## State up

Brainstarters 3

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901234

690123456

Triangular and square numbers

Circus towers:

triangular towers

- 1 List the first ten even numbers.
- 2 Sort these numbers, putting all the even numbers in one group, and the odd numbers in another:

17	2002	371	134
60 023	2	748	691
90 704 006	1	95	13
2074	1 000 000	99 999	1256

- 3 Find all the numbers that divide into 6.
- 4 Find all the numbers that divide into 24.
- 5 Find all the even numbers that divide into 36.
- 6 Find all the odd numbers that divide into 90.
- 7 How can you tell if a number is even without dividing it?
- 8 How can you recognise an odd number?

9 Write the next three numbers in each of these patterns:

a 8, 10, 12, <u> </u>	b 27, 30, 33,,,
c 101, 103, 105,,,	d 39, 37, 35,,,
e 44, 39, 34, <u> </u>	f 7, 15, 23,,,
W7L + 1 0 to the	

- 10 What is 8 squared?
- 11 What is  $\sqrt[3]{27}$ ?
- 12 Find two numbers that have a product of 48.

## 3-01 Special number patterns

The numbers 1, 2, 3, 4, 5, ... are called the **counting numbers**. There are groups of counting numbers which make special patterns. We will investigate some of them.

## Exercise 3401

1 Triangular numbers are shown in the diagram below.





			★	
		★	★	
	★	★	★	
	$\star$	★	$\star$	
10				

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- a Why are they called 'triangular numbers'?
- b Work out all the triangular numbers less than 100.
- c Complete four more lines of this pattern:

$$1 = 1$$

$$1 + 2 = 3$$

1+2+3=6

1+2+3+4=10

- d Describe how the pattern in part c works.
- e Use what you have worked out to help you find the 100th triangular number. (Hint: Do you know a quick way to add up all the numbers from 1 to 100?)
- 2 Square numbers are shown in the diagram below.





- a Why are these called 'square numbers'?
- b Work out all the square numbers up to 100.

4

c Complete four more lines of this pattern:

$$1 = 1$$

$$1 + 3 = 4$$

$$1 + 3 + 5 = 9$$

1 + 3 + 5 + 7 = 16

- d Describe how the pattern works.
- e Work out another pattern to help you find the square numbers. What is the 50th square number?

11

f Complete four more lines of these patterns:  $1 = 1^2$ 

, and a second

 $1 + 2 + 1 = 2^2$  $1 + 2 + 3 + 2 + 1 = 3^2$ 

 $3^2 = 2^2 + (2+3)$  $4^2 = 3^2 + (3 + 4)$ 

 $2^2 = 1^2 + (1+2)$ 

g Each square number is said to be the sum of two consecutive triangular numbers.



Show that this is true for the square numbers up to 100.

h Find two numbers that are both triangular numbers and square numbers.

TLF 1, 1935 Circus towers: souare stacks 45678 0123456 678901234567890



3 Leonardo Fibonacci was an Italian mathematician who lived in the early 13th century,
He discovered this pattern when studying the breeding habits of rabbits:
1, 1, 2, 3, 5, 8, 13, 21, ...

The diagram below illustrates this. The vertical arrows labelled 'Birth' indicate the new offspring of a pair of rabbits every two months. The unlabelled arrows indicate the same pair of rabbits. After each month, the number of pairs is a term in Fibonacci's pattern.

Original pair of rabbits pair Birth 1st generation pair 2nd generation 3rd generation Ath generation

- a How is the Fibonacci pattern formed?
- **b** Add five more lines to this pattern:
  - $1 \\ 1 \\ 1 + 1 = 2 \\ 1 + 2 = 3$
  - 1 + 2 = 32 + 3 = 5
  - 2+3=33+5=8
  - ) <del>1</del> ) = 0
- c Write the first 20 Fibonacci numbers.
  - i Write every *third* Fibonacci number, beginning with 2. What number divides evenly into all these numbers?
  - ii Write every *fourth* Fibonacci number, beginning with 3. What number divides evenly into all these numbers?
  - iii Write every *fiftb* Fibonacci number, beginning with 5. What number divides evenly into all these numbers?
- d i Find any triangular numbers in the Fibonacci numbers up to 100.
  - ii Find any square numbers in the Fibonacci numbers up to 100.
- e Pairs of Fibonacci numbers are found by counting along the spirals on pine cones. Investigate how and where else Fibonacci numbers occur in nature.

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4 Blaise Pascal, a French mathematician who lived in the 17th century, studied a triangle of numbers known to the Chinese as the Yanghui triangle. Each row of the triangle is created using the numbers in the row above it. The triangle is known as **Pascal's triangle**. The first seven rows are shown at the right.



- a Complete the next four rows of Pascal's triangle.
- b Describe how the pattern works.
- c Add each row in Pascal's triangle. What do you notice?
- d The diagonals in Pascal's triangle produce some interesting patterns. Write the triangular numbers using Pascal's triangle.
- e We can even find Fibonacci numbers in this pattern. Rewrite the triangle above as a rightangled triangle.

Add along the arrows to find the Fibonacci numbers.

ascal's triangle

5 A palindrome is a word, number or sentence that reads the same forward and backward. The following number, words and sentence are all palindromes: noon 151 Able was I ere I saw Elba (Napoleon Bonaparte)



a Select the palindromes from these numbers.

447 373 656 281 37 22 899 191 797 516 b Find the numbers between 1000 and 2000 that are palindromes.

c The following steps change any number into a palindrome:

- choose any number to start with  $\rightarrow 64$ • reverse the digits and add 46
- 110 • reverse the digits and add 011
- repeat until you get a palindrome. 121



1

5

1

6

10

10

5



45678

0123 378901234567

9012345