Student Number: _____

2016 ASSESSMENT TASK 3

Mathematics

General Instructions

- Working time – 50 minutes
- Write using black pen
- Board-approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions 4–5, show relevant mathematical reasoning and/or calculations

Total Marks – 33

Section I Page 2

3 marks

- Attempt Questions 1–3
- Allow about 5 minutes for this section

Section II Pages 3–4

30 marks

- Attempt Questions 4–5
- Allow about 45 minutes for this section

Section I

3 marks
Attempt Questions 1–3
Allow about 5 minutes for this section

Use the multiple-choice answer sheet for Questions 1–3.

1 If $\log_n a = 0.77$ and $\log_n b = 3.08$, what is the value of $\log_n\left(\frac{a}{b}\right)$?

(A) –2.31 (B) 0.25 (C) 2.31 (D) 4.00

2 Ariella correctly solved the equation $\sqrt{2}\cos x - 1 = 0$ for the domain $0 \le x \le 2\pi$. Which of the following is her solution?

(A) $\frac{\pi}{4}$ or $\frac{5\pi}{4}$ (B) $\frac{\pi}{4}$ or $\frac{7\pi}{4}$ (C) $\frac{\pi}{3}$ or $\frac{2\pi}{3}$ (D) $\frac{\pi}{3}$ or $\frac{5\pi}{3}$

3 Which of the following is a solution to the equation $e^{2x} - 4e^x - 5 = 0$?

(A) 4 (B) 5 (C) ln 4 (D) ln 5

Section II

30 marks
Attempt Questions 4 – 5
Allow about 45 minutes for this section

Answer each question on the writing paper provided. Start each question on a new page. Extra writing paper is available.

In Questions 4 – 5, your responses should include relevant mathematical reasoning and/or calculations.

Question 4 (15 marks) Start a new page

(a) A quadrant with radius r has an arc length of 10 cm. Find the exact value of the radius. **1**

(b) Differentiate the following with respect to x.

(i) $\ln 3x$ **1**

(ii) $3xe^x$ **2**

(c) Find the following indefinite integrals.

(i) $\int \frac{2x}{x^2+3}\,dx$ **1**

(ii) $\int e^x(e^x+1)\,dx$ **2**

(d) Find the area swept out by the 6 cm minute hand of a clock as it moves between 10:50 am and 11:45 am. Give your answer in terms of π (exact form). **2**

(e) If $\log_a P = x$, $\log_a Q = y$ and $\log_a R = z$, write an expression in terms of x, y and z for each of the following.

(i) $\log_a \frac{PR}{Q}$ **1**

(ii) $\log_a \frac{P^2\sqrt{Q}}{R^3}$ **1**

Question 4 (continued)

(f) Draw a large, neat sketch of the graph of $y = 2 - 4\ln x$. On your sketch, indicate the coordinates of the point(s) where the curve crosses the coordinate axes. **2**

(g) If $\sin x = -\frac{\sqrt{3}}{2}$ and $\pi < x < \frac{3\pi}{2}$, find the exact value of:

(i) $\tan x$ **1**

(ii) $\sec^2 x$ **1**

Question 5 (15 marks) Start a new page

(a) Using Simpson's rule with 5 function values, find an approximation for $\int_0^2 e^{x^2}\,dx$. Give your answer correct to 2 decimal places. **3**

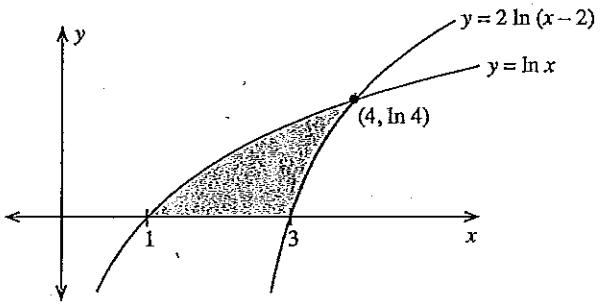
(b) (i) Show that $\frac{d}{dx}[(\ln x)^2] = \frac{2\ln x}{x}$. **1**

(ii) Hence, or otherwise, evaluate the definite integral $\int_e^{e^2} \frac{\ln x}{x}\,dx$. **3**

(c) The area under the curve $y = \sqrt{e^{2x}+1}$ between $x = 0$ and $x = 1$ is rotated around the x-axis to create a solid of revolution. Find the volume of this solid. Give your answer in exact form. **3**

(d) (i) If $y = 2\ln(x-2)$, show that $x = e^{\frac{y}{2}} + 2$. **2**

(ii)



The graphs of the curves $y = \ln x$ and $y = 2\ln(x-2)$ are shown in the diagram above. The curves intersect at $(4, \ln 4)$. The area bound by the two curves and the x-axis is shaded in the diagram. **3**

Calculate the exact area of the shaded region.

End of paper

Kambala 2016 Mathematics

Assessment Task 3.

Multiple Choice.

1. $\log_n a = 0.77$

$\log_n b = 3.08$

$\log_n\left(\frac{a}{b}\right) = 0.77 - 3.08 = -2.31$ =A

2. $\sqrt{2}\cos x - 1 = 0$

$\cos x = \frac{1}{\sqrt{2}}$

S	A
T	C

$x = \pi/4, 7\pi/4$ = B

3. $e^{2x} - 4e^x - 5 = 0$

let $u = e^x$

$u^2 - 4u - 5 = 0$

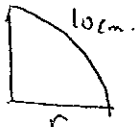
$(u-5)(u+1) = 0$

$u = -1, 5$

$5 = e^x$

$\ln 5 = x$. =D

4. a)



ie $\frac{\pi d}{4} = 10$

but $d = 2r$

ie $\frac{2r\pi}{4} = 10$

$2r\pi = 40$

$r\pi = 20$

$r = \frac{20}{\pi}$

b) i) $\frac{d}{dx}(\ln 3x)$

$= \frac{3}{3x}$

ii) $\frac{d}{dx} 3xe^x$

= product rule

$u = 3x \quad u' = 3$

$v = e^x \quad v' = e^x$

$= 3xe^x + 3e^x$

$= 3e^x(x+1)$

c) i) $\int \frac{2x}{x^2+3}$

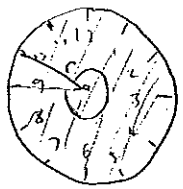
$= \ln(x^2+3) + C$

ii) $\int e^x(e^x+1)\,dx$

$= \int (e^x)^2 + e^x\,dx$

$= \int e^{2x} + e^x\,dx$ [since $(e^x)^2 = e^{2x}$]

$= \left[\frac{e^{2x}}{2} + e^x + C \right]$

d) 

$r = 6$

ie $\frac{11}{12}$ area of circle with radius 6cm

Since all sectors are equal in area (12).

ie $= \frac{11}{12}\pi r^2$ where $r = 6$

$= \frac{11}{12}(36)\pi$

$= 33\pi\ cm^2$

e) $\log_a P = x$

$\log_a Q = y$

$\log_a R = z$

i) $\log_a\left(\frac{PR}{Q}\right)$

$= \log_a(PR) - \log_a(Q)$

$= \log_a(P) + \log_a(R) - \log_a(Q)$

$= x + z - y$

ii) $\log_a \frac{P^2\sqrt{Q}}{R^3} = \log_a(P^2\sqrt{Q}) - \log_a(R^3)$

$= \log_a(P^2) + \log_a(\sqrt{Q}) - 3\log_a(R)$

$= 2\log_a(P) + \frac{1}{2}\log_a(Q) - 3\log_a R$.

$= 2x + \frac{y}{2} - 3z$

f) $y = 2 - 4\ln x$

Important points

domain $x > 0$

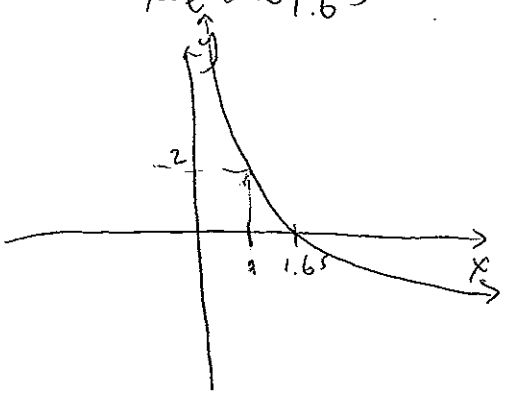
x intercept

$0 = 2 - 4\ln x$

$4\ln x = 2$

$\ln x = \frac{1}{2}$

$x = e^{\frac{1}{2}} \approx 1.65$



9). $\sin x = -\frac{\sqrt{3}}{2}$ $\pi < x < \frac{3\pi}{2}$.

S	A
T	C

$x = 4\pi/3$

i) $\tan \frac{4\pi}{3} = \sqrt{3}$

ii) $\sec^2\left(\frac{4\pi}{3}\right) = \frac{1}{\cos^2(\frac{4\pi}{3})}$

$= (-0.5)^2 = 0.25$

b) $\frac{d}{dx}[\ln(x)]^2 = \frac{2\ln x}{x}$

$= 2\ln(x) * \frac{d}{dx}\ln(x)$

$= 2\ln x \times \frac{1}{x} = \frac{2\ln x}{x}$.

i; $\int_e^{e^2} \frac{\ln x}{x} dx$

$= \frac{1}{2}\int_e^{e^2} \frac{2\ln x}{x} dx$

$= \frac{1}{2}\left[[\ln(x)]^2\right]_e^{e^2}$ from (i)

$= \frac{1}{2}\left[2^2 - 1^2\right]$

$= \frac{1}{2}[4-1] = \frac{3}{2}$.

5. $\int_0^2 e^{x^2} dx$ $f(x) = e^{x^2}$.

5 values

x	0	0.5	1	1.5	2.
f(x)	1	1.284	2.72	9.5	54.6

$= \frac{0.5}{3}\left(1 + 4(1.284) + 2(2.72) + 4(9.5) + 1(54.6)\right)$

e^0 $e^{0.5^2}$ e^{1^2} $e^{1.5^2}$ e^{2^2}

≈ 17.36 units^2.

c). $y = \sqrt{e^{2x}+1}$ $x=0, x=1$. (d ii)

Cross sectional area is then $\pi\left[e^{2x}+1\right]$

ie Volume is $\pi \int_0^1 e^{2x}+1\, dx$.

$= \pi\left[\frac{e^{2x}}{2} + x\right]_0^1$

$= \pi\left[\left(\frac{e^2}{2}+1\right) - \left(\frac{1}{2}\right)\right]$

$= \pi\left[\frac{e^2}{2} + \frac{1}{2}\right] = \frac{\pi(e^2+1)}{2}$ units^3

d) $y = 2\ln(x-2)$

$\frac{y}{2} = \ln(x-2)$

$e^{\frac{y}{2}} = (x-2)$

$e^{\frac{y}{2}} + 2 = x$ (as required)

$\int_0^{\ln 4} \left(e^{\frac{y}{2}}+2\right) - \left(e^y\right) dy$.

$= \left[2e^{\frac{y}{2}} + 2y - e^y\right]_0^{\ln 4}$

$2e^{\frac{\ln 4}{2}} + 2\ln 4 - e^{(\ln 4)}$

$= 2e^{\frac{2\ln 2}{2}} + 4\ln 2 - 4$

$= 4 + 4\ln 2 - 4$

$= 4\ln 2$ units^2