MARCELLIN COLLEGE RANDWICK

YEAR 11
EXTENSION 1

PRELIMINARY ASSESSMENT TASK 1
2016

STUDENT NAME: _____ MARK /31

TEACHER: _____

TIME ALLOWED: 45 minutes
WEIGHTING: 40 % (of Preliminary Mark)

Directions:

- Answer multiple choice questions on the page provided.
- Use a new sheet for additional questions.
- Show all necessary working. Where more than one mark is allocated to a question, full marks may not be awarded for answers only.
- Marks may not be awarded for careless or badly arranged work.
- Calculators may be used

Question 1 (16 marks)

Marks

a. Solve

i) $\frac{2}{1-x} > -1$ 2

ii) $\sqrt{4y+5} + y = 10$ 3

iii) $\frac{x+2}{2x-1} - 2 = \frac{1-2x}{x+2}$ 3

b. Graph the curve 3

$$y = \frac{x}{x^2 - 1}$$

c. A is the point $(-1, 5)$ and B is the point $(2, -3)$. Find the coordinates of the point P that divides the interval AB internally in the ratio of 1: 2 2

d. i) Show that the point of intersection of the curves $y = x^2 - 1$ and $y = x^2 - 4x + 3$ occurs when $x = 1$ 1

ii) Hence or otherwise, find the acute angle between the two curves (to the nearest degree) 2

Question 2 (15 marks)

a. Find the exact value of $\sin 15°$ 2

b. i) Express $\sin x + \sqrt{3}\cos x$ in the form $r\sin(x+\alpha)$ where $0 \leq \alpha \leq 90°$ 2

ii) Hence or otherwise, solve $\sin x + \sqrt{3}\cos x = 1$ 2

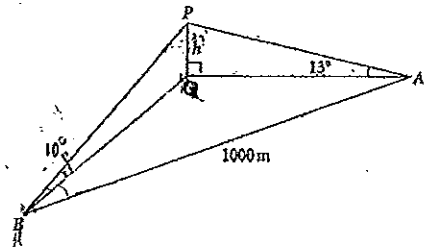
c. i) Show that

$$tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

where $\cos 2\theta \neq -1$ 2

ii) Hence or otherwise, find the exact value of $\tan 22.5°$ 2

d. A tower PQ of height h metres has an angle of elevation of 13° at a point A due east of it. From another point B, the bearing of the tower is 050° and the angle of elevation 10°. The points A and B are 1000 metres apart on the same level as the towers base Q



i) Show that $\angle AQB = 140°$ 2

ii) Calculate h to the nearest metre 3

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Q1 $\frac{2}{1-x} > -1$

a) i) $2(1-x) > -(1-x)^2$

$2(1-x) + (1-x)^2 > 0$

$(1-x)[2+(1-x)] > 0$

$(1-x)(3-x) > 0$

$x = 1, 3$ ✗

$x < 1, x > 3$ ✓ 2

ii) $\sqrt{4y+5} = 10-y$

$4y+5 = (10-y)^2$

$4y+5 = 100 - 20y + y^2$

$0 = y^2 - 24y + 95$

$\frac{24 \pm \sqrt{576-380}}{2}$

$= \frac{24 \pm 14}{2}$

$= 19$ or 5

$19\ \sqrt{4(19)+5} = 10-19$

does not work

$\sqrt{25} = 5$

5 is the solution ✓ 3

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a)ii) $\frac{x+2}{2x-1} - 2 = \frac{1-2x}{x+2}$

let $y = \frac{x+2}{2x-1}$

$y - 2 = \frac{-1}{y}$

$y^2 - 2y = -1$

$y^2 - 2y + 1 = 0$

$(y-1)(y-1) = 0$

$y = 1$

$1 = \frac{x+2}{2x-1}$

$2x - 1 = x + 2$

$x = 3$ ✓ 3

b) $y = \frac{x}{x^2-1}$ $x \neq \pm 1$

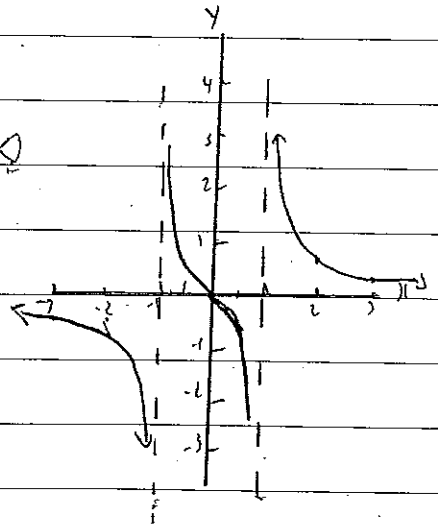
y-int $= 0$ $\lim_{x \to \infty} \frac{\frac{x}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}} = 0$

x-int $= 0$

$f(-x) = \frac{-x}{x^2-1}$

$-f(x) = -\left(\frac{x}{x^2-1}\right)$ ✗

∴ odd function



✓ 3

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c) A(−1, 5) B(2, −3)

1 : 2

$x = \frac{2(-1)+1(2)}{3}$ $y = \frac{-3+10}{3}$

$= 0$ $= \frac{7}{3}$

$\therefore (0, 2\frac{1}{3})$ ✓ 2

d)i) $y = x^2 - 1$ $y = x^2 - 4x + 3$

$x^2 - 1 = x^2 - 4x + 3$

$4x - 4 = 0$

$4(x-1) = 0$

$x = 1$ ✓ 1

ii) $y' = 2x$ $y' = 2x - 4$

At $x = 1$ At $x = 1$

$y' = 2$ $y' = -2$

$\tan\theta = \left|\frac{4}{-3}\right|$

$\tan\theta = \frac{4}{3}$

$\theta = 53°$ (nearest degree) ✓ 2

Q2) a) $\sin 15 = \sin(60 - 45)$

$= \sin 60 \cos 45 - \cos 60 \sin 45$

$= \left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2} \times \frac{1}{\sqrt{2}}\right)$ 2

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}} \times \frac{2\sqrt{2}}{2\sqrt{2}} = \frac{2\sqrt{6} - 2\sqrt{2}}{8} = \frac{2(\sqrt{6}-\sqrt{2})}{8}$

b) i) $\sin x + \sqrt{3}\cos x \quad 0 \le x \le 90$

$a = 1 \quad b = \sqrt{3}$

$R = \sqrt{1+(\sqrt{3})^2} \quad 2\sin(x+60^\circ)$ ✓

$= 2$ 2

$\tan\theta = \sqrt{3}$

$\theta = 60^\circ$

ii)

Assume.

$0 \le x \le 360^\circ$ or $0 \le x \le 2\pi$

S	A
T	C.

$\sin(x+60^\circ) = \frac{1}{2}$ → $(\frac{11\pi}{6}, \frac{\pi}{2})$

$x = 330^\circ$ or 90° (1)

c) i) $\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$

R.H.S $\frac{1-(1-2\sin^2\theta)}{1+(2\cos^2\theta-1)} = \frac{2\sin^2\theta}{2\cos^2\theta} = \tan^2\theta$ = L.H.S ✓ 2

ii) let $t = \tan\frac{\theta}{2}$

$\tan\theta = 22.5$

$\tan\theta = \sqrt{2}-1$ ✓ 2

$\tan\frac{\theta}{2} = 22.1$

$\therefore \theta = 45^\circ$

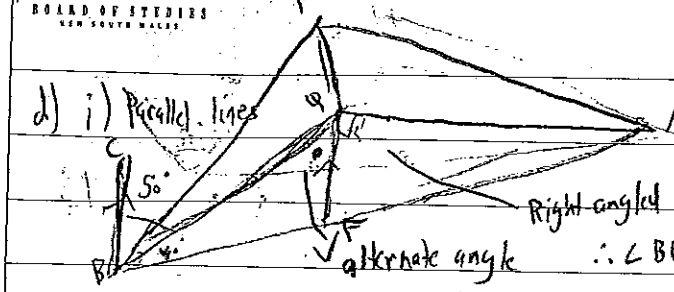
$\frac{1-\cos 45}{1+\cos 45} = \frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}} \quad \frac{\sqrt{2}-1}{\sqrt{2}+1}$

$\cos\theta = \frac{1-t^2}{1+t^2}$

$1+t^2 = 1-t^2$

$2t^2 = 0$

d) i) Parallel lines



$\therefore \angle BQA = 50 + 90$

$= 140^\circ$ ✓ 2

$\angle CBQ = \angle BQF$ (alt $\angle$s, parallel lines) $= 50^\circ$

$\angle FQA = 90^\circ$

ii) $\tan 50 = \frac{QB}{h} \quad \tan 77 = \frac{AQ}{h}$

$QB = h\tan 50 \quad AQ = h\tan 77$

$1000^2 = h^2\tan^2 50 + h^2\tan^2 77 - (2\times h\tan 50\times h\tan 77\times\cos 140)$

$1000^2 = h^2(\tan^2 50 + \tan^2 77 - 2\tan 50\tan 77(\cos 140)$

$h^2 = \frac{1000^2}{\tan^2 50 + \tan^2 77 - 2\tan 50\tan 77(\cos 140)}$

$h = 106$m (nearest metre ✓ 3