REDLANDS
LET YOUR LIGHT SHINE

Student Number:________________

Teacher:________________________

# Year 12 HSC
## Term 1 Assessment
## February 2016
# Mathematics

**Time allowed: 45 minutes**

**Instructions to Students:**

- Board approved calculators may be used.
- To obtain full marks, well set out logical reasoning or explanations must accompany your answers.
- Include your student number/name at the top of each purple sheet.
- Start each question on a new sheet of purple paper.

**Total:** 32 marks

| | | |
|---|---|---|
| Q1 | 14 | |
| Q2 | 18 | |
| | 32 | |



**Question 1** [Maximum mark: 14] *[Start on a new purple sheet]*

a) Find the equation of the locus of a point $P(x,y)$ that moves so that its distance from the origin is 4 units. (1)

b) i) Find the locus of a point $P(x,y)$ that moves so that $PA = 2PB$, given that $A(-2,4)$ and $B(4,1)$. (3)

ii) Show that the locus is a circle. State the radius and the coordinates of its centre. (3)

c) Find the equation of the parabola with vertex (3,1) and directrix the line $x = 2$ and hence find the coordinates of the focus. (3)

d) Find $f(x)$ given $f'(x) = 2x - 2$ and $f(1) = 4$ (2)

e) Find the gradient of the tangent to the curve $x^2 = -8y$ at the point $x = 1$. (2)

**Question 2** [Maximum mark: 18] *[Start on a new purple sheet]*

a) Consider the function $f(x) = x^3(4 - x)$

i) Find the coordinates of the points where the curve crosses the axes. (2)

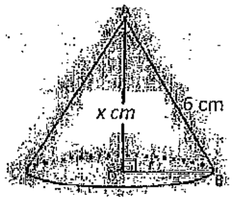
ii) Find any stationary points and determine their nature. (4)

iii) Find the coordinates of the points of inflexion. (2)

iv) Sketch the graph of $y = f(x)$, indicating clearly the intercepts, stationary points and points of inflexion. (3)

v) For what values of $x$ is the curve concave down? (1)

b) The slant edge $AB$ of a right circular cone is 6 cm. The vertical height of the cone is $x$ cm.




i) Express the radius of the base in terms of $x$. (1)

ii) Using the formula for volume of a cone, $V = \frac{1}{3}\pi r^2 h$, show that the volume of the cone can be expressed as $V = \frac{\pi x(36-x^2)}{3}$ (2)

iii) Hence find the vertical height of the cone when the volume is a maximum. (3)

**End of Assessment**



SOLUTIONS – YR 12 TERM 1 ASSESSMENT – MATHEMATICS

1.

a) $x^2 + y^2 = 16$

b) i) $PA^2 = 4PB^2$

$(x+2)^2 + (y-4)^2 = 4[(x-4)^2 + (y-1)^2]$

$x^2 + 4x + 4 + y^2 - 8y + 16 = 4(x^2 - 8x + 16 + y^2 - 2y + 1)$

$3x^2 - 36x + 3y^2 + 48 = 0$

$x^2 - 12x + y^2 + 16 = 0$

ii) $x^2 - 12x + (-6)^2 + y^2 = -16 + (-6)^2$

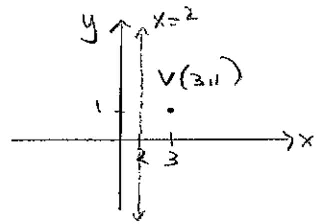
$(x-6)^2 + y^2 = 20$

∴ Locus is a circle as is of general form $(x-h)^2 + (y-k)^2 = r^2$

$C = (6, 0)$

$r = \sqrt{20} = 2\sqrt{5}$ units

c)




$(y-k)^2 = 4a(x-h)$ $V = (3, 1)$ $a = 1$

$(y-1)^2 = 4(x-3)$

Focus $= (4, 1)$

d) $f'(x) = 2x - 2$

$f(x) = x^2 - 2x + c$

$f(1) = 4$

$4 = (1)^2 - 2(1) + c$

$4 = 1 - 2 + c$

$\therefore c = 5$

$\therefore f(x) = x^2 - 2x + 5$

e) $x^2 = -8y$

$\therefore y = \frac{x^2}{-8}$

$\frac{dy}{dx} = \frac{2x}{-8}$

$= -\frac{x}{4}$

when $x = 1$

$\frac{dy}{dx} = -\frac{1}{4}$

$\therefore m = -\frac{1}{4}$

2. a)

i) x-int, let $y=0$

$0 = x^3(4-x)$

$\therefore x=0$ or $x=4$.

y-int. let $x=0$

$y = (0)^3(4-0)$
$= 0$

$\therefore (0,0)$ and $(4,0)$

$f'(x) = 12x^2 - 4x^3$

let $f'(x)=0$ for stat. points.

$12x^2 - 4x^3 = 0$

$4x^2(3-x) = 0$

$\therefore x=0$, or $x=3$

when $x=0$, $y=0$

when $x=3$, $y=(3)^3(4-3)=27$

Stat pts are $(0,0)$ $(3,27)$

$f''(x) = 24x - 12x^2$

$f''(0) = 24(0) - 12(0)^2$
$= 0$

$\therefore (0,0)$ is possible point of inflexion

| $x$ | $-1$ | $0$ | $1$ |
|---|---|---|---|
| $f''(x)$ | $-36$ | $0$ | $12$ |

$\therefore$ change in concavity $\therefore (0,0)$ is a p.o.I

$f''(3) = 24(3) - 12(3)^2$
$= -36$
$< 0$

$\therefore (3,27)$ is a maximum.

(iii) let $f''(x)=0$ for p.o.I

$24x - 12x^2 = 0$

$12x(2-x) = 0$

$\therefore x=0$, or $x=2$

$\therefore (0,0)$ is a p.o.I from (ii)

when $x=2$, $y = 2^3(4-2)$
$= 16$

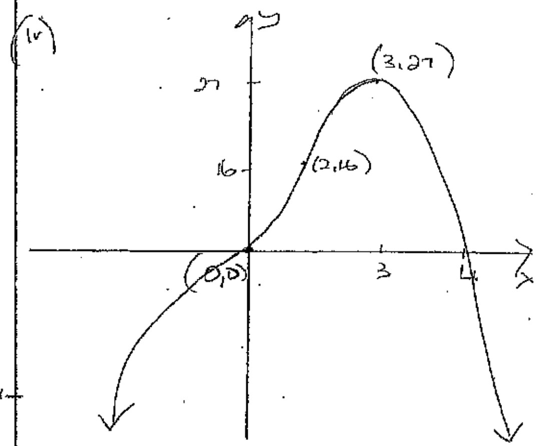
$\therefore (2,16)$ is a poss p.o.I.

| $x$ | $1$ | $2$ | $2.5$ |
|---|---|---|---|
| $y''$ | $12$ | $0$ | $-15$ |

$\therefore$ change in concavity

$\therefore (2,16)$ is a p.o.I.

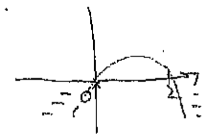
(iv)




(v) $f''(x) < 0$ for concave down

$24x - 12x^2 < 0$.

$12x(2-x) < 0$




$\therefore x<0$ or $x>2$,

$f(x)$ is concave down.

(b)

i) $6^2 = x^2 + r^2$

$r^2 = 36 - x^2$

$r = \sqrt{36-x^2}$ ✓

ii) $V = \frac{1}{3}\pi r^2 h$ $\boxed{h=x}$

$= \frac{1}{3}\pi(\sqrt{36-x^2})^2 . x$ ✓

$= \frac{\pi x(36-x^2)}{3}$ ✓

iii) $V = \frac{36\pi x}{3} - \frac{\pi x^3}{3}$

$= 12\pi x - \frac{\pi x^3}{3}$

$V' = 12\pi - \pi x^2$

let $V'=0$ for stationary points.

$0 = 12\cancel{\pi} - \cancel{\pi}x^2$

$x^2 = 12$

$x = \sqrt{12}$ ($x>0$ as height)

$x = 2\sqrt{3}$

$V'' = -2\pi x$

When $x = 2\sqrt{3}$

$V'' = -2 \times \pi \times 2\sqrt{3}$
$= -4\sqrt{3}\pi$
$< 0$

$\therefore$ Maximum at $x = 2\sqrt{3}$

$\therefore$ the vertical height of the cone which gives a maximum volume is $2\sqrt{3}$ cm.