1. Write down all the prime numbers between 1 and 100.
2. What is the 12^{th} triangular number?
3. What is the 15^{th} Fibonacci number?
4. What is the 100^{th} square number?
5. Use the following list of numbers in this question:

   27, 30, 225, 437, 2021, 4767, 5568, 6625, 2 376 000, 8 722 890.

   From the list write down the numbers that are divisible by:

   (a) 25
   (b) 3
   (c) 2
   (d) 5
   (e) 9
   (f) 4
   (g) 11
   (h) 8

6. Write down the factors of the following numbers:
   (a) 72
   (b) 462
   (c) 41
7. Find the HCF of the following numbers:
   (a) 3564 and 11 375
   (b) 6800 and 9900
8. Find the LCM of 1080 and 9900.
9. Evaluate:
   (a) $\sqrt{3136}$
   (b) $\sqrt[3]{1\,157\,625}$

Number Theory 2 - Soln

1)

~~1~~	(2)	(3)	~~4~~	(5)	~~6~~	(7)	~~8~~	~~9~~	~~10~~
(11)	~~12~~	(13)	~~14~~	~~15~~	~~16~~	(17)	~~18~~	(19)	~~20~~
~~21~~	~~22~~	(23)	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	(29)	~~30~~
(31)	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	(37)	~~38~~	~~39~~	~~40~~
(41)	~~42~~	(43)	~~44~~	~~45~~	~~46~~	(47)	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	(53)	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	(59)	~~60~~
(61)	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	(67)	~~68~~	~~69~~	~~70~~
(71)	~~72~~	(73)	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	(79)	~~80~~
~~81~~	~~82~~	(83)	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	(89)	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	(97)	~~98~~	~~99~~	~~100~~

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37,
41, 43, 47, 53, 59, 61, 67, 71,
73, 79, 83, 89, 97. ✓✓

2)

0, 1, 3, 6, 10, 15, 21, 28, 36, 45,
55, (66), 78

∴ 12th triangular number is 66 ✓✓
ie (0+1+2+3+4+5+6+7+8+9+10+11)

3)

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610

∴ 15th Fibonacci number is 610 ✓✓

4) $99^2 = (100-1)^2$
$= 10\,000 - 200 + 1$
$= 9801$

∴ The 100th square number is 9801. ✓✓

		a f	b e	c d	Odd digits	Even digits	g Diff	h
5)	27	27	9	7	2	7	5	027
	30	30	3	0	3	0	3	030
	225	25	9	5	7	2	5	225
	437	37	14	7	11	3	8	437
	2021	21	5	1	4	1	3	021
	4767	67	24	7	10	14	4	767
	5568	68	24	8	11	13	2	568
	6625	25	19	5	8	11	3	625
	2 376 000	00	18	0	9	9	0	000
	8 722 890	90	36	0	18	18	0	890

a) 225, 6625, 2 376 000 ✓

b) 27, 30, ~~225,~~ 4767, 5568, 2 376 000, 8 722 890 ✓

c) 30, 5568, 2 376 000, 8 722 890 ✓

d) 30, 225, 6625, 2 376 000, 8 722 890 ✓

e) 27, 225, 2 376 000, 8 722 890 ✓

f) 5568, 2 376 000 ✓

g) 2 376 000, 8 722 890 ✓

h) 5568, 2 376 000 ✓

6) a)
2 | 72
2 | 36
2 | 18
3 | 9
3 | 3
1

∴ $72 = 2 \times 2 \times 2 \times 3 \times 3$

Factors; 1, 2, 3, 2×2, 2×3, 3×3,
2×2×2, 2×3×3, 2×2×3,
2×2×2×3, 2×2×3×3,
2×2×2×3×3
= 1, 2, 3, 4, 6, 8, 9, 12, 18
24, 36, 72

✓✓

b)
2 | 462
3 | 231
7 | 77
11 | 11
1

$462 = 2 \times 3 \times 7 \times 11$

Factors; 1, 2, 3, 7, 11, 2×3, 2×7, 2×11, 3×7, 3×11, 7×11, $2\times3\times7$, $2\times3\times11$, $2\times7\times11$, $3\times7\times11$, $2\times3\times7\times11$

= 1, 2, 3, 6, 7, 11, 14, 21, 22, 33, 42, 66, 77, 154, 231, 462 ✓✓

c) 1, 41 ✓✓

7a)
2 | 3564
2 | 1782
3 | 891
3 | 297
3 | 99
3 | 33
11 | 11 ✓
1

5 | 11375
5 | 2275
5 | 455
7 | 91
13 | 13 ✓
1

No prime factors in common!
∴ HCF = 1 ✓

b)
2 | 6800
2 | 3400
2 | 1700
2 | 850
5 | 425
5 | 85
17 | 17 ✓
1

2 | 9900
2 | 4950
3 | 2475
3 | 825
5 | 275
5 | 55
11 | 11 ✓
1

∴ $6800 = 2\times2\times2\times2\times5\times5\times17$ // $9900 = 2\times2\times3\times3\times5\times5\times11$

∴ HCF = $2\times2\times5\times5$
= 100. ✓

8)
2 | 1080
2 | 540
2 | 270
3 | 135
3 | 45
3 | 15
5 | 5 ✓
1

∴ $9900 = 2^2\times3^2\times5^2\times11$ // $1080 = 2^3\times3^3\times5$

LCM = $5^2\times11\times2^3\times3^3$ ✓
= $100\times2\times11\times27$
= 200×297
= 59 400 ✓

27 × 11 = 297

9a)
2 | 3136
2 | 1569
2 | 784
2 | 392
2 | 196
2 | 98
7 | 49
7 | 7 ✓
1

∴ $3136 = 2^6\times7^2$
$= 2^3\times7\times2^3\times7$
$= 8\times7\times8\times7$
$= 56\times56$

∴ $\sqrt{3136} = 56$ ✓

b)
3 | 1157625
3 | 385875
3 | 128625
5 | 42875
5 | 8575
5 | 1715
7 | 343
7 | 49
7 | 7 ✓
1

∴ $1157625 = 3^3\times5^3\times7^3$
$= 3\times5\times7\times3\times5\times7\times3\times5\times7$
$= 105\times105\times105$

∴ $\sqrt[3]{1157625} = 105$ ✓