Centre Number

Student Number

QATs
Quality Assessed Tasks

2016
PRELIMINARY EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Draw diagrams using pencil
- Board-approved calculators may be used
- A reference sheet of formulae is provided
- All necessary working should be shown in every question

Total marks 70

Section I Pages 2-6
10 marks
- Attempt Questions 1-10
- Allow about15 minutes for this section

Section II Pages 7-10
60 marks
- Attempt Questions 11-14
- Allow about 1 hour 45 minutes for this section

2016 PRELIMINARY EXAMINATION
MATHEMATICS EXTENSION 1

Use the multiple-choice answer sheet provided for Questions 1-10

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2+4=$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you have changed your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

A ⊗ B ⊗ ← *correct* C ○ D ○

1 Solve $\dfrac{1}{x+2} \geq 0$.

(A) $x > -0.5$

(B) $x \geq -0.5$

(C) $x > -2$

(D) $x \geq -2$

2 Solve $\frac{2}{|1+x|}=\frac{3}{2}$.

(A) $\frac{4}{3}$

(B) $\frac{1}{3}$

(C) $\frac{-7}{3}$ or $\frac{1}{3}$

(D) $\frac{-7}{3}$ or $\frac{4}{3}$

3 Solve for x, $\frac{x^2-6x+5}{x-1}\le 0$

(A) $x=1$ or 5

(B) $1\le x\le 5$

(C) $1<x<5$

(D) $x\le 5,\quad x\ne 1.$

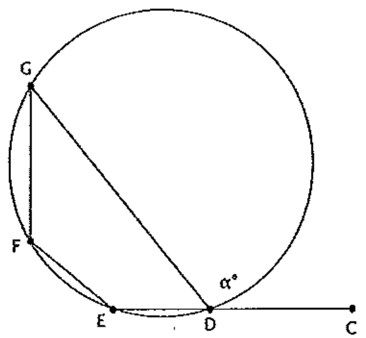
4 In the circle below, angle $\alpha°$ is

(A) the supplement of $\angle EFG$

(B) the supplement of $\angle FGD$

(C) twice the size of $\angle FGD$

(D) the same size as $\angle EFG$



5 $\tan 20° =$.

(A) $\sec^2 10°$

(B) $2\tan 10°$

(C) $\dfrac{2\tan 10°}{2-\sec^2 10°}$

(D) None of the above

6 The acute angle between lines with equations $y = x+1$ and $5y = x+1$ is:

(A) $\tan^{-1}(\frac{2}{3})$

(B) $\tan^{-1}(1\frac{1}{2})$

(C) $\tan^{-1}(\frac{1}{5})$

(D) $50°$

7 Which of the following is true?

(A) ${}^{n}C_{k-n} = {}^{n}C_{n-k}$

(B) ${}^{n}C_{n-k} = {}^{n}C_{k}$

(C) ${}^{n+1}C_{k} = {}^{n}C_{k}$

(D) ${}^{n-k}C_{k} = {}^{n}C_{k}$

8. A cubic polynomial in x of the form $y = ax^3,\ a \neq 0$.

(A) Cannot be a function.

(B) Is an odd function.

(C) Is neither an even, nor odd, function.

(D) An even function.

9 In circle geometry, the angle at the centre:

(A) Is $90°$

(B) Is equal to the angle between the chord and the tangent.

(C) Is double the angle at the circumference standing on the same arc.

(D) Is cointerior.

10 The graph of the equation $y = \dfrac{(x-3)(x^2-4)}{x+2}$ could be described as:

(A) A cubic curve that is undefined for $x = -2$.

(B) A quadratic curve that is undefined for $x = -2$.

(C) A straight line that is undefined for $x = -2$.

(D) A straight line .

Question 11 (15 marks) Use a SEPARATE sheet of paper or booklet.

(a) Draw a one third page sketch of the parallelogram with vertices O(0,0) A(0,1) B($\sqrt{3}$,0) C($\sqrt{3}$,-1).

(i) Find the equation of diagonal AC and show that it intersects the other diagonal OB at $P(\frac{\sqrt{3}}{2},0)$. **2**

(ii) Using the properties of parallelograms or otherwise, show that the diagonals form two pairs of congruent triangles, but all have the same area. **2**

(b) Consider the points $A(x_1,y_1)$, $B(4,-1)$ and $P(10,-3)$

Given that P divides AB externally in the ratio $p:q$ derive an expression for x_1 in terms of p and q. **2**

(c) Solve the equation $(x+\frac{1}{x})^2-11=-10(x+\frac{1}{x})$ leaving your answers in exact form. **4**

(d) Three digits are chosen with <u>no</u> repetition, to form a single number using the digits 0 to 9.

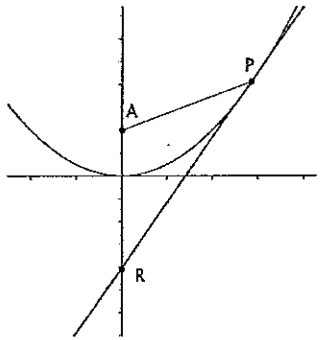
(i) Determine how many numbers are less than or equal to 100. **1**

(ii) If a number is chosen at random what is the probability than it is greater than 100. **2**

(e) Minh plays a game in which he has to guess the 5 numbers that are chosen randomly from the numbers 1 to 40 and repetition **<u>is allowed</u>**. Determine the probability that the numbers chosen are 1 2 3 4 5 in that order. **2**

Question 12 (15 marks) Use a SEPARATE sheet of paper or booklet.

(a) Consider the Parabola below, with tangent at point $P(2at,at^2)$, and focus at $A(0,a)$. The tangent at P meets the y-axis at R.



(i) Eliminate parameter t to determine the Cartesian Equation of the parabola shown above, with a general point $P(2at,at^2)$. **2**

(ii) Derive the gradient of the tangent at $P(2at,at^2)$ and hence determine its equation. **3**

(iii) Determine the coordinates of Point R, where the tangent at P, intersects the axis of the parabola. **1**

(iv) Find an expression for the area of triangle APR. **2**

(b) The curve $f(x)=x^3+9$ has a zero at approximately -2. Use one application of Newton's method to show that a better approximation is $-2\frac{1}{12}$. **3**

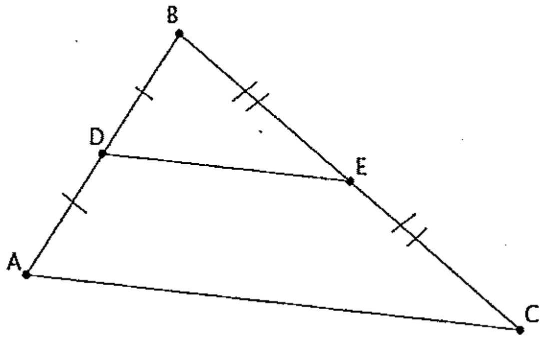
(c) Consider the polynomial. $P(x)=(x^2+x+1)^2$.

(i) After expanding, express P(x) in descending powers of x. **2**

(ii) Determine any real zeros of $P(x)$ and justify your answer. **2**

Question 13 (15 marks) Use a SEPARATE sheet of paper or booklet.

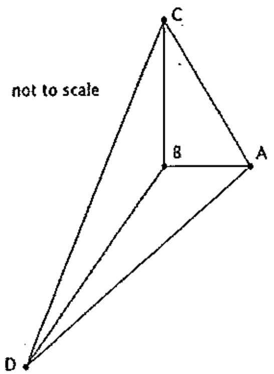
(a) In triangle ABC below, interval DE joins the midpoints of sides AB and BC. Prove that the smaller triangle DBE occupies $\frac{1}{4}$ the area of the larger triangle ABC. **4**



(b) In the diagram below, points D, B and A are on level ground. **4**

C is the top of a vertical tower, with base B, $\angle BAC = 50^\circ$, $\angle BDC = 30^\circ$

and $\angle ABD = 90^\circ$. Calculate the size of $\angle BDA$ correct to the nearest minute.



(c) Consider the Polynomial $P(x) = ax^3 + bx^2 + cx + d$ whose roots are $\alpha, -\alpha$ and β. Express $\beta - \alpha^2$ in terms of the coefficients of $P(x)$. **3**

(d) Consider the Polynomial $P(x) = 4x^3 - 3x + 1$.

(i) Express $\frac{d}{dx}P(x)$ in factored form. **2**

(ii) Hence determine the double root of $P(x) = 0$. **1**

(iii) Determine the third root of $P(x) = 0$. **1**

Question 14 (15 marks) Use a SEPARATE sheet of paper or booklet.

(a) Solve the equation $\cos 2x - \sin x = 1$ and express your answer as a general solution. [where *x* is in degrees] **5**

(b) Given $\tan\frac{\theta}{2} = t$, $t \neq \pm 1$ and $\tan\theta = \frac{2t}{1-t^2}$.

(i) Explain why $t \neq \pm 1$. **1**

(ii) Show that $\cos\theta = \frac{1-t^2}{1+t^2}$. **1**

(iii) Rewrite the equation $\cos^2\theta = \frac{1}{4}$ *for* $0 \leq \theta \leq 360^\circ$ in terms of *t*. **1**

(iv) Solve the rewritten equation for *t*. **4**

(v) Hence show that $\theta = 60^\circ, 120^\circ, 240^\circ, 300^\circ$. **3**

QATs
Quality Assessment Tasks

2016 PRELIMINARY EXAMINATION

MATHEMATICS EXTENSION 1 – MAPPING GRID

Exam Section	Question	Marks	Syllabus/Course Outcomes	Content	Targeted Performance Bands	Answer
Section I: Multiple Choice	1	1	PE3	Algebra	E2	C
	2	1	PE6, P3	Algebra	E2	C
	3	1	PE6, P3	Algebra	E2	D
	4	1	PE3	Circle geometry	E3	D
	5	1	PE6	Further trigonometry	E3	C
	6	1	PE6	Further trigonometry	E3	A
	7	1	PE3	Permutations & Combinations	E3	B
	8	1	PE3	Polynomials	E3	B
	9	1	PE3	Circle geometry	E3	C
	10	1	PE6, P3	Real Functions	E3	B

2016 PRELIMINARY EXAMINATION

MATHEMATICS EXTENSION 1
MARKING GUIDELINES

10 marks
Questions 1-10 (1 mark each)

Question	Correct Response	Outcomes Assessed	Targeted Performance Bands
1	C	PE3	E2
2	C	PE6, P3	E2
3	D	PE6, P3	E2
4	D	PE3	E3
5	C	PE6	E3
6	A	PE6	E3
7	B	PE3	E3
8	B	PE3	E3
9	C	PE3	E3
10	B	PE6, P3	E3

Question 11 (15 marks)
11 (a) (i) (2 marks)
Outcomes Assessed: PE1
Targeted Performance Bands: E2, E3

Criteria	Marks
• Achieves a version of the equation below $y=\frac{-2}{\sqrt{3}}x+1$	1
• Substitutes y=0 to find $x=\frac{\sqrt{3}}{2}$	1

11 (a) (ii) (2 marks)
Outcomes Assessed: PE1
Targeted Performance Bands: E3, E4

Criteria	Marks
• Lists and justifies 3 pieces of relevant information for a congruency proof such as :- with diagonals meeting at Point P, then in triangles AOP, COB, OP=BP, AP=CP, diagonals bisect in a parallelogram, AO=CB, opposite sides are equal in a parallelogram	1
• Has a congruency proof for one pair of triangles and states that the other pair are similarly congruent	1

11 (b) (2 marks)
Outcomes Assessed: PE1
Targeted Performance Bands: E2, E3

Criteria	Marks
• Using ratio formula:- $\frac{4p-x_1q}{p+q}=10$	1
• Rearranging:- $x_1=\frac{-6p-10q}{q}$	1

11 (c) (4 marks)
Outcomes Assessed: PE2
Targeted Performance Bands: E2, E3, E4

Criteria	Marks
• $((x+\frac{1}{x})+11)((x+\frac{1}{x})-1)=0$	1
• $(x+\frac{1}{x})=-11 \text{ or } 1$	1
• $x^2+11x+1=0 \text{ and } x^2-x+1=0$	1
• $x=\frac{-11\pm\sqrt{117}}{2}$ *and 2nd equation has no solution*	1

11 (d)(i) (1 mark)
Outcomes Assessed: PE3
Targeted Performance Bands: E2

Criteria	Mark
• With no repeats it is not possible to have the number 100 so the only possible numbers use a zero as first digit and there will be 72 possible numbers formed	1

11 (d) (ii) (2 marks)
Outcomes Assessed: PE3
Targeted Performance Bands: E2, E3

Criteria	Marks
• $P(N\leq 100)=\frac{72}{648}=\frac{1}{9}$	1
• $P(N>100)=1-P(N\leq 100)$ $=\frac{8}{9}$	1

11 (e) (2 marks)
Outcomes Assessed: PE3
Targeted Performance Bands: E3, E4

Criteria	Marks
• There is only one way to choose those numbers in that order	1
• There are $40\times40\times40\times40\times40$ possible numbers, so the probability is $\frac{1}{40\times40\times40\times40\times40}$	1

Question 12 (15 marks)
12 (a) (i) (2 marks)
Outcomes Assessed: PE4
Targeted Performance Bands: E2, E3

Criteria	Marks
• From $x=2at$, make t the subject and attempt to substitute $t=\frac{x}{2a}$ into $y=at^2$	1
• $y=a.(\frac{x}{2a})^2=\frac{x^2}{4a}$ [do not accept $y=\frac{x^2}{4a}$ without evidence of substitution]	1

12 (a) (ii) (3 marks)
Outcomes Assessed: PE4
Targeted Performance Bands: E2, E3, E4

Criteria	Marks
• $\frac{d}{dx}(\frac{x^2}{4a})=\frac{x}{2a}$	1
• Substitute $x=2at$ to get $\frac{dy}{dx}=\frac{2at}{2a}=t$ alternatively for 2 marks $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}=2at\times\frac{1}{2a}=t$	1
• $y-at^2=t(x-2at)$ $\therefore \quad y=tx-at^2$	1

12 (a) (iii) (1 mark)
Outcomes Assessed: PE4
Targeted Performance Bands: E3, E4

Criteria	Mark
• $R(0,-at^2)$ NOTE: accept any attempt to substitute $x=0$ into their equation of tangent and then find **both** x and y coordinates	1

12 (a) (iv) (2 marks)
Outcomes Assessed: PE4
Targeted Performance Bands: E2, E3

Criteria	Marks
• $AR = a + at^2$	1
• Area $= \frac{2at(a+at^2)}{2} u^2$ or equivalent	1

12 (b) (3 marks)
Outcomes Assessed: PE3, PE5
Targeted Performance Bands: E2, E3, E4

Criteria	Marks
• $f'(x) = 3x^2 3$ $\therefore f'(-2) = 12$	1
• $x = \frac{-1}{12} - 2$	1
• $x = \frac{-25}{12}$ $= -2\frac{1}{12}$ is a better approximation of the root	1

12 (c) (i) (2 marks)
Outcomes Assessed: PE3
Targeted Performance Bands: E2

Criteria	Marks
• $= x^4 + x^3 + x^2 + x^3 + x^2 + x + x^2 + x + 1$	1
• $x^4 + 2x^3 + 3x^2 + 2x + 1$	1

12 (c) (ii) (2 marks)
Outcomes Assessed: PE3
Targeted Performance Bands: E3, E4

Criteria	Marks
• There are no real roots	1
• Because the discriminate is -3 for $x^2 + x + 1$	1

Question 13 (15 marks)
13 (a) (4 marks)
Outcomes Assessed: PE2
Targeted Performance Bands: E2, E3, E4

Criteria	Marks
In ΔABC, ΔDBE • let $AB = 2\alpha$ then $DB = \alpha$ (given D is midpoint of AB)	1
Similarly let $CB = 2\beta$ • then $EB = \beta$ also $\angle B$ is common	1
• Area $\Delta ABC = \frac{1}{2} \times 2\alpha \times 2\beta \times \sin B$ $= \Delta ABC = 2\alpha\beta \sin B$	1
$\Delta DBE = \frac{1}{2} \times \alpha \times \beta \times \sin B$ • $= \Delta ABC = \frac{1}{2}\alpha\beta \sin B$ $= \frac{1}{4}$ area ΔABC	1

13 (b) (4 marks)
Outcomes Assessed: PE1, PE2, PE6
Targeted Performance Bands: E2, E3

Criteria	Marks
In ΔBAC, • $AB = h\tan 40^\circ$	1
In ΔDBC, • $DB = h\tan 60^\circ$	1
In ΔABC, • $\tan\angle BDA = \frac{h\tan 40^\circ}{h\tan 60^\circ}$	1
• $\angle BDA = 25^\circ 51'$	1

13 (c) (3 marks)
Outcomes Assessed: PE3
Targeted Performance Bands: E2, E3

Criteria	Marks
• $-\alpha+\alpha+\beta=\frac{-b}{a}$ $\therefore \quad \beta=\frac{-b}{a}$	1
• $-\alpha\beta+\alpha\beta-\alpha^2=\frac{c}{a}$	1
• $\beta-\alpha^2=\frac{c-b}{a}$	1

13 (d) (i) (2 marks)
Outcomes Assessed: PE3
Targeted Performance Bands: E3

Criteria	Marks
• $P'(x)=12x^2-3$	1
• $=3(2x-1)(2x+1)$	1

13 (d) (ii) (1 mark)
Outcomes Assessed: PE3
Targeted Performance Bands: E3

Criteria	Mark
• $P'(\frac{1}{2})=P(\frac{1}{2})=0$ the double root is $x=\frac{1}{2}$	1

13 (d) (iii) (1 mark)
Outcomes Assessed: PE3
Targeted Performance Bands: E4

Criteria	Mark
• The sum of the roots = 0 We know that two roots are $\frac{1}{2}$ and $\frac{1}{2}$ Thus the third root is -1	1

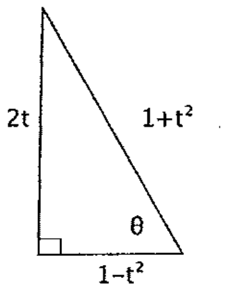
Question 14 (15 marks)
14 (a) (5 marks)
Outcomes Assessed: PE6
Targeted Performance Bands: E2, E3, E4

Criteria	Marks
• $1-2\sin^2 x-\sin x=1$	1
• $\sin x(2\sin x+1)=0$	1
• $\sin x=0$ *or* $-\frac{1}{2}$	1
• $x=n180^\circ$, *for* $n\in\{.....-2,-1,0,1,2,....\}$ *and* $x=n180^\circ-(-1)^n30^\circ$, *for* $n\in\{.....-2,-1,0,1,2,....\}$	2

14 (b) (i) (1 mark)
Outcomes Assessed: PE6
Targeted Performance Bands: E2

Criteria	Mark
• $\tan\theta=\frac{2t}{1-t^2}$ hence for $t=\pm1$ $\tan\theta=\frac{\pm2}{0}$ $=undefined \ \therefore t\neq\pm1$	1

14 (b) (ii) (1 mark)
Outcomes Assessed: PE6
Targeted Performance Bands: E2

Criteria	Mark
• By Pythagoras $(2t)^2+(1-t^2)^2=(1+t^2)^2$ $\therefore \cos\theta=\frac{1-t^2}{1+t^2}$ accept $[\frac{(1-t^2)}{(1+t^2)}]^2=\frac{1}{4}$ • Alternatively provide the sketch below 	1

14 (b) (iii) (1 mark)
Outcomes Assessed: PE6
Targeted Performance Bands: E3

Criteria	Mark
• $\cos\theta=\pm\frac{1}{2}$ $\therefore\frac{1-t^2}{1+t^2}=\pm\frac{1}{2}$	1

14 (b) (iv) (4 marks)
Outcomes Assessed: PE6
Targeted Performance Bands: E3, E4

Criteria	Marks
• $2-2t^2=1+t^2,\ 2-2t^2=-1-t^2$	1
• $1=3t^2,\ 3=t^2$ $t=\pm\frac{1}{\sqrt{3}},\quad t=\pm\sqrt{3}.$	1 2

14 (b) (v) (3 marks)
Outcomes Assessed: PE6
Targeted Performance Bands: E4

Criteria	Marks
• $\frac{\theta}{2}=30°, 210°,\ 150°, 330°\ \therefore\theta=60°, 300°$ *or* $\frac{\theta}{2}=60°, 240°,\ \ 120°, 300°\ \therefore\theta=120°, 240°$	2
• $\theta=60°,\ 120,\ 240°,\ 300°$	1