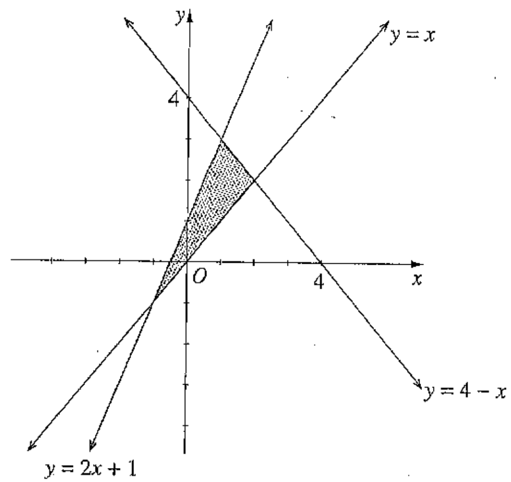
A. $\operatorname{cosec}\theta + \sec\theta$

B. $\sec\theta \operatorname{cosec}\theta$

C. 2

D. 1

8 The region enclosed by $y = 4 - x, y = x$ and $y = 2x + 1$ is shaded in the diagram below.



Which of the following defines the shaded region?

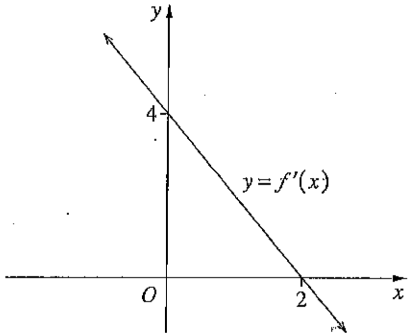
A. $y \le 2x + 1, \quad y \le 4 - x, \quad y \ge x$

B. $y \ge 2x + 1, \quad y \le 4 - x, \quad y \ge x$

C. $y \le 2x + 1, \quad y \ge 4 - x, \quad y \ge x$

D. $y \ge 2x + 1, \quad y \ge 4 - x, \quad y \ge x$

9 The graph of $y = f'(x)$ is shown.



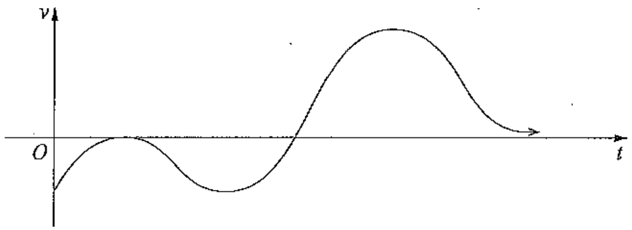
The curve $y = f(x)$ has a maximum value of 12.

What is the equation of the curve $y = f(x)$?

A. $y = x^2 - 4x + 12$
B. $y = 4 + 4x - x^2$
C. $y = 8 + 4x - x^2$
D. $y = x^2 - 4x + 16$

10 A particle is moving along a straight line.

The graph shows the velocity, v, of the particle for time $t \geq 0$.



How many times does the particle change direction?

A. 1
B. 2
C. 3
D. 4

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet.

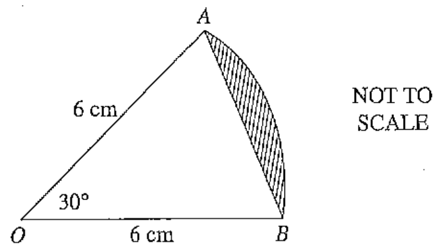
(a) Rationalise the denominator of $\dfrac{2}{\sqrt{5}-1}$. **2**

(b) Find $\displaystyle\int (2x+1)^4\,dx$. **1**

(c) Differentiate $\dfrac{\sin x}{x}$. **2**

(d) Differentiate $x^3 \ln x$. **2**

(e) In the diagram, OAB is a sector of the circle with centre O and radius 6 cm, where $\angle AOB = 30°$.

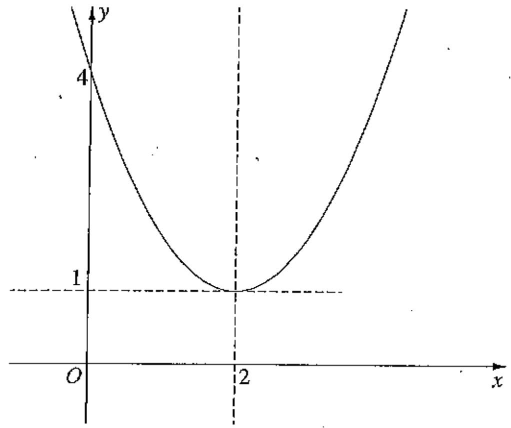


(i) Find the exact value of the area of the triangle OAB. **1**

(ii) Find the exact value of the area of the shaded segment. **1**

Question 11 continues on page 7

(f) Determine the equation of the parabola shown. Write your answer in the form $(x-h)^2 = 4a(y-k)$. **2**



(g) Solve $|3x-1| = 2$. **2**

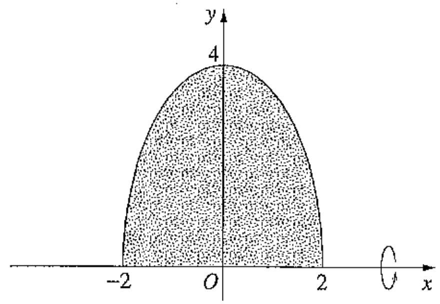
(h) Find the domain of the function $f(x) = \sqrt{3-x}$. **2**

End of Question 11

Question 12 (15 marks) Use the Question 12 Writing Booklet.

(a) Find the equation of the tangent to the curve $y = x^2 + 4x - 7$ at the point $(1, -2)$. **2**

(b) The diagram shows the region bounded by $y = \sqrt{16 - 4x^2}$ and the x-axis. **3**



The region is rotated about the x-axis to form a solid.

Find the exact volume of the solid formed.

(c) In an arithmetic series, the fifth term is 200 and the sum of the first four terms is 1200. **3**

Find the value of the tenth term.

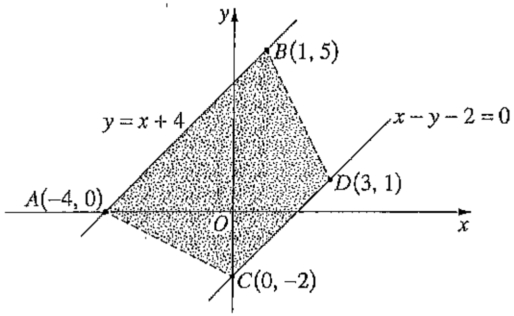
Question 12 continues on page 9

(d) The points $A(-4, 0)$ and $B(1, 5)$ lie on the line $y = x + 4$.

The length of AB is $5\sqrt{2}$.

The points $C(0, -2)$ and $D(3, 1)$ lie on the line $x - y - 2 = 0$.

The points A, B, D, C form a trapezium as shown.

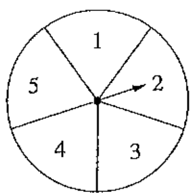


NOT TO SCALE

(i) Find the perpendicular distance from point $A(-4, 0)$ to the line $x - y - 2 = 0$. **1**

(ii) Calculate the area of the trapezium. **2**

(e) A spinner is marked with the numbers 1, 2, 3, 4 and 5. When it is spun, each of the five numbers is equally likely to occur.



The spinner is spun three times.

(i) What is the probability that an even number occurs on the first spin? **1**

(ii) What is the probability that an even number occurs on at least one of the three spins? **1**

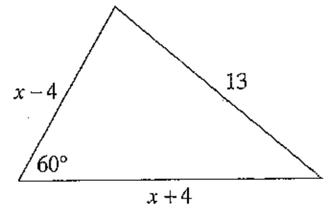
(iii) What is the probability that an even number occurs on the first spin and odd numbers occur on the second and third spins? **1**

(iv) What is the probability that an even number occurs on exactly one of the three spins? **1**

End of Question 12

Question 13 (15 marks) Use the Question 13 Writing Booklet.

(a) Using the cosine rule, find the value of x in the following diagram. **3**



NOT TO SCALE

(b) Consider the curve $y = 2x^3 + 3x^2 - 12x + 7$.

(i) Find the stationary points of the curve and determine their nature. **4**

(ii) Sketch the curve, labelling the stationary points. **2**

(iii) Hence, or otherwise, find the values of x for which $\frac{dy}{dx}$ is positive. **1**

(c) By letting $m = t^{\frac{1}{3}}$, or otherwise, solve $t^{\frac{2}{3}} + t^{\frac{1}{3}} - 6 = 0$. **2**

(d) The rate at which water flows into a tank is given by **3**

$$\frac{dV}{dt} = \frac{2t}{1 + t^2},$$

where V is the volume of water in the tank in litres and t is the time in seconds.

Initially the tank is empty.

Find the exact amount of water in the tank after 10 seconds.

Question 14 (15 marks) Use the Question 14 Writing Booklet.

(a) Sketch the curve $y = 4 + 3\sin 2x$ for $0 \le x \le 2\pi$. **3**

(b) (i) Find the exact value of $\int_0^{\frac{\pi}{3}} \cos x \, dx$. **1**

(ii) Using Simpson's rule with one application, find an approximation to the integral **2**

$$\int_0^{\frac{\pi}{3}} \cos x \, dx,$$

leaving your answer in terms of π and $\sqrt{3}$.

(iii) Using parts (i) and (ii), show that **1**

$$\pi \approx \frac{18\sqrt{3}}{3+4\sqrt{3}}.$$

Question 14 continues on page 12

(c) Carbon-14 is a radioactive substance that decays over time. The amount of carbon-14 present in a kangaroo bone is given by

$$C(t) = Ae^{kt},$$

where A and k are constants, and t is the number of years since the kangaroo died.

(i) Show that $C(t)$ satisfies $\frac{dC}{dt} = kC$. **1**

(ii) After 5730 years, half of the original amount of carbon-14 is present. **2**

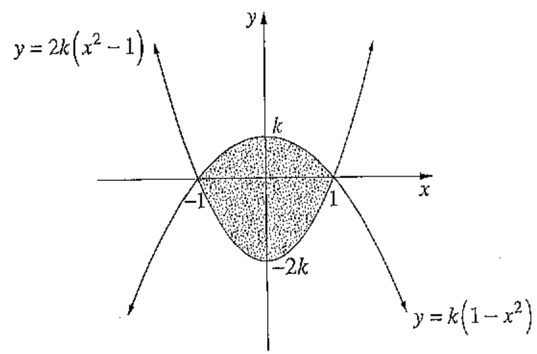
Show that the value of k, correct to 2 significant figures, is -0.00012.

(iii) The amount of carbon-14 now present in a kangaroo bone is 90% of the original amount. **2**

Find the number of years since the kangaroo died. Give your answer correct to 2 significant figures.

(d) The shaded region shown is enclosed by two parabolas, each with x-intercepts at $x = -1$ and $x = 1$. **3**

The parabolas have equations $y = 2k(x^2 - 1)$ and $y = k(1 - x^2)$, where $k > 0$.

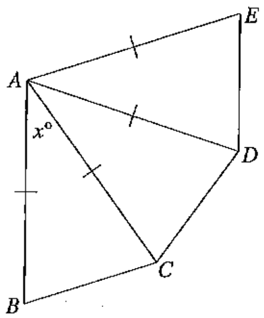


Given that the area of the shaded region is 8, find the value of k.

End of Question 14

Question 15 (15 marks) Use the Question 15 Writing Booklet.

(a) The triangle ABC is isosceles with $AB = AC$ and the size of $\angle BAC$ is $x°$. Points D and E are chosen so that $\triangle ABC$, $\triangle ACD$ and $\triangle ADE$ are congruent, as shown in the diagram. **3**



Find the value of x for which AB is parallel to ED, giving reasons.

(b) Anita opens a savings account. At the start of each month she deposits $\$X$ into the savings account. At the end of each month, after interest is added into the savings account, the bank withdraws \$2500 from the savings account as a loan repayment. Let M_n be the amount in the savings account after the n^{th} withdrawal.

The savings account pays interest of 4.2% per annum compounded monthly.

(i) Show that after the second withdrawal the amount in the savings account is given by **2**

$$M_2 = X\left(1.0035^2 + 1.0035\right) - 2500\left(1.0035 + 1\right).$$

(ii) Find the value of X so that the amount in the savings account is \$80 000 after the last withdrawal of the fourth year. **3**

Question 15 continues on page 14

(c) Two particles move along the x-axis.

When $t = 0$, particle P_1 is at the origin and moving with velocity 3.

For $t \geq 0$, particle P_1 has acceleration given by $a_1 = 6t + e^{-t}$.

(i) Show that the velocity of particle P_1 is given by $v_1 = 3t^2 + 4 - e^{-t}$. **2**

When $t = 0$, particle P_2 is also at the origin.

For $t \geq 0$, particle P_2 has velocity given by $v_2 = 6t + 1 - e^{-t}$.

(ii) When do the two particles have the same velocity? **2**

(iii) Show that the two particles do not meet for $t > 0$. **3**

End of Question 15

Question 16 (15 marks) Use the Question 16 Writing Booklet.

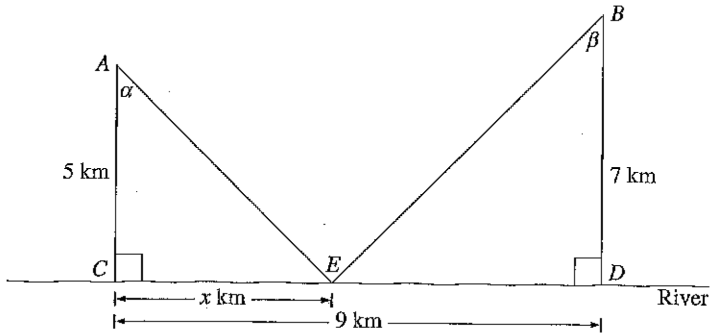
(a) John's home is at point A and his school is at point B. A straight river runs nearby.

The point on the river closest to A is point C, which is 5 km from A.

The point on the river closest to B is point D, which is 7 km from B.

The distance from C to D is 9 km.

To get some exercise, John cycles from home directly to point E on the river, x km from C, before cycling directly to school at B, as shown in the diagram.



The total distance John cycles from home to school is L km.

(i) Show that $L = \sqrt{x^2 + 25} + \sqrt{49 + (9 - x)^2}$. **1**

(ii) Show that if $\frac{dL}{dx} = 0$, then $\sin\alpha = \sin\beta$. **3**

(iii) Find the value of x that makes $\sin\alpha = \sin\beta$. **2**

(iv) Explain why this value of x gives a minimum for L. **1**

Question 16 continues on page 16

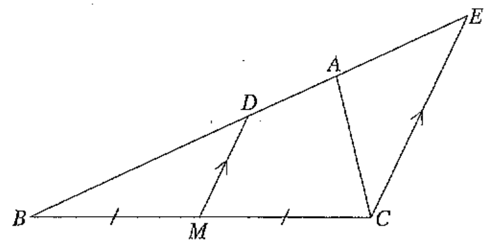
(b) A geometric series has first term a and limiting sum 2. **3**

Find all possible values for a.

(c) In the triangle ABC, the point M is the mid-point of BC. The point D lies on AB and

$$BD = DA + AC.$$

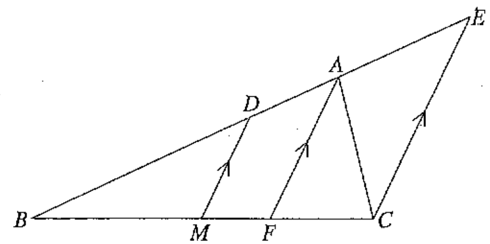
The line that passes through the point C and is parallel to MD meets BA produced at E.



Copy or trace this diagram into your writing booklet.

(i) Prove that $\triangle ACE$ is isosceles. **3**

(ii) The point F is chosen on BC so that AF is parallel to DM. **2**



Show that AF bisects $\angle BAC$.

End of paper

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2017 HSC Mathematics Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	A
2	D
3	C
4	A
5	B
6	D
7	B
8	A
9	C
10	A

Section II

Question 11 (a)

Criteria	Marks
• Provides correct solution	2
• Recognises a conjugate surd, or equivalent merit	1

Sample answer:

$$\frac{2}{\sqrt{5}-1}\times\frac{\sqrt{5}+1}{\sqrt{5}+1}$$

$$=\frac{2(\sqrt{5}+1)}{(\sqrt{5})^2-1^2}$$

$$=\frac{2(\sqrt{5}+1)}{5-1}$$

$$=\frac{2(\sqrt{5}+1)}{4}$$

Question 11 (b)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\int(2x+1)^4\,dx$$

$$=\frac{(2x+1)^5}{2(5)}+c$$

$$=\frac{(2x+1)^5}{10}+c$$

Question 11 (c)

Criteria	Marks
• Provides correct derivative	2
• Attempts to use the quotient rule, or equivalent merit	1

Sample answer:

$$\frac{d}{dx}\left(\frac{\sin x}{x}\right)$$

$$=\frac{x(\cos x)-\sin x(1)}{x^2}$$

$$=\frac{x\cos x-\sin x}{x^2}$$

$$u=\sin x$$

$$\frac{du}{dx}=\cos x$$

$$v=x$$

$$\frac{dv}{dx}=1$$

Question 11 (d)

Criteria	Marks
• Provides correct derivative	2
• Attempts to use the product rule, or equivalent merit	1

Sample answer:

$$\frac{d}{dx}(x^3\times\ln x)$$

$$=x^3\times\frac{1}{x}+3x^2\times\ln x$$

$$=x^2+3x^2\times\ln x$$

$$u=x^3$$

$$\frac{du}{dx}=3x^2$$

$$v=\ln x$$

$$\frac{dv}{dx}=\frac{1}{x}$$

Question 11 (e) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$A=\frac{1}{2}ab\sin c$$

$$=\frac{1}{2}\times 6\times 6\times\sin 30°$$

$$=\frac{1}{2}\times 36\times\frac{1}{2}$$

$\therefore$ Area = 9 cm^2

Question 11 (e) (ii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

Shaded area = Area of sector AOB – Area of $\triangle OAB$

$$=\frac{\pi}{12}\left(6^2\right)-9$$

$$=\frac{36\pi}{12}-9$$

$\therefore$ Area = $(3\pi-9)$ cm^2

Question 11 (f)

Criteria	Marks
• Provides correct equation	2
• Identifies the vertex, or equivalent merit	1

Sample answer:

$$v = (2,\ 1)$$

$$pt = (0,\ 4)$$

$$(x-h)^2 = 4a(y-k)$$

$$(x-2)^2 = 4a(y-1)$$

$$(0-2)^2 = 4a(4-1)$$

$$4 = 4a(3)$$

$$4 = 12a$$

$$a = \frac{1}{3}$$

$$\therefore (x-2)^2 = \frac{4}{3}(y-1)$$

Question 11 (g)

Criteria	Marks
• Provides correct solution	2
• Attempts to deal with the absolute value, or equivalent merit	1

Sample answer:

$$|3x-1| = 2$$

$+(3x-1)=2$	$-(3x-1)=2$
$3x-1=2$	$-3x+1=2$
$3x=3$	$-3x=1$
$x=1$	$x=-\frac{1}{3}$

Question 11 (h)

Criteria	Marks
• Provides correct domain	2
• Attempts to obtain an inequality, or equivalent merit	1

Sample answer:

$$f(x) = \sqrt{3-x}$$

$$3-x \geq 0$$

$$3 \geq x$$

$$\therefore\ x \leq 3$$

Question 12 (a)

Criteria	Marks
• Provides correct solution	2
• Finds the slope of the curve at the given point, or equivalent merit	1

Sample answer:

$$y = x^2 + 4x - 7$$

$$y' = 2x + 4$$

when $x = 1$ $\quad y' = 2 + 4 = 6$

$\therefore\ m = 6$

$$y - y_1 = m(x - x_1)$$

$$y - -2 = 6(x - +1)$$

$$y = 6x - 8$$

Question 12 (b)

Criteria	Marks
• Provides correct solution	3
• Obtains the primitive of an integrand for the volume, involving the square of the function, or equivalent merit	2
• Obtains an integral for the volume involving the square of the function, or equivalent merit	1

Sample answer:

$$V = \pi \int_{-2}^{2} \left(\sqrt{16-4x^2}\right)^2 dx$$

$$= 2\pi \int_{0}^{2} \left(16-4x^2\right) dx$$

$$= 2\pi \left[16x - \frac{4x^3}{3}\right]_0^2$$

$$= 2\pi \left[32 - \frac{32}{3} - 0 + 0\right]$$

$$\text{Volume} = \frac{128\pi}{3} \text{ units}^3$$

Question 12 (c)

Criteria	Marks
• Provides correct solution	3
• Finds the three correct equations and attempts to solve, or equivalent merit	2
• Finds a valid equation linking a and d, or equivalent merit	1

Sample answer:

$T_5 = a + 4d = 200$

$S_5 = a + a + d + a + 2d + a + 3d = 1200$

$$\begin{aligned} a + 4d &= 200 \quad \text{———} ① \\ 4a + 6d &= 1200 \quad \text{———} ② \\ 4a + 16d &= 800 \quad \text{———} ① \times 4 \end{aligned}$$

Subtracting

$$\begin{aligned} 10d &= -400 \\ d &= -40 \end{aligned}$$

$$\begin{aligned} a + -160 &= 200 \\ a &= 360 \end{aligned}$$

$$\begin{aligned} T_{10} &= 360 + 9 \times -40 \\ &= 0 \end{aligned}$$

Question 12 (d) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

Point A (–4, 0) to the line $y = x - 2$

$$x - y - 2 = 0$$

$$a = 1 \quad b = -1 \quad c = -2$$

$$\text{Perpendicular distance} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|1 \times -4 + -1 \times 0 - 2|}{\sqrt{1^2 + (-1)^2}}$$

$$= \frac{|-4 - 2|}{\sqrt{2}} = \frac{6}{\sqrt{2}}$$

$$= \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{6\sqrt{2}}{2}$$

$$= 3\sqrt{2}$$

Question 12 (d) (ii)

Criteria	Marks
• Provides correct solution	2
• Finds the length of DC, or equivalent merit	1

Sample answer:

$$A = \frac{1}{2}(a + b)h$$

$h = 3\sqrt{2}$ and one length $(AB) = 5\sqrt{2}$

$$\text{Other length is distance } CD = \sqrt{(3-0)^2 + (1 - -2)^2}$$

$$= \sqrt{9+9}$$

$$= \sqrt{18}$$

$$= 3\sqrt{2}$$

$$\text{So } A = \frac{1}{2}\left(5\sqrt{2} + 3\sqrt{2}\right) \times 3\sqrt{2}$$

$$= \frac{1}{2} \times 8\sqrt{2} \times 3\sqrt{2}$$

$$= \frac{24 \times 2}{2}$$

$$\therefore \text{Area} = 24 \text{ units}^2$$

Question 12 (e) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$P \text{ (even number)} = \frac{2}{5}$$

Question 12 (e) (ii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

Probability (at least 1 even number)

$= 1 -$ Probability (all odd numbers)

$= 1 - \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$

$= \frac{98}{125}$

Question 12 (e) (iii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

P (even, odd, odd) $= \frac{2}{5} \times \frac{3}{5} \times \frac{3}{5}$

$= \frac{18}{125}$

Question 12 (e) (iv)

Criteria	Marks
• Provides correct answer	1

Sample answer:

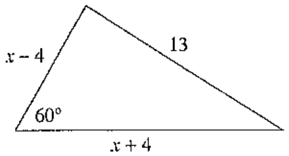
P (exactly one even) $= 3 \times \frac{2}{5} \times \frac{3}{5} \times \frac{3}{5}$

$= \frac{54}{125}$

Question 13 (a)

Criteria	Marks
• Provides correct solution	3
• Attempts to expand each term and evaluate the cosine term, or equivalent merit	2
• Correctly uses the cosine rule, or equivalent merit	1

Sample answer:



Using cosine rule:

$$13^2 = (x+4)^2 + (x-4)^2 - 2(x+4)(x-4)\cos 60°$$

$$169 = x^2 + 8x + 16 + x^2 - 8x + 16 - 2(x^2 - 16) \times \frac{1}{2}$$

$$= 2x^2 + 32 - x^2 + 16$$

$$= x^2 + 48$$

$$121 = x^2$$

$$\therefore \quad x = \pm\sqrt{121}$$

$$= \pm 11$$

But $x - 4 > 0$ since it is a length

So $x > 4$

$\therefore \; x = 11$

Question 13 (b) (i)

Criteria	Marks
• Provides correct solution	4
• Determines the x-coordinates of the two stationary points and determines the nature of one of them, or equivalent merit	3
• Obtains x-values of the stationary points, or equivalent merit	2
• Differentiates and sets $\frac{dy}{dx} = 0$, or equivalent merit	1

Sample answer:

$$y = 2x^3 + 3x^2 - 12x + 7$$

$$\frac{dy}{dx} = 6x^2 + 6x - 12$$

$$= 6(x^2 + x - 2)$$

$$\frac{d^2y}{dx^2} = 6(2x + 1)$$

For stationary point, $\frac{dy}{dx} = 0$

$\therefore \quad x^2 + x - 2 = 0$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(-2)}}{2}$$

$$= \frac{-1 \pm 3}{2}$$

$$= 1 \quad \text{or} \quad -2$$

When $x = 1$, $y = 2(1)^3 + 3(1)^2 - 12(1) + 7$

$$= 0$$

$$\frac{d^2y}{dx^2} = 6(2(1) + 1) = 18 > 0$$

$\therefore$ Concave up

$\therefore$ (1, 0) is a local minimum.

When $x = -2$, $y = 2(-2)^3 + 3(-2)^2 - 12(-2) + 7$

$$= 27$$

$$\frac{d^2y}{dx^2} = 6(2(-2) + 1) = -18 < 0$$

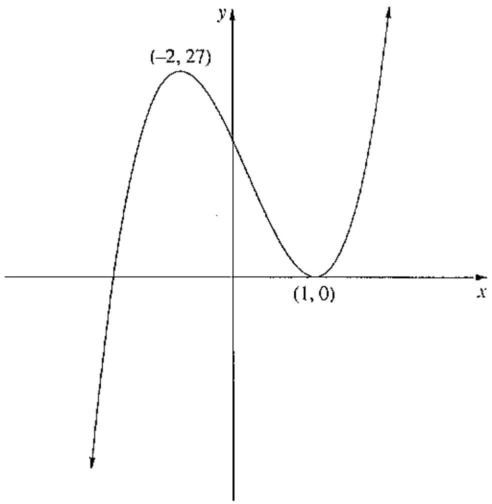
$\therefore$ Concave down

$\therefore$ (–2, 27) is a local maximum.

Question 13 (b) (ii)

Criteria	Marks
• Provides correct sketch	2
• Sketches a cubic curve, or equivalent merit	1

Sample answer:



Question 13 (b) (iii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

From the graph, $\frac{dy}{dx}$ is positive when $x > 1$ or $x < -2$

Question 13 (c)

Criteria	Marks
• Provides correct solution	2
• Obtains the quadratic in m, or equivalent merit	1

Sample answer:

$m = t^{\frac{1}{3}}, \quad m^2 = \left(t^{\frac{1}{3}}\right)^2 = t^{\frac{2}{3}}$

$t^{\frac{2}{3}} + t^{\frac{1}{3}} - 6 = 0$

$\therefore \quad m^2 + m - 6 = 0$

$(m-2)(m+3) = 0$

$\therefore \ m = 2 \quad \text{or} \quad m = -3$

$\therefore \ t^{\frac{1}{3}} = 2 \quad \text{or} \quad t^{\frac{1}{3}} = -3$

$t = 8 \qquad\qquad t = -27$

Question 13 (d)

Criteria	Marks
• Provides correct solution	3
• Provides correct primitive, or equivalent merit	2
• Attempts to integrate the expression, or equivalent merit	1

Sample answer:

$\dfrac{dV}{dt} = \dfrac{2t}{1+t^2}$ when $t = 0, \ V = 0$

$$V = \int_0^{10} \frac{2t}{1+t^2}\,dt$$

$$= \left[\ln(1+t^2)\right]_0^{10}$$

$$= \ln(101) - \ln 1$$

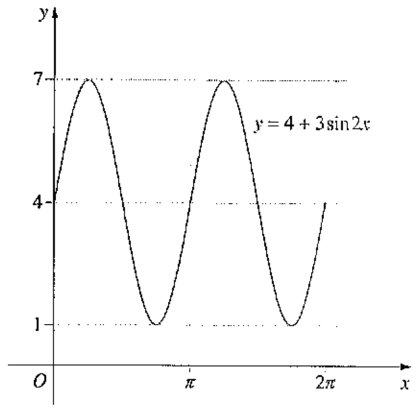
$$= \ln(101) \qquad (\doteqdot 4.615\ldots)$$

After 10 seconds the volume of water in the tank is $\ln 101$ litres.

Question 14 (a)

Criteria	Marks
• Provides correct sketch	3
• Indicates correct amplitude and period, or equivalent merit	2
• Indicates correct amplitude, or equivalent merit	1

Sample answer:



Question 14 (b) (i)

Criteria	Marks
• Provides correct answer	1

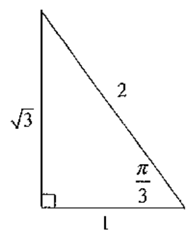
Sample answer:

$$\int_0^{\frac{\pi}{3}} \cos x\,dx$$

$$= \left[\sin x\right]_0^{\frac{\pi}{3}}$$

$$= \sin\left(\frac{\pi}{3}\right) - \sin(0)$$

$$= \frac{\sqrt{3}}{2}$$



Question 14 (b) (ii)

Criteria	Marks
• Provides correct solution	2
• Attempts to apply Simpson's rule, or equivalent merit	1

Sample answer:

$$\int_0^{\frac{\pi}{3}} \cos x\,dx = \frac{\frac{\pi}{3}-0}{6}\left[\cos(0)+4\cos\left(\frac{\pi}{6}\right)+\cos\frac{\pi}{3}\right]$$

$$=\frac{\pi}{18}\left[1+4\left(\frac{\sqrt{3}}{2}\right)+\frac{1}{2}\right]$$

$$=\frac{\pi}{18}\left[\frac{3}{2}+\frac{4\sqrt{3}}{2}\right]$$

$$=\frac{\pi}{18}\left[\frac{3+4\sqrt{3}}{2}\right]$$

Question 14 (b) (iii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\frac{\sqrt{3}}{\not{2}} \doteq \frac{\pi}{18}\left[\frac{3+4\sqrt{3}}{\not{2}}\right]$$

$$18\sqrt{3} \doteq \pi\left[3+4\sqrt{3}\right]$$

$$\pi \doteq \frac{18\sqrt{3}}{3+4\sqrt{3}}$$

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Question 14 (c) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$C(t) = Ae^{kt}$$

$$\frac{dC}{dt} = k \times Ae^{kt}$$

$$= kC, \text{ as required}$$

Question 14 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Recognises the significance of the half-life, or equivalent merit	1

Sample answer:

$t = 5730 \qquad C = \frac{1}{2}C_0$

$$C(0) = A$$

$$\frac{1}{2}C(0) = \frac{1}{2}A$$

$$\therefore \quad \frac{1}{2}\not{A} = \not{A}e^{5730k}$$

$$\ln\frac{1}{2} = 5730k$$

$$k = \ln\left(\frac{1}{2}\right) \div 5730$$

$$k = -0.00012$$

Question 14 (c) (iii)

Criteria	Marks
• Provides correct solution	2
• Obtains a correct exponential equation for t, or equivalent merit	1

Sample answer:

$C(t) = 0.9A \qquad t = ?$

$$0.9A = Ae^{kt}$$

$$\ln(0.9) = kt$$

$$t = \frac{\ln(0.9)}{k}$$

$$t = \frac{\ln(0.9)}{k}$$

$$= \frac{\ln(0.9)}{-0.00012\ldots}$$

$$\approx 870.9777\ldots$$

$$= 870 \text{ years}$$

or using $k = -0.00012$

$$t = 878.0042\ldots$$

$$t = 880 \text{ years}$$

Question 14 (d)

Criteria	Marks
• Provides correct solution	3
• Obtains an expression for the area involving k with integration complete, or equivalent merit	2
• Attempts to use integration to find the area of the shaded region, or equivalent merit	1

Sample answer:

$$\text{Area} = 2\int_0^1 \left(k\left(1-x^2\right) - 2k\left(x^2-1\right)\right)dx$$

$$= 2\int_0^1 \left(3k - 3kx^2\right)dx$$

$$= 6k\int_0^1 \left(1-x^2\right)dx$$

$$= 6k\left[x - \frac{1}{3}x^3\right]_0^1$$

$$= 4k$$

$$\therefore 4k = 8$$

$$k = 2$$

Question 15 (a)

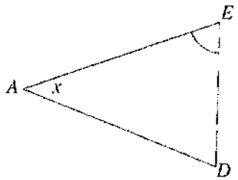
Criteria	Marks
• Provides correct solution	3
• Finds a correct equation for x, or equivalent merit	2
• Makes some progress	1

Sample answer:

$\triangle ABC$, $\triangle ACD$, $\triangle ADE$ are congruent (given)

So angles $\angle BAC$, $\angle CAD$, and $\angle DAE$, are equal (corresponding angles in congruent triangles)

$\angle AED$ is the base angle of an isosceles triangle.



$2 \times \angle AED + x = 180$ (angle sum of a triangle)

So $\angle AED = \dfrac{180 - x}{2}$

$$\angle AED + \angle EAB = 180 \quad (\text{cointerior angle} - AB \parallel ED)$$

$$\frac{180 - x}{2} + 3x = 180$$

$$180 - x + 6x = 360$$

$$5x = 180$$

$$x = 36$$

Question 15 (b) (i)

Criteria	Marks
• Provides correct solution	2
• Obtains $M_1 = X(1.0035) - 2500$, or equivalent merit	1

Sample answer:

$$M_1 = X \times \left(1 + \frac{0.042}{12}\right) - 2500$$

$$= X(1.0035) - 2500$$

$$M_2 = \big((X(1.0035) - 2500) + X\big)1.0035 - 2500$$

$$= X(1.0035)^2 + X(1.0035) - 2500(1.0035) - 2500$$

$$= X\left(1.0035^2 + 1.0035\right) - 2500(1.0035 + 1)$$

Question 15 (b) (ii)

Criteria	Marks
• Provides correct solution	3
• Obtains an expression for M_{48} with at least one series summed, or equivalent merit	2
• Obtains an expression for M_{48}, or equivalent merit	1

Sample answer:

At the end of 4 years = 48 months, $M_{48} = 80\,000$

$$X\left(1.0035^{48} + \cdots + 1.0035\right) - 2500\left(1.0035^{47} + \cdots + 1\right) = 80\,000$$

$$X\left[\frac{1.0035\left(1.0035^{48} - 1\right)}{0.0035}\right] - \frac{2500\left(1.0035^{48} - 1\right)}{0.0035} = 80\,000$$

$$X\left[\frac{1.0035\left(1.0035^{48} - 1\right)}{0.0035}\right] = 210\,421.2054$$

$$52.351X = 240\,421.2054$$

$$X = 4019.42$$

Question 15 (c) (i)

Criteria	Marks
• Provides correct solution	2
• Finds correct primitive, or equivalent merit	1

Sample answer:

Particle 1 when $t = 0$ $x_1 = 0$ $v_1 = 3$ and $a_1 = 6t + e^{-t}$

v_1 is a primitive of a_1

$$v_1 = \frac{6t^2}{2} + -e^{-t} + k$$
$$= 3t^2 - e^{-t} + k$$

When $t = 0$ $v_1 = 3$

$$3 = 3 \times 0 - e^0 + k$$
$$= -1 + k$$
$$4 = k$$

So $v_1 = 3t^2 + 4 - e^{-t}$

Question 15 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Equates velocities, or equivalent merit	1

Sample answer:

$$v_1 = v_2$$
$$3t^2 + 4 - e^{-t} = 6t + 1 - e^{-t}$$
$$3t^2 - 6t + 3 = 0$$
$$3\left(t^2 - 2t + 1\right) = 0$$
$$3(t-1)^2 = 0$$
$$t = 1$$

Question 15 (c) (iii)

Criteria	Marks
• Provides correct solution	3
• Obtains cubic equation in t, or equivalent merit	2
• Finds x_1 or x_2, or equivalent merit	1

Sample answer:

Show that the particles do not meet for $t > 0$

$$v_1 = 3t^2 + 4 - e^{-t} \quad \text{so}$$
$$x_1 = \frac{3t^3}{3} + 4t + e^{-t} + C$$

When $t = 0$ $x_1 = 0$

$$0 = 0 + 0 + 1 + C$$

So $C = -1$

$$x_1 = t^3 + 4t + e^{-t} - 1$$
$$v_2 = 6t + 1 - e^{-t}$$
$$x_2 = \frac{6t^2}{2} + t + e^{-t} + k$$
$$= 3t^2 + t + e^{-t} + k$$

When $t = 0$ $x_2 = 0$

$$3 \times 0^2 + 0 + 1 + k = 0$$
$$k = -1$$

So $x_2 = 3t^2 + t + e^{-t} - 1$

If particles meet $x_1 = x_2$

So $t^3 + 4t + e^{-t} - 1 = 3t^2 + t + e^{-t} - 1$

$$t^3 - 3t^2 + 4t - t = 0$$
$$t^3 - 3t^2 + 3t = 0$$
$$t\left(t^2 - 3t + 3\right) = 0$$

$t = 0$ (at origin)

or $t = \dfrac{3 \pm \sqrt{9 - 4 \times 3}}{2}$

$9 - 12 < 0$ so quadratic has no solutions.

$\therefore$ particles do not meet for $t > 0$

(alternative method)

Since both particles start at the origin,

$$v_1 = 3t^2 + 3 + \left(1 - e^{-t}\right)$$
$$= 3t^2 + 3 + \left(v_2 - 6t\right)$$
$$= v_2 + 3\left(t^2 - 2t + 1\right)$$
$$= v_2 + 3(t-1)^2$$

So P_1 is never slower than P_2.

They start together. P_1 starts faster than P_2 and never gets slower.

$\therefore$ P_1 will always be ahead of P_2

$\therefore$ The particles never meet ($t > 0$).

Question 16 (a) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

By Pythagoras' theorem

$AE = \sqrt{x^2 + 25}$

$DE = 9 - x$, so

$BE = \sqrt{49 + (9-x)^2}$

$\therefore \quad L = \sqrt{x^2+25} + \sqrt{49+(9-x)^2}$

Question 16 (a) (ii)

Criteria	Marks
• Provides correct solution	3
• Obtains $\dfrac{x}{\sqrt{x^2+25}} = \dfrac{9-x}{\sqrt{49+(9-x)^2}}$, or equivalent merit	2
• Correctly differentiates $\sqrt{x^2+25}$, or equivalent merit	1

Sample answer:

$$\frac{dL}{dx} = \frac{x}{\sqrt{x^2+25}} - \frac{(9-x)}{\sqrt{49+(9-x)^2}}$$

$$\frac{dL}{dx} = 0 \quad \Rightarrow \quad \frac{x}{\sqrt{x^2+25}} = \frac{9-x}{\sqrt{49+(9-x)^2}}$$

$\Rightarrow \quad \sin\alpha = \sin\beta$

since in $\triangle ACE$, $\sin\alpha = \dfrac{CE}{AE}$

$$= \frac{x}{\sqrt{25+x^2}}$$

in $\triangle BDE$, $\sin\beta = \dfrac{ED}{EB}$

$$= \frac{9-x}{\sqrt{(9-x)^2+49}}$$

Question 16 (a) (iii)

Criteria	Marks
• Provides correct solution	2
• Makes some progress	1

Sample answer:

$\sin\alpha = \sin\beta \quad \Rightarrow \quad \alpha = \beta \quad$ (since α, β acute)

$\therefore \quad \triangle ACE \,|||\, \triangle BDE$ (equiangular)

$$\therefore \quad \frac{x}{5} = \frac{9-x}{7}$$

$$\Rightarrow \quad 7x = 45 - 5x$$

$$x = \frac{45}{12}$$

$$= 3\frac{3}{4}$$

Question 16 (a) (iv)

Criteria	Marks
• Provides correct solution	1

Sample answer:

From parts (ii) and (iii) there is only one value of x for which $\dfrac{dL}{dx} = 0$ for $0 < x < 9$

For $x = 0 \qquad \dfrac{dL}{dx} = 0 - \dfrac{9}{\sqrt{130}} < 0$

For $x = 9 \qquad \dfrac{dL}{dx} = \dfrac{9}{\sqrt{106}} > 0$

Hence, since $x = 3\frac{3}{4}$ lies between $x = 0$ and $x = 9$ then this must be a minimum.

Question 16 (b)

Criteria	Marks
• Provides correct solution	3
• Obtains $-1 < 1 - \frac{a}{2} < 1$, or equivalent merit	2
• Recognises $\frac{a}{1-r} = 2$, or equivalent merit	1

Sample answer:

$S_\infty = \frac{a}{1-r} = 2$

$\therefore \quad \frac{a}{2} = 1 - r$

$\therefore \quad r = 1 - \frac{a}{2}$

Now $|r| < 1$ so $\left|1 - \frac{a}{2}\right| < 1$

$-1 < 1 - \frac{a}{2} < 1$

$-2 < 2 - a < 2$

$2 > a - 2 > -2$

$4 > a > 0$

$\therefore \quad 0 < a < 4$

Question 16 (c) (i)

Criteria	Marks
• Provides correct solution	3
• Shows $BD = DE$, or equivalent merit	2
• Shows $\triangle BDM$ is similar to $\triangle BEC$, or equivalent merit	1

Sample answer:

Since $BM = MC$ (given)
then $BD = DE$ (equal intercept)

$\therefore \quad BD = DA + AE$
and $BD = DA + AC$ (given)

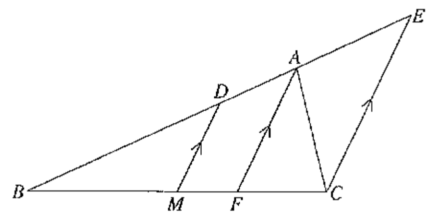
ie $DA + AE = DA + AC$
$\therefore \quad AE = AC$

Hence, $\triangle ACE$ is isosceles (two equal sides)

Question 16 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Shows $\angle FAC = \angle ACE$, or equivalent merit	1

Sample answer:



Since $AF \parallel DM$ and
$EC \parallel DM$ then
$AF \parallel EC$

$\angle BAF = \angle AEC$ (corresponding angles, parallel lines $AF \parallel EC$)
$\angle AEC = \angle ACE$ (base angles of isosceles triangle ACE)
$\angle ACE = \angle CAF$ (alternate angles, parallel lines AF and EC)

So $\angle BAF = \angle CAF$, ie AF bisects $\angle BAC$