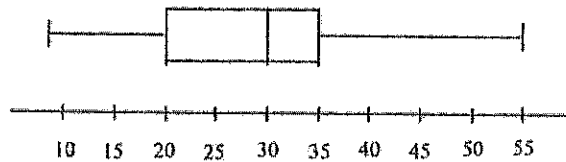# 2017 Higher School Certificate Examination Paper

## Mathematics General 2

Section I

25 marks   **Attempt Questions 1–25**   **Allow about 35 minutes for this section**

1   The box-and-whisker plot for a set of data is shown.

What is the median of this set of data?

(A)   15   (B)   20   (C)   30   (D)   35

2   A car is travelling at 95 km/h.

How far will it travel in 2 hours and 30 minutes?

(A)   38 km   (B)   41.3 km   (C)   218.5 km   (D)   237.5 km

3   The graph shows the relationship between infant mortality rate (deaths per 1000 live births) and life expectancy at birth (in years) for different countries.
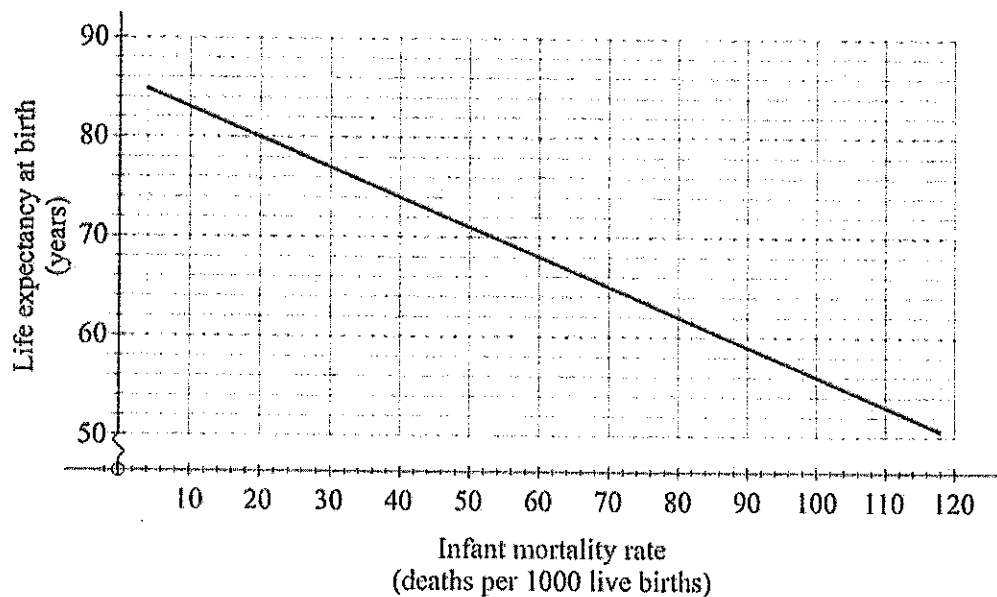
What is the life expectancy at birth in a country which has an infant mortality rate of 60?

(A)   68 years   (B)   69 years   (C)   86 years   (D)   88 years

4   A factory's quality control department has tested every 50th item produced for possible defects.

What type of sampling has been used?

(A)   Random   (B)   Stratified   (C)   Systematic   (D)   Quantitative

5   In a survey of 200 randomly selected Year 12 students it was found that 180 use social media.

Based on this survey, approximately how many of 75 000 Year 12 students would be expected to use social media?

(A)   60 000   (B)   67 500   (C)   74 980   (D)   75 000

6   Tom earns a weekly wage of $1025. He also receives an additional allowance of $87.50 per day when handling toxic substances.

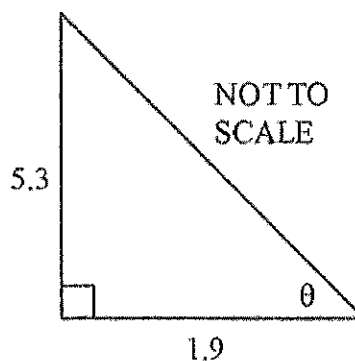What is Tom's income in a fortnight in which he handles toxic substances on 5 separate days?

(A) $1112.50   (B) $1462.50   (C) $2225.00   (D) $2487.50

7   It is given that $I = \dfrac{3}{2}MR^2$.

What is the value of I when $M = 26.55$ and $R = 3.07$, correct to two decimal places?

(A)   375.35   (B)   3246.08   (C)   9965.45   (D)   14 948.18

8   The diagram shows a right-angled triangle.

What is the value of θ, to the nearest minute?

(A)   70°16'   (B)   70°17'   (C)   70°27'   (D)   70°28'

9   What is the value of x in the equation $\dfrac{5-x}{3} = 6$?

(A)   −13   (B)   −3   (C)   3   (D)   13

10 A single amount of $10 000 is invested for 4 years, earning interest at the rate of 3% per annum, compounded monthly.

Which expression will give the future value of the investment?

(A) $10\,000 \times (1 + 0.03)^4$

(B) $10\,000 \times (1 + 0.03)^{48}$

(C) $10\,000 \times \left(1 + \dfrac{0.03}{12}\right)^4$

(D) $10\,000 \times \left(1 + \dfrac{0.03}{12}\right)^{48}$

11 A new car was bought for $19 900 and one year later its value had depreciated to $16 300.

What is the approximate depreciation, expressed as a percentage of the purchase price?

(A) 18%   (B) 22%   (C) 78%   (D) 82%

12 Which of the data sets graphed below has the largest positive correlation coefficient value?
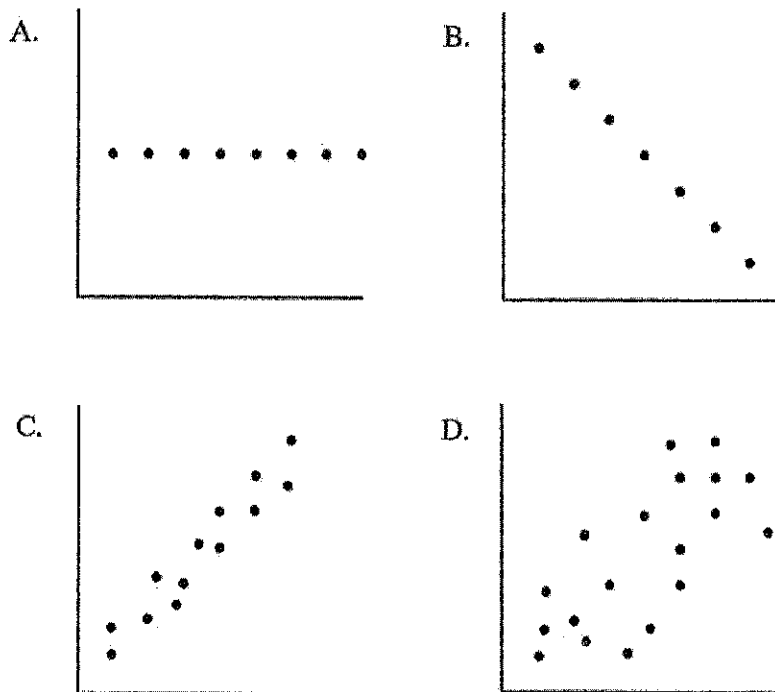
13 The heights of Year 12 girls are normally distributed with a mean of 165 cm and a standard deviation of 5.5 cm.

What is the z-score for a height of 154 cm?

(A) -2 (B) -0.5 (C) 0.5   (D) 2

14   Kate is comparing two different models of car. Car *A* uses fuel at the rate of 9 L/100 km. Car *B* uses 3.5 L/100 km.

Suppose Kate plans on driving 8000 km in the next year.

How much less fuel will she use driving car *B* instead of car *A*?

(A)   280 L   (B)   440 L   (C)   720 L   (D)   1000 L

15   The faces on a twenty-sided die are labelled $0.05, $0.10, $0.15, ... , $1.00.

The die is rolled once.

What is the probability that the amount showing on the upper face is more than 50 cents but less than 80 cents?

(A) $\dfrac{1}{4}$ (B) $\dfrac{3}{10}$ (C) $\dfrac{7}{20}$ (D) $\dfrac{1}{2}$

16   The benchmark for annual greenhouse gas emissions from the residential sector is 3292 kg of carbon dioxide per person per year.

A new building, planned to house 6 people, has been designed to achieve a 25% reduction on this benchmark.

What is the maximum amount of carbon dioxide per year, to the nearest kilogram, that this building is designed to emit when fully occupied?
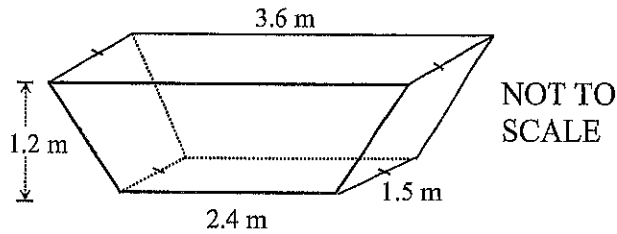
(A)   823 kg   (B)   2469 kg   (C)   4938 kg   (D)   14 814 kg

17   The graph of the line with equation $y = 6 - 2x$ is shown.
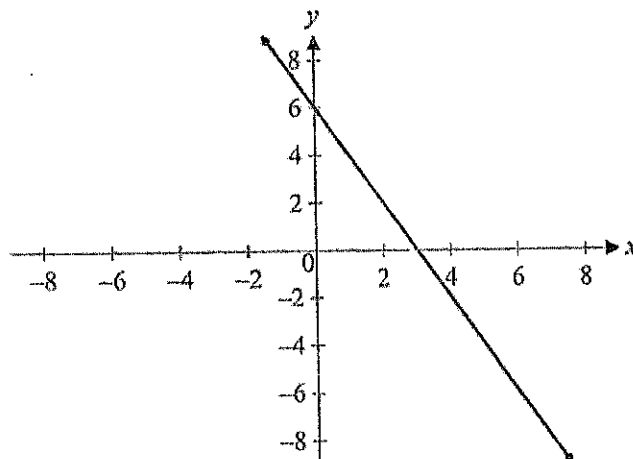
When the graph of the line with equation $y = x + 3$ is also drawn on this number plane, what will be the point of intersection of the two lines?

(A) $(0, 6)$ (B) $(1, 4)$ (C) $(2, 2)$ (D) $(3, 0)$

18 A skip bin is in the shape of a trapezoidal prism, with dimensions as shown.

3.6 m

NOT TO
SCALE

1.2 m

1.5 m

2.4 m

What is the volume of the skip bin?

(A)   5.4 m³   (B)   7.776 m³   (C)   10.8 m³   (D)   15.552 m³

19 Young's formula, shown below, is used to calculate the dosage of medication for children aged 1–12 years based on the adult dosage.
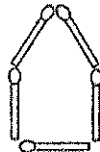
$$D = \frac{yA}{y+12}$$

where D = dosage for children aged 1-12 years
   y = age of child (in years)
   A = adult dosage
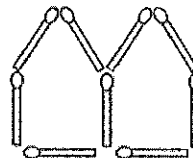
A child's dosage is calculated to be 20 mg, based on an adult dosage of 40 mg.
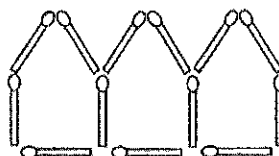
How old is the child in years?

(A)   6   (B)   8   (C)   10   (D)   12

20 A pentagon is created using matches.

By adding more matches, a row of two pentagons is formed.

Continuing to add matches, a row of three pentagons can be formed.

Continuing this pattern, what is the maximum number of complete pentagons that can be formed if 100 matches in total are available?

(A)   25   (B)   24   (C)   21   (D)   20

21 The length of a netball court is measured to be 30.50 metres, correct to the nearest centimetre.

What is the lower limit for the length of the netball court?

(A)   30.45 m   (B)   30.49 m   (C)   30.495 m   (D)   30.499 m

22 A concrete water pipe is manufactured in the shape of an annular cylinder. The dimensions are shown in the diagrams.

Cross-section
(annulus)

0.35m   0.1 m

Water pipe

NOT TO SCALE

2.8 m
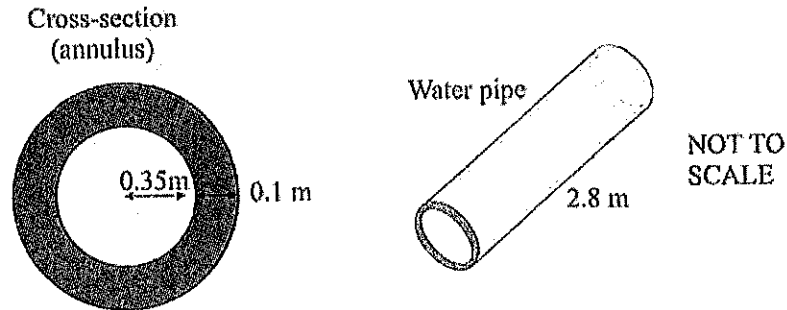
What is the approximate volume of concrete needed to make the water pipe?

(A)   0.06 m³   (B)   0.09 m³   (C)   0.70 m³   (D)   0.99 m³

23 How many bits are there in 2 terabytes?

(A) 2^{40} (B) 2^{41} (C) 2^{43} (D) 2^{44}

24 A deck of 52 playing cards contains 12 picture cards. Two cards from the deck are drawn at random and placed on a table.

What is the probability, correct to four decimal places, that exactly one picture card is on the table?

(A)   0.0498   (B)   0.1810   (C)   0.3550   (D)   0.3620

25 In the circle, centre O, the area of the quadrant is 100 cm².

O $A = 100$ cm² l
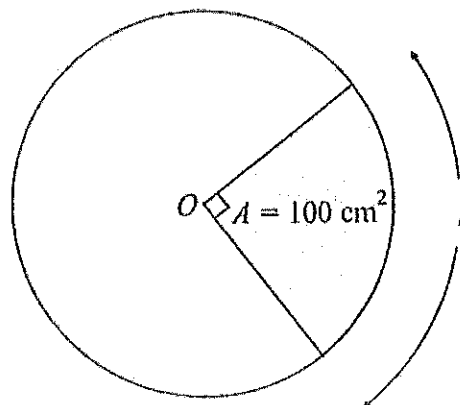
What is the arc length l, correct to one decimal place?

(A)   8.9 cm   (B)   11.3 cm   (C)   17.7 cm   (D)   25.1 cm

Section II

75 marks   **Attempt Questions 26-30**   **Allow about 1 hour and 55 minutes for this section**
Your responses should include relevant mathematical reasoning and/or calculations.

Question 26 (15 marks)

(a)   Electricity costs $0.27 per kWh.   **1**

How much does 20 kWh cost?

(b)   Toby's mobile phone plan costs $20 per month, plus the cost of all calls. Calls are   **2**
charged at the rate of 70 cents per 30 seconds, or part thereof. There is also a call
connection fee of 50c per call.

Here is a record of all his calls in July.

Date	Call duration
5 July	20 seconds
12 July	40 seconds
23 July	2 minutes 15 seconds

How much is Toby's mobile phone bill for July?

(c)   A farmer needed to estimate the number of goats on his property. He tagged 80 of his   **2**
goats. Later, he collected a random sample of 45 goats and found that 16 of these had
tags.

Estimate the number of goats the farmer has on his property.

(d)   A sewer pipe needs to be placed into the ground so that it has a 2° angle of depression.   **2**
The length of the pipe is 15 000 mm.
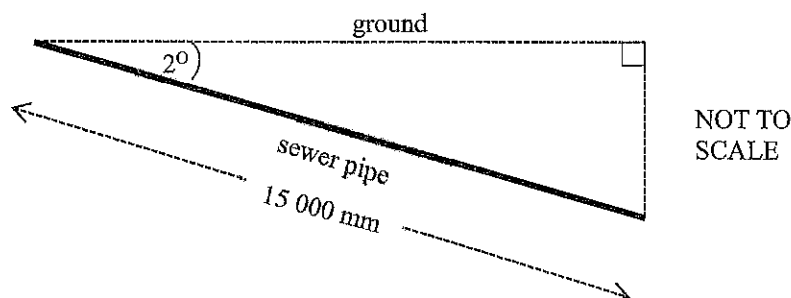
How much deeper should one end of the pipe be compared to the other end?
Answer to the nearest mm.

(e)   Sam purchased 500 company shares at $3.20 per share. Brokerage fees were 1.5% of   **3**
the purchase price.

Sam is paid a dividend of 26 cents per share, then immediately sells the shares for $4.80
each.

If he pays no further brokerage fees, what is Sam's total profit?

(f) The area chart shows the number of goals scored by three hockey teams, *A*, *B* and *C*, in the first 4 rounds.
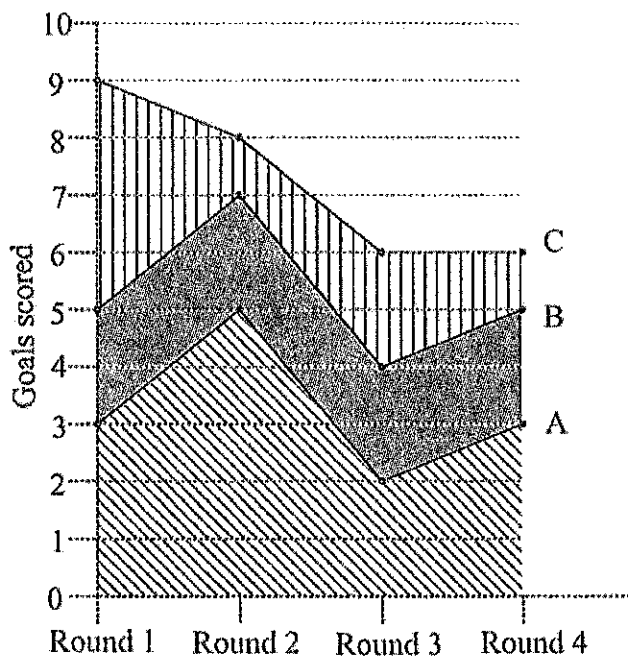
(i) How many goals were scored by team *C* in round 1?   **1**

(ii) In which round did all three teams score the same number of goals?   **1**

(g) Rachel bought a motorcycle advertised for $7990. She paid a $500 deposit and took out a flat-rate loan to repay the balance. Simple interest was charged at a rate of 7% per annum on the amount borrowed. She repaid the loan over 2 years, making equal weekly repayments.   **3**

Calculate the weekly repayment.

Question 27 (15 marks)

(a) Jamal surveyed eight households in his street. He asked them how many kilolitres (kL) of water they used in the last year. Here are the results.

$$220, 105, 101, 450, 37, 338, 151, 205$$

(i) Calculate the mean of this set of data.   **1**

(ii) What is the standard deviation of this set of data, correct to one decimal place?   **1**

(b) How many 20 megabyte files can fit on a 3 terabyte external hard disc?   **2**

(c) A table of future value interest factors for an annuity of $1 is shown.

Table of future value interest factors					
Period	Interest rate per period				
	1%	2%	3%	4%	5%
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019

An annuity involves contributions of $12 000 per annum for 5 years. The interest rate is 4% per annum, compounded annually.

(i) Calculate the future value of this annuity.   1

(ii) Calculate the interest earned on this annuity.   1

(d) Island *A* and island *B* are both on the equator. Island *B* is west of island *A*. The longitude of island *A* is 5°E and the angle at the centre of Earth (*O*), between *A* and *B*, is 30°.

NOT TO SCALE
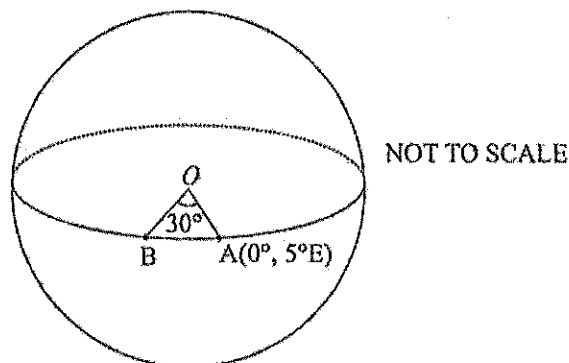
(i) What is the longitude of island B?   1

(ii) What time is it on island B when it is 10 am on island A?   1

(iii) A ship leaves island A and travels west along the equator to island B. It travels at a constant speed of 40 km/h.   3

How long will the ship take to arrive at island B? Give your answer in days and hours to the nearest hour.

(e) Rhys is drinking low alcohol beer at a party over a five-hour period. He reads on the label of the low alcohol beer bottle that it is equivalent to 0.8 of a standard drink.   4

Rhys weighs 90 kg.

What is the maximum number of complete bottles of the low alcohol beer he can drink to remain under a Blood Alcohol Content (BAC) of 0.05?

Question 28 (15 marks)

(a)   Temperature can be measured in degrees Celsius (C) or degrees Fahrenheit (F).
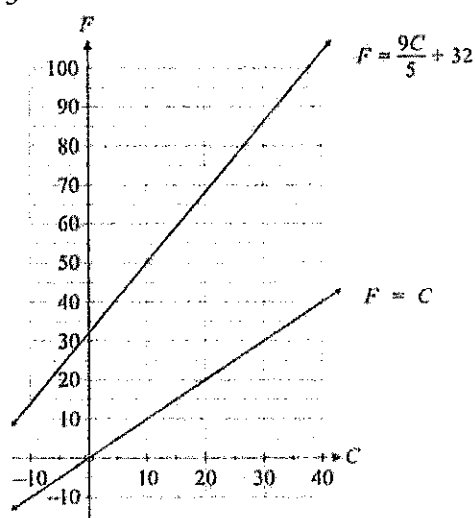
The two temperature scales are related by the equation $F = \dfrac{9C}{5} + 32$.

    (i)   Calculate the temperature in degrees Fahrenheit when it is -20 degrees Celsius.   **1**

    (ii)   Solve the following equations simultaneously, using either the substitution   **2**
method or the elimination method.

$$F = \dfrac{9C}{5} + 32$$
$$F = C$$

    (iii)   The graphs of $F = \dfrac{9C}{5} + 32$ and $F = C$ are shown below.   **1**

What does the result from part (ii) mean in the context of the graph?

(b)   Five people are in a team. Two of them are selected at random to attend a competition.

    (i)   How many different groups of two can be selected?   **1**

    (ii)   If Mary is one of the five people in the team, what is the probability that she is   **1**
selected to attend the competition?

(c)   Michelle borrows $100 000. The interest rate charged is 12% per annum compounded   **3**
monthly. The monthly payment is $1029 and the first repayment is made after one
month.

What is the amount outstanding immediately after the SECOND monthly repayment is
made?

(d)   Make y the subject of the equation $x = \sqrt{yp - 1}$.   **2**

(e)   A movie theatre has 200 seats. Each ticket currently costs $8.

The theatre owners are currently selling all 200 tickets for each session. They decide to increase the price of tickets to see if they can increase the income earned from each movie session.

It is assumed that for each one dollar increase in ticket price, there will be 10 fewer tickets sold.

A graph showing the relationship between an increase in ticket price and the income is shown below.
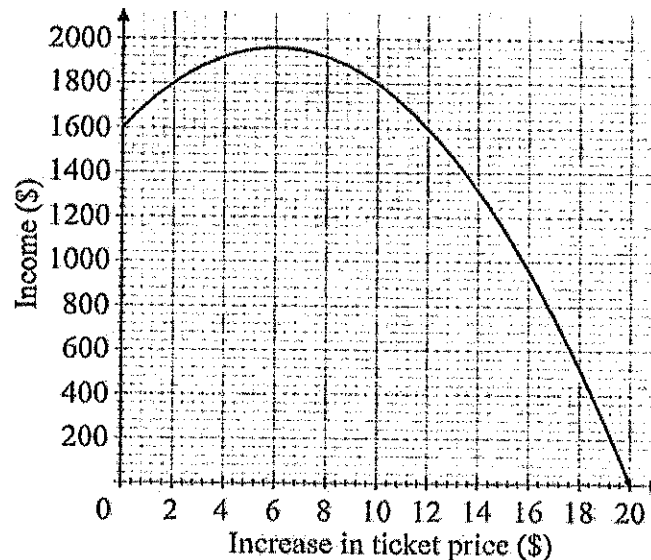
(i)   What ticket price should be charged to maximise the income from a movie session?   **1**

(ii)   What is the number of tickets sold when the income is maximised?   **1**

(iii)   The cost to the theatre owners of running each session is $500 plus $2 per ticket sold.   **2**

Calculate the profit earned by the theatre owners when the income earned from a session is maximised.

Question 29 (15 marks)

(a)   A new 200-metre long dam is to be built.

The plan for the new dam shows evenly spaced cross-sectional areas.
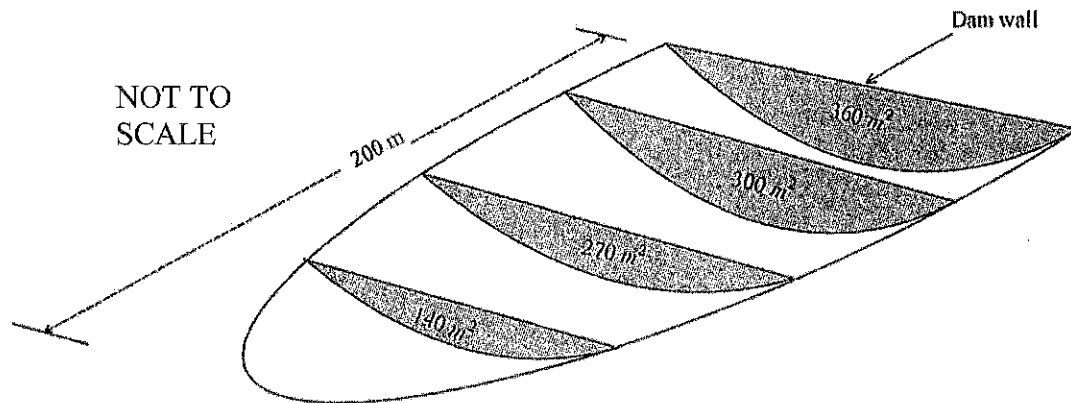
(i)   Using TWO applications of Simpson's rule, show that the volume of the dam   **2**
        approximately 44 333 m^3.

(ii)   It is known that the catchment area for this dam is 2 km^2.   **2**

        Calculate how much rainfall is needed, to the nearest mm, to fill the dam.

(b)   Sabrina's taxable income is $86 725 in a particular year.   **3**

The table below is used to calculate her tax payable. In addition, she pays the
Medicare levy, which is 2% of her taxable income.

| Taxable income ($)	Tax payable
$0 – $18 200	Nil
$18 201 – $37 000	19c for each $1 over $18 200
$37 001 – $87 000	$3572 plus 32.5c for each $1 over $37 000
$87 001 – $180 000	$19 822 plus 37c for each $1 over $87 000
$180 001 and over	$54 232 plus 45c for each $1 over $180 000

Calculate Sabrina's net income in that year.

(c) A group of Year 12 students was surveyed. The students were asked whether they live in the city or the country. They were also asked if they have ever waterskied. The results are recorded in the table.

	Have waterskied	Have never waterskied
Live in the city	150	2500
Live in the country	70	800

    (i) A person is selected at random from the group surveyed. **2**

        Calculate the probability that the person lives in the city and has never waterskied.

    (ii) A newspaper article claimed that Year 12 students who live in the country are more likely to have waterskied than those who live in the city. **2**

        Is this true, based on the survey results? Justify your answer with relevant calculations.

(d) All the students in a class of 30 did a test.

The marks, out of 10, are shown in the dot plot.
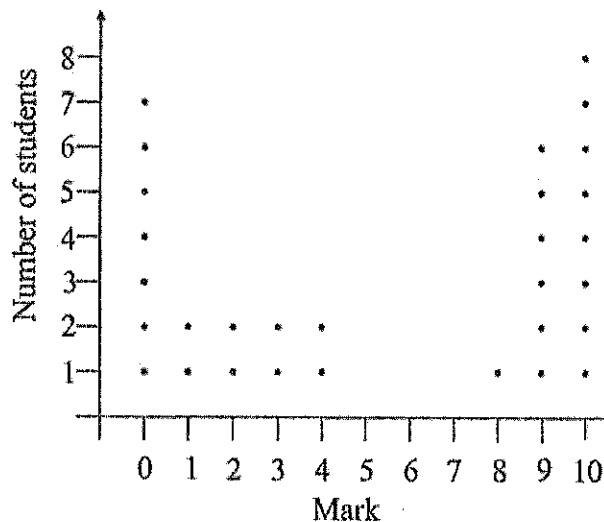
    (i) Find the median test mark. **1**

    (ii) The mean test mark is 5.4. The standard deviation of the test marks is 4.22. **2**

        Using the dot plot, calculate the percentage of the marks which lie within one standard deviation of the mean.

    (iii) A student states that for any data set, 68% of the scores should lie within one standard deviation of the mean. **1**

        With reference to the dot plot, explain why the student's statement is NOT relevant in this context.

Question 30 (15 marks)

(a) A set of data has a lower quartile (Q_L) of 10 and an upper quartile (Q_U) of 16. **2**

What is the maximum possible range for this set of data if there are no outliers?

(b) The cost of a jewellery box varies directly with the cube of its height. **2**

A jewellery box with a height of 10 cm costs $50.

Calculate the cost of a jewellery box with a height of 12 cm.

(c) The diagram shows the location of three schools. School A is 5 km due north of school B, school C is 13 km from school B and $\angle ABC$ is 135°.
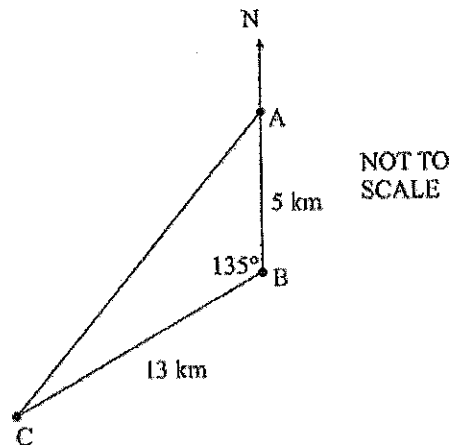
(i) Calculate the shortest distance from school A to school C, to the nearest kilometre. **2**

(ii) Determine the bearing of school C from school A, to the nearest degree. **3**

(d) In an investigation, students used different numbers of identical small solar panels to power model cars. The cars were then tested and their speed measured in km/h. The results are summarised in the table.

	Mean	Standard deviation
Number of solar panels (x)	2.9	0.8
Speed (y)	8.2	2

The equation of the least-squares line of best fit, relating the speed and the number of solar panels, has been calculated to be

$$y = 2.125x + 2.0375.$$

(i) What would be the speed of a car powered by 5 solar panels, based on this equation? **1**

(ii) Calculate the correlation coefficient, r, between the number of solar panels and the speed of a car. **2**

(e)   A solid is made up of a sphere sitting partially inside a cone.   **3**

The sphere, centre O, has a radius of 4 cm and sits 2 cm inside the cone. The solid has a
total height of 15 cm. The solid and its cross-section are shown.

Solid   Cross-section
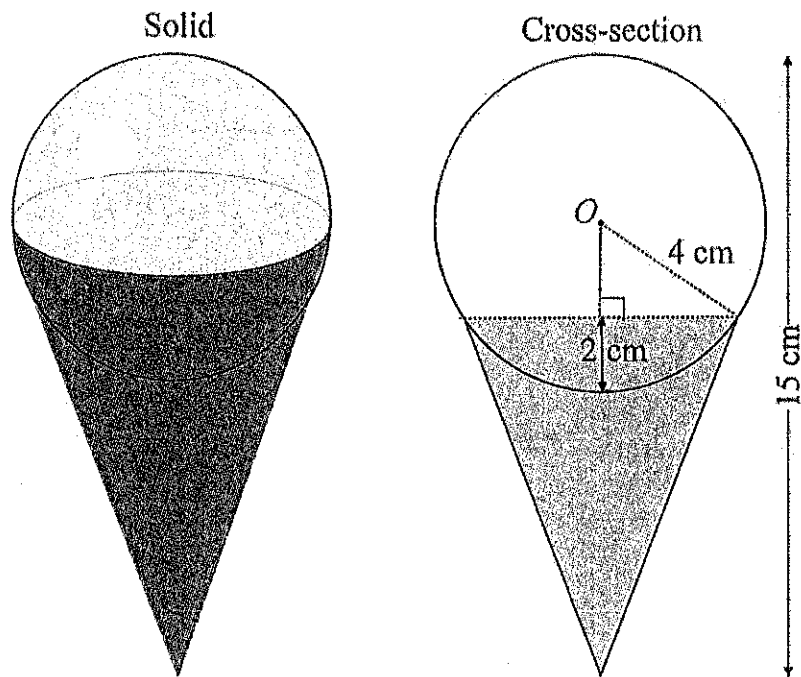
What is the volume of the cone, correct to the nearest cm^3?

End of paper

2017 Higher School Certificate Solutions Mathematics General 2
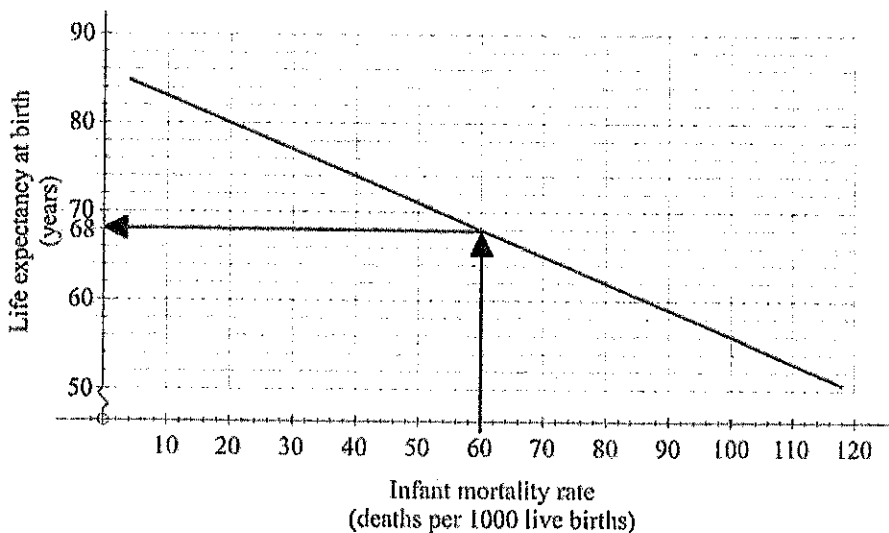
Section I
Multiple Choice Summary

1	C	2	D	3	A	4	C	5	B	6	D	7	A	8	B	9	A	10	D
11	A	12	C	13	A	14	B	15	A	16	D	17	B	18	A	19	D	20	B
21	C	22	C	23	D	24	D	25	C												

Multiple Choice Solutions

1. **C**

Median is indicated by the vertical line inside the box.

2. **D**

$d = s \times t$

$\quad = 95 \times 2.5$

$\quad = 237.5 \, \text{km}$

3. **A**

4. **C**

A systematic sample is one where one in every *XXX* is sampled, in this case one in every 50.

5. **B**

$\dfrac{180}{200} \times 75\,000 = 67\,500$

6. **D**

$$\text{Wage} \times 2 \text{ weeks} + \text{Allowance} \times 5 \text{ days} = 1025 \times 2 + 87.50 \times 5$$
$$= \$2487.50$$

7. **A**

$$I = \frac{3}{2} MR^2$$
$$= \frac{3}{2} \times 26.55 \times 3.07^2$$
$$= 375.3466425$$
$$\approx 375.35 \ (2 \ \text{d.p.})$$

8. **B**

$$\tan \theta = \frac{opposite}{adjacent}$$
$$\tan \theta = \frac{5.3}{1.9}$$
$$\theta = \tan^{-1}\left(\frac{5.3}{1.9}\right) = 70°16'\ 39.8''$$
$$\approx 70°17' \quad \text{(nearest minute)}.$$

9. **A**

$$\frac{5-x}{3} = 6 \qquad \}\times 3$$
$$5-x = 18 \qquad \}-5$$
$$-x = 13 \qquad \}\times -1$$
$$x = -13$$

10. **D**

$$FV = PV(1+r)^n \qquad \text{(from Formulae and Data Sheet)}$$
$$= 10000\left(1+\frac{0.03}{12}\right)^{4 \times 12}$$
$$= 10000\left(1+\frac{0.03}{12}\right)^{48}$$

11. **A**

$$\frac{\text{purchase price} - \text{depreciated value}}{\text{purchase price}} = \frac{19900-16300}{19900} \times 100$$
$$= 18\%$$

12. **C**

The dots need to be as close as possible to a straight line, with a positive gradient.

13. **A**

$$z = \frac{x - \bar{x}}{s} \qquad \text{(from Formulae and Data Sheet)}$$

$$= \frac{154 - 165}{5.5}$$

$$= -2$$

14. **B**

Car A: $\qquad \dfrac{8000}{100} \times 9 = 720 \text{ L}$

Car B: $\qquad \dfrac{8000}{100} \times 3.5 = 280 \text{ L}$

Difference in fuel: $\quad 720 - 280 = 440 \text{ L}$

15. **A**

Could land on: $0.55, $0.60, $0.65, $0.70, $0.75
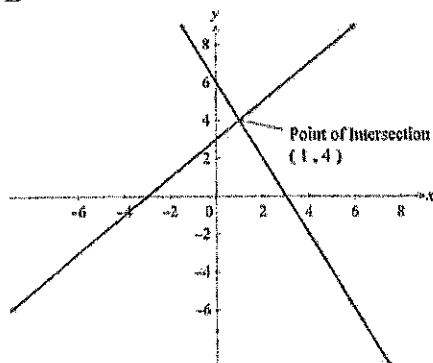
Which is 5 options out of a total of 20.

$$\therefore P(\$0.50 < X < \$0.80) = \frac{5}{20} = \frac{1}{4}$$

16. **D**

$$3292 \times 6 \times (1 - 0.25) = 3292 \times 6 \times 0.75$$

$$= 14814 \text{ kg}$$

17. **B**

Point of intersection (1, 4)

18. **A**

$$\text{Area of Trapezium} = \frac{h}{2}(a + b) \qquad \text{(from Formulae and Data Sheet)}$$

$$= \frac{1.2}{2}(2.4 + 3.6) = 3.6$$

$$\text{Volume} = \text{Area} \times \text{Depth} = 3.6 \times 1.5$$

$$= 5.4 \text{ m}^3$$

19.   **D**

$$D = \frac{yA}{y+12}$$

$$20 = \frac{y \times 40}{y+12} \quad \left.\right\} \times (y+12)$$

$$20y + 240 = 40y \qquad \left.\right\} - 20y$$

$$240 = 20y \qquad \left.\right\} \div 20$$

$$y = 12$$

20.   **B**

Pentagon *(P)*	1	2	3	?
Matches *(M)*	5	9	13	100

$$M = 4P + 1$$

$$100 = 4P + 1 \qquad \left.\right\} - 1$$

$$99 = 4P \qquad \left.\right\} \div 4$$

$$P = 24.75$$

$$\therefore P = 24 \text{ whole pentagons}$$

21.   **C**

Limit of accuracy $= \pm 0.5$ unit of measurement

$0.5 \times 0.01 = 0.005$

Therefore, lower limit is $30.50 - 0.005 = 30.495$ m

22.   **C**

$$V = Ah$$

$$= \pi(R^2 - r^2)h \qquad \text{(from Formulae and Data Sheet)}$$

$$= \pi\left[(0.35 + 0.1)^2 - 0.35^2\right] \times 2.8$$

$$= 0.703716754...$$

$$\approx 0.70 \text{ m}^3$$

23.   **D**

$$8 \times 2^{40} \times 2 = 2^3 \times 2^{40} \times 2^1 \qquad \text{(from Formulae and Data Sheet)}$$

$$= 2^{44}$$

24.   **D**

$$\frac{12}{52} \times \frac{40}{51} \times 2 = 0.3620$$

25. C

$$A = \frac{\theta}{360}\pi r^2 \quad \text{(from Formulae and Data Sheet)}$$

$$100 = \frac{90}{360}\pi r^2 = \frac{1}{4}\pi r^2 \quad \}\times 4$$

$$400 = \pi r^2 \quad \}\div \pi$$

$$\frac{400}{\pi} = r^2 \quad \}\sqrt{}$$

$$r = \sqrt{\frac{400}{\pi}}$$

$$l = \frac{\theta}{360}2\pi r \quad \text{(from Formulae and Data Sheet)}$$

$$= \frac{90}{360}\times 2\pi r$$

$$= \frac{1}{4}\times 2 \times \pi \times \sqrt{\frac{400}{\pi}}$$

$$= 17.72453851...$$

$$\approx 17.7 \text{ cm (1 d.p.)}$$

Section II

Question 26

(a) $\text{Cost} = 0.27 \times 20$

$= \$5.40$

(b) $\text{Cost} = 20 + (0.50 + 0.70) + (0.50 + 0.70 \times 2) + (0.50 + 0.70 \times 5)$

$= \$27.10$

(c)   Let g be the number of goats on the property.

$$\frac{g}{80} = \frac{45}{16} \quad \}\times 80$$

$$g = \frac{45}{16}\times 80$$

$$= 225$$

(d)   Let d be the depth of the pipe

$$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\sin 2° = \frac{d}{15000} \quad \}\times 15000$$

$$d = 15000 \times \sin 2°$$

$$= 523.4924505$$

$$\approx 523 \text{ mm} \quad \text{(nearest mm)}$$

(e) $\text{Cost} = 500 \times 3.20 + \dfrac{1.5}{100} \times 500 \times 3.20$

$\quad = \$1624$

$\text{Dividend} = 0.26 \times 500$

$\quad = \$130$

$\text{Sale} = 500 \times 4.80$

$\quad = \$2400$

$\text{Profit} = \text{Dividend} + \text{Sale} - \text{Cost}$

$\quad = 130 + 2400 - 1624$

$\quad = \$906$

(f)   (i)   Round 1 $= 9 - 5$

$\qquad\qquad\quad = 4 \text{ goals}$

       (ii)

	Team A	Team B	Team C
Round 1:	Team A = 3,	Team B = 2,	Team C = 4
Round 2:	Team A = 5,	Team B = 2,	Team C = 1
Round 3:	Team A = **2**,	Team B = **2**,	Team C = **2**
Round 4:	Team A = 3,	Team B = 2,	Team C = 1

All three teams scored the same number of goals (2) in Round 3.

(g)   Cost $= \$7990$

Amount Borrowed $= 7990 - 500$

$\qquad\qquad\qquad\quad = \$7490$

$I = PRN$ (from Formulae and Data Sheet)

$\quad = 7490 \times 0.07 \times 2$

$\quad = \$1048.60$

Amount to Repay $= 7490 + 1048.60$

$\qquad\qquad\qquad\quad = \$8538.60$

Weekly Repayments $= 8538.60 \div (52 \times 2)$

$\qquad\qquad\qquad\qquad = 82.10192308$

$\qquad\qquad\qquad\qquad \approx \$82.10$

Question 27

(a)   (i) $\bar{x} = \dfrac{\text{sum of scores}}{\text{number of scores}}$ (from Formulae and Data Sheet)

$\qquad\qquad = \dfrac{220 + 105 + 101 + 450 + 37 + 338 + 151 + 205}{8}$

$\qquad\qquad = 200.875 \text{ kL}$

       (ii)   Using the statistics mode on the calculator:

$\qquad\qquad \sigma_x = 127.3572117...$
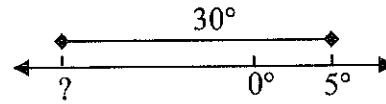
$\qquad\qquad\qquad \approx 127.4 \text{ kL (1 d.p.)}$

(b) 3 terabytes $= 3 \times 1024 \times 1024$ (from Formulae and Data Sheet)

$= 3\,145\,728$ megabytes

$$\text{Number of files} = \frac{3\,145\,728}{20}$$

$= 157\,286.4$

$= 157\,286$ complete files

(c) (i) $FV = 12\,000 \times 5.4163$

$= \$64\,995.60$

  (ii) $I = 64\,995.60 - 5 \times 12\,000$

$= \$4\,995.60$

(d) (i) $A(0°, 5°E)$ $B(0, ?)$
Using number line:
Longitude $= 5 - 30$

$= -25$

$= 25°W$

(ii) From the Formula and Data Sheet: $15°$ longitude $= 1$ hour time difference

$30° = 2$ hours

Time on Island B $= 10 - 2$

$= 8$ am

(iii) $d = \dfrac{\theta}{360} \times 2\pi r$ (from Formulae and Data Sheet)

$= \dfrac{30}{360} \times 2 \times \pi \times 6400$

$= 3551.032164... \approx 3351$ km (nearest km)

$T = \dfrac{D}{S}$ (from Formulae and Data Sheet)

$= \dfrac{3351.03...}{40}$

$= 83.7758...$

≈ 83 h 46 min 33 secs

≈ 3 days 11 hrs 47 min

≈ 3 days 12 hrs (nearest hour)

(e) $BAC_{Male} = \dfrac{10N - 7.5H}{6.8M}$ (from Formulae and Data Sheet)

$0.05 > \dfrac{10 \times (0.8x) - 7.5 \times 5}{6.8 \times 90}$

$0.05 > \dfrac{8x - 37.5}{612}$ $\}\times 612$

$30.6 > 8x - 37.5$ $\} + 37.5$

$68.1 > 8x$

$8x < 68.1$ $\} \div 8$

$x < \dfrac{68.1}{8}$

$x < 8.5125$

$\therefore$ Rhys can drink a maximum of 8 complete bottles of beer to remain under 0.05.

Question 28

(a) (i) $F = \dfrac{9C}{5} + 32$

$= \dfrac{9 \times -20}{5} + 32$

$= -4°F$

(ii)

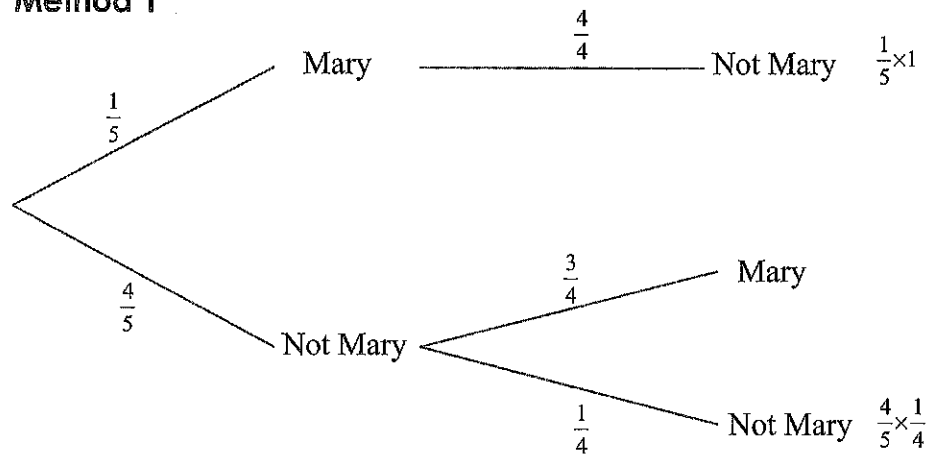| **Method 1:** By substitution	**Method 2:** By elimination
① $\quad F = \dfrac{9C}{5} + 32$	① $\quad F = \dfrac{9C}{5} + 32$
② $\quad F = C$	② $\quad F = C$
Sub ② into ①	① - ②
$C = \dfrac{9C}{5} + 32 \quad \}\times 5$	$0 = \dfrac{4C}{5} + 32 \quad \}\times 5$
$5C = 9C + 160 \quad \} - 5C - 160$	$0 = 4C + 160 \quad \} - 160$
$-160 = 4C \quad \} \div 4$	$4C = -160 \quad \} \div 4$
$C = -40°C$	$C = -40°C$
Sub into ② $\;F = -40°F$	Sub into ② $\;F = -40°F$
$\therefore C = -40°C$ and $F = -40°F$	$\therefore C = -40°C$ and $F = -40°F$

(iii) The two graphs will meet at the point $(-40, -40)$, which is where $F = C$.

(b) (i) **Method 1**

$$\text{Number of Groups} = \frac{5 \times 4}{2}$$
$$= 10$$

Method 2

$$\text{Number of Groups} = {}^5C_2$$
$$= 10$$

(ii) **Method 1**

$$P(\text{Mary}) = P(\text{Mary and then Not Mary}) + P(\text{Not Mary and then Mary})$$
$$= \frac{1}{5} \times \frac{4}{4} + \frac{4}{5} \times \frac{1}{4}$$
$$= \frac{4}{20} + \frac{4}{20}$$
$$= \frac{8}{20}$$
$$= \frac{2}{5}$$

Method 2

Let the 5 people be A, B, C, D and M when M represents Mary. There are 10 groups of 2 that can be listed:

AB   BC   CD   (DM)

AC   BD   (CM)

AD   (BM)

(AM)

$$P(\text{Mary}) = \frac{4}{10}$$
$$= \frac{2}{5}$$

(c) $P = \$100\,000$ $r = 12\%$ p.a. $= 1\%$ per month   Monthly Repayment $= \$1029$

Let A_n be the outstanding amount after n months,
where $A = P(1+r)$ (from Formulae and Data Sheet).

$A_1 = 100\,000(1+0.01) - 1029$
$\quad = \$99\,971$

$A_2 = 99\,971(1+0.01) - 1029$
$\quad = \$99\,941.71$

So the amount outstanding after 2 months is \$99 941.71

(d) $x = \sqrt{yp-1}$ } square both sides
$x^2 = yp-1$ } $+1$
$x^2 + 1 = yp$ } $\div p$
$y = \dfrac{x^2+1}{p}$

(e)   (i)   Take maximum increase in ticket price from the graph: \$6
Ticket price $= 8 + 6$
$\quad\quad\quad = \$14$

(ii)   **Method 1**
Each division on the y-axis represents \$40 ($\$200 \div 5$), so an increase in \$6 is an income of \$1960 (Reading up from 6 on the horizontal axis)

$\text{Tickets sold} = \dfrac{\text{income}}{\text{cost of ticket}}$
$\quad\quad\quad\quad = \dfrac{1960}{14}$
$\quad\quad\quad\quad = 140$ tickets

Method 2
$200 - 10 \times (14-8) = 200 - 10 \times 6$
$\quad\quad\quad\quad\quad\quad\quad = 140$ tickets

(iii)   **Method 1**
Cost $= 500 + 2 \times 140$ Income $= \$1960$ (from graph)
$\quad\quad = 500 + 280$ Profit $= 1960 - 780$
$\quad\quad = \$780$ $\quad\quad\quad = \$1180$

Method 2
Profit per ticket $= 14 - 2$ Total Profit $=$ Profit per ticket $\times$ tickets sold in (ii)
$\quad\quad\quad\quad\quad = \$12$ $\quad\quad\quad\quad\quad\quad\quad\quad$ $-$ General Costs
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad = 12 \times 140 - 500 = \$1180$

Question 29

(a)   (i) $V \approx \dfrac{h}{3}\{A_L + 4A_M + A_R\}$ (from Formulae and Data Sheet)

$$V_1 \approx \frac{200 \div 4}{3}\{0 + 4(140) + 270\}$$
$$\approx 13\,833.333...$$

$$V_2 \approx \frac{200 \div 4}{3}\{270 + 4(300) + 360\}$$
$$\approx 30\,500$$

$$V_T \approx 13\,833.333... + 30\,500$$
$$\approx 44\,333.33$$
$$\approx 44\,333\,\text{m}^3$$

(ii) $A = 2 \times 1000^2$
$$= 2\,000\,000\,\text{m}^2$$

$$\left. \begin{array}{l} V = Ah \\ \\ \end{array} \right\} \div A$$
$$h = \frac{V}{A}$$
$$= \frac{44\,333}{2\,000\,000}$$
$$= 0.0221665\,\text{m}$$
$$= 2.21665\,\text{cm}$$
$$= 22.1665\,\text{mm}$$
$$\approx 22\,\text{mm (nearest mm)}$$

(b)   Taxable income $= \$86\,725$

$$\text{Medicare levy} = \frac{2}{100} \times 86\,725$$
$$= \$1734.50$$

$$\text{Income tax} = 3772 + \left[0.325 \times (86\,725 - 37\,000)\right]$$
$$= \$19\,732.63$$
$$\text{Net income} = 86\,725 - (1734.50 + 19\,732.63)$$
$$= \$65\,257.87$$

(c) (i) Adding totals to the table first, then identifying required cells:

	Have waterskied	Have never waterskied	*Total*
Live in the city	150	2500	**2650**
Live in the country	70	800	**870**
Total	**220**	**3300**	**3520**

$$P(\text{city \& never waterskied}) = \frac{2500}{3520}$$
$$= \frac{125}{176}$$

(ii) $P(\text{city \& have waterskied}) = \frac{150}{2650} \approx 6\%$

$P(\text{country \& have waterskied}) = \frac{70}{870} \approx 8\%$

Based on the probabilities above, a greater proportion of students in the country have waterskied compared to those that live in the city, therefore the statement is true.

(d) (i) Since there is an even number of scores, the median lies midway between the 15th and 16th marks (with values 4 and 8), giving a median of $\frac{4+8}{2} = 6$.

(ii) $5.4 - 4.22 = 1.18$

$5.4 + 4.22 = 9.62$

There are 13 marks within this range (with marks of 2, 3, 4, 8 or 9), therefore:

$\frac{13}{30} \times 100 = 43.\dot{3}\%$

(iii) The data is not normally distributed since it does not have the typical bell-shape of a normal distribution.

Question 30

(a) $IQR = 16 - 10$

$\quad = 6$

Upper Outlier $= Q_u + 1.5 \times IQR$

$\quad\quad = 16 + 1.5 \times 6$

$\quad\quad = 25$

Lower Outlier $= Q_L - 1.5 \times IQR$

$\quad\quad = 10 + 1.5 \times 6$

$\quad\quad = 1$

$\therefore$ Maximum Range $= 25 - 1 = 24$

(b) $C = kh^3$

$50 = k \times 10^3$ $\} \div 10^3$

$k = \dfrac{50}{10^3}$

$= 0.05$

So $C = 0.05h^3$

$= 0.05 \times 12^3$

$= \$86.40$

(c)   (i) $c^2 = a^2 + b^2 - 2ab \cos C$ (from Formulae and Data Sheet)

$AC^2 = 5^2 + 13^2 - 2 \times 5 \times 13 \times \cos 135°$

$= 285.9238816...$ $\} \sqrt{\ }$

$AC = \sqrt{285.9238816...}$

$= 16.9092883$

$\approx 17 \, \text{km (to the nearest km)}$

(ii)

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$ (from Formulae and Data Sheet)

$\dfrac{\sin A}{13} = \dfrac{\sin 135°}{17}$ $\} \times 13$

$\sin A = 13 \times \dfrac{\sin 135°}{17}$

$= 0.540728715...$

$A = \sin^{-1} 0.540728715...$

$= 32.73325942...$

$\approx 33° \text{ (nearest degree)}$

Bearing $= 180° + 33° = 213°\text{T}$

(d)   (i) $y = 2.125x + 2.0375$

$= 2.125 \times 5 + 2.0375$
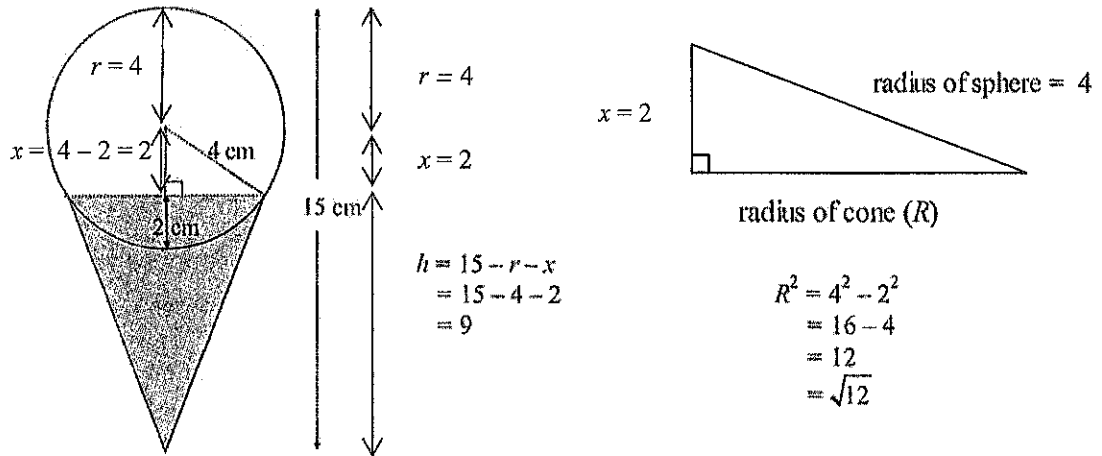
$= 12.6625 \, \text{km/h}$

(ii) $\text{gradient} = r \times \dfrac{\sigma_y}{\sigma_x}$ (from Formulae and Data Sheet)

$2.125 = r \times \dfrac{2}{0.8}$ $\} \div \dfrac{2}{0.8}$

$r = 2.125 \div \dfrac{2}{0.8}$

$r = 0.85$

(e)

$$V = \frac{1}{3} Ah \qquad \text{(from Formulae and Data Sheet)}$$

$$= \frac{1}{3}\left(\pi R^2\right)h$$

$$= \frac{1}{3} \times \pi \times 12 \times 9$$

$$= 113.0973355$$

$$\approx 113 \, \text{cm}^3 \text{ (nearest cm}^3).$$

End of Mathematics General 2 solutions