Expanding Brackets

A term outside a bracket is multipled with all terms inside a bracket, so that:

$$a(b+c) = a\times b + a\times c = ab + ac$$

This property of multiplication with brackets is called the 'distribution property'. Here are some examples:

Expand the following

a) $3(2+x)$

3×2, $3\times x$

$= 6 + 3x$

b) $x(5+2x+3y)$

$x\times 5$, $x\times 2x$, $x\times 3y$

$= 5x + 2x^2 + 3xy$

There is no difference if the number on the outside is negative. Always multiply the term outside with all the terms inside.

Expand the following

a) $-4(y-5)$

$-4\times y$, -4×-5

$= -4y + 20$

b) $-3p(2q+6p-9)$

$-3p\times 2q$, $-3p\times 6p$, $-3p\times -9$

$= -6pq - 18p^2 + 27p$

Sometimes like terms can be simplified after expanding brackets.

Expand the following

a) $4(2x - x^2) + 3x(5 - 2x)$

$= 8x - 4x^2 + 15x - 6x^2$ (Like terms: $8x$, $15x$; Like terms: $-4x^2$, $-6x^2$)

$= 23x - 10x^2$

b) $-2q(1 + 4p - 3pq) + q(2p - 5pq)$

$= -2q - 8pq + 6pq^2 + 2pq - 10pq^2$ (Like terms: $-8pq$, $2pq$; Like terms: $6pq^2$, $-10pq^2$)

$= -2q - 6pq - 4pq^2$

Multiplying Brackets

Brackets can also be multiplied together. Both terms in the first bracket are multiplied with both terms in the second bracket.

$$(a+b)(c+d)$$

$$\text{①}\quad\text{②}\quad\text{③}\quad\text{④}$$
$$= ac + ad + bc + bd$$

The product of two brackets can also be thought of in this way:

$$(a+b)(c+d) = a(c+d) + b(c+d)$$
$$= ac + ad + bc + bd$$

Find the following products:

a) $(x+3)(x+2)$

$= (x\times x) + (x\times 2) + (3\times x) + (3\times 2)$

$= x^2 + 2x + 3x + 6$ (Like terms: $2x$, $3x$)

$= x^2 + 5x + 6$

b) $(2y+3)(3y-4)$

$= (2y\times 3y) + (2y\times -4) + (3\times 3y) + (3\times -4)$

$= 6y^2 - 8y + 9y - 12$ (Like terms: $-8y$, $9y$)

$= 6y^2 + y - 12$

c) $(2p+3q)(p-4q)$

$= (2p\times p) + (2p\times -4q) + (3q\times p) + (3q\times -4q)$

$= 2p^2 - 8pq + 3pq - 12q^2$

$= 2p^2 - 5pq - 12q^2$

d) $(3a-5b)(3c+2d)$

$= (3a\times 3c) + (3a\times 2d) + (-5b\times 3c) + (-5b\times 2d)$

$= 9ac + 6ad - 15bc - 10bd$

1. Expand these brackets:

a) $5(2y+3)$

b) $6(4-3t)$

c) $-4(-3-5m)$

d) $-3(9-4x)$

e) $\frac{1}{2}(8x-2y)$

f) $-\frac{1}{4}(16n-24m)$

g) $2x(4x-2y)$

h) $x^2(4xy-3y)$

i) $5p(q+2pq+3p)$

j) $3xy(2x-2y+1)$

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1. Expand these brackets:

a) $5(2y+3)$

$= 10y+15$

b) $6(4-3t)$

$= 24-18t$

c) $-4(-3-5m)$

$= 12+20m$

d) $-3(9-4x)$

$= 27+12x$

e) $\frac{1}{2}(8x-2y)$

$= 4x-y$

f) $-\frac{1}{4}(16n-24m)$

$= -4n+6m$

g) $2x(4x-2y)$

$= 8x^2-4xy$

h) $x^2(4xy-3y)$

$= 4x^3y-3yx^2$

i) $5p(q+2pq+3p)$

$= 5pq+10p^2q+15p^2$

j) $3xy(2x-2y+1)$

$= 6x^2y-6xy^2+3xy$

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2. Expand these brackets:

a) $x(2+3x-4y+4xy)$

$= 2x+3x^2-4xy+4x^2y$

b) $-2abc(3-ab-4ac+2a^2b-5bc+3c^2)$

$= -6abc+2a^2b^2c+8a^2bc^2-4a^3b^2c+10ab^2c^2-6abc^3$

2. Expand these brackets:

a) $x(2 + 3x - 4y + 4xy)$

b) $-2abc(3 - ab - 4ac + 2a^2b - 5bc + 3c^2)$

3. Expand then simplify using like terms:

a) $4(x - 3x^2) + 2x(7x - 5)$

b) $d^2(1 - 2c + 3cd) + cd(5d - 3cd^2 - 2)$

c) $5p(3q - 2pq^2) - 4q(2p^2q + 2p - 3)$

d) $-3ab(a^2b - 2b^2 + 6a) - b(8a^3b + 2ab^2 - 4a^2) + a^2b$

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1. Expand these brackets:

a) $5(2y + 3)$

$= 10y + 15$

b) $6(4 - 3t)$

$= 24 - 18t$

c) $-4(-3 - 5m)$

$= 12 + 20m$

d) $-3(9 - 4x)$

$= 27 + 12x$

e) $\frac{1}{2}(8x - 2y)$

$= 4x - y$

f) $-\frac{1}{4}(16n - 24m)$

$= -4n + 6m$

g) $2x(4x - 2y)$

$= 8x^2 - 4xy$

h) $x^2(4xy - 3y)$

$= 4x^3y - 3yx^2$

i) $5p(q + 2pq + 3p)$

$= 5pq + 10p^2q + 15p^2$

j) $3xy(2x - 2y + 1)$

$= 6x^2y - 6xy^2 + 3xy$

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2. Expand these brackets:

a) $x(2 + 3x - 4y + 4xy)$

$= 2x + 3x^2 - 4xy + 4x^2y$

b) $-2abc(3 - ab - 4ac + 2a^2b - 5bc + 3c^2)$

$= -6abc + 2a^2b^2c + 8a^2bc^2 - 4a^3b^2c + 10ab^2c^2 - 6abc^3$

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3. Expand then simplify using like terms:

a) $4(x-3x^2)+2x(7x-5)$
$=4x-12x^2+14x^2-10x$
$=2x^2-6x$

b) $d^2(1-2c+3cd)+cd(5d-3cd^2-2)$
$=d^2-2cd^2+3cd^3+5cd^2-3c^2d^3-2cd$
$=3cd^3+3cd^2+d^2-3c^2d^3-2cd$

c) $5p(3q-2pq^2)-4q(2p^2q+2p-3)$
$=15pq-10p^2q^2-8p^2q^2-8pq+12q$
$=7pq+12q-18p^2q^2$

d) $-3ab(a^2b-2b^2+6a)-b(8a^3b+2ab^2-4a^2)+a^2b$
$=-3a^3b^2+6ab^3-18a^2b-8a^3b^2-2ab^3+4a^2b+a^2b$
$=4ab^3-13a^2b-11a^3b^2$

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4. Expand and simplify:

a) $(2x+2)(x-1)$
$=2x^2-2x+2x-2$
$=2x^2-2$

b) $(2st+3t)(4t-2s)$
$=8st^2-4s^2t+12t^2-6st$

c) $(4k-1)(3-2k)$
$=12k-8k^2-3+2k$
$=-8k^2+14k-3$

d) $(ab-2a)(3b^2+6ab)$
$=3ab^3+6a^2b^2-6ab^2-12a^2b$

e) $(x+y)(x-y)$
$=x^2-xy+xy-y^2$
$=x^2-y^2$

f) $(ab-cd)(ab+cd)$
$=a^2b^2+abcd-abcd-c^2d^2$
$=a^2b^2-c^2d^2$

g) $(3x^2y-4xy^2)(xy+2x)$
$=3x^3y^2+6x^3y-4x^2y^3-8x^2y^2$

h) $(3ab^3c-4a^2bc^2)(5abc-2a^2b^2c^2)$
$=15a^2b^4c^2-6a^3b^5c^3-20a^3b^3c^3+8a^4b^3c^4$

4. Expand and simplify:

a) $(2x+2)(x-1)$

b) $(2st+3t)(4t-2s)$

c) $(4k-1)(3-2k)$

d) $(ab-2a)(3b^2+6ab)$

e) $(x+y)(x-y)$

f) $(ab-cd)(ab+cd)$

g) $(3x^2y-4xy^2)(xy+2x)$

h) $(3ab^3c-4a^2bc^2)(5abc-2a^2b^2c^2)$

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3. Expand then simplify using like terms:

a) $4(x-3x^2)+2x(7x-5)$
$=4x-12x^2+14x^2-10x$
$=2x^2-6x$

b) $d^2(1-2c+3cd)+cd(5d-3cd^2-2)$
$=d^2-2cd^2+3cd^3+5cd^2-3c^2d^3-2cd$
$=3cd^3+3cd^2+d^2-3c^2d^3-2cd$

c) $5p(3q-2pq^2)-4q(2p^2q+2p-3)$
$=15pq-10p^2q^2-8p^2q^2-8pq+12q$
$=7pq+12q-18p^2q^2$

d) $-3ab(a^2b-2b^2+6a)-b(8a^3b+2ab^2-4a^2)+a^2b$
$=-3a^3b^2+6ab^3-18a^2b-8a^3b^2-2ab^3+4a^2b+a^2b$
$=4ab^3-13a^2b-11a^3b^2$

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4. Expand and simplify:

a) $(2x+2)(x-1)$
$=2x^2-2x+2x-2$
$=2x^2-2$

b) $(2st+3t)(4t-2s)$
$=8st^2-4s^2t+12t^2-6st$

c) $(4k-1)(3-2k)$
$=12k-8k^2-3+2k$
$=-8k^2+14k-3$

d) $(ab-2a)(3b^2+6ab)$
$=3ab^3+6a^2b^2-6ab^2-12a^2b$

e) $(x+y)(x-y)$
$=x^2-xy+xy-y^2$
$=x^2-y^2$

f) $(ab-cd)(ab+cd)$
$=a^2b^2+abcd-abcd-c^2d^2$
$=a^2b^2-c^2d^2$

g) $(3x^2y-4xy^2)(xy+2x)$
$=3x^3y^2+6x^3y-4x^2y^3-8x^2y^2$

h) $(3ab^3c-4a^2bc^2)(5abc-2a^2b^2c^2)$
$=15a^2b^4c^2-6a^3b^5c^3-20a^3b^2c^3+8a^4b^3c^4$

Shortcuts for Multiplying Brackets

A product of two brackets it called a 'binomial product'. Some binomial products can be found more easily than others. Look at this example:

$(a+b)(a-b)$
$=a^2-ab+ab-b^2$

The middle terms cancel each other away

$=a^2-b^2$

Can you see the shortcut? Their product is the square of the first term minus the square of the second term. Here are some examples using this shortcut:

Find these products:

a) $(x+2)(x-2)$
$=(x)^2-(2)^2$
(1st term squared) (2nd term squared)
$=x^2-4$

b) $(3y-5)(3y+5)$
$=(3y)^2-(5)^2$
(1st term squared) (2nd term squared)
$=9y^2-25$

What about the perfect square of a bracket? Look at this example:

$(a+b)^2=(a+b)(a+b)$
$=a^2+ab+ab+b^2$
$=a^2+2ab+b^2$
(1st term squared) (2 X product of terms) (2nd term squared)

Can you see the shortcut? Here are some examples using this shortcut:

Find these products:

a) $(x+4)^2$
$=(x^2)+2(x)(4)+(4)^2$
(1st term squared) (2 X product of terms) (2nd term squared)
$=x^2+8x+16$

b) $(4p-2q)^2$
$=(4p)^2+2(4p)(-2q)+(-2q)^2$
(1st term squared) (2 X product of terms) (2nd term squared)
$=16p^2-16pq+4q^2$

These shortcuts may seem silly for these easier examples, but they are especially useful for more complicated examples.

Find these products:

a $\left(\frac{4}{x}+x\right)\left(\frac{4}{x}-x\right)$

$=\left(\frac{4}{x}\right)^2-(x)^2$

$=\frac{16}{x^2}-x^2$

b $(3x^2y+4pq^3)(3x^2y-4pq^3)$

$=(3x^2y)^2-(4pq^3)^2$

$=9x^4y^2-16p^2q^6$

c $\left(5y-\frac{3}{2y}\right)^2$

$=(5y)^2+2(5y)\left(-\frac{3}{2y}\right)+\left(-\frac{3}{2y}\right)^2$

$=25y^2+2\left(-\frac{15y}{2y}\right)+\frac{9}{4y^2}$

$=25y^2-15+\frac{9}{4y^2}$

d $(2m^3n^2-3n)^2$

$=(2m^3n^2)^2+2(2m^3n^2)(-3n)+(-3n)^2$

$=4m^6n^4+2(-6m^3n^3)+9n^2$

$=4m^6n^4-12m^3n^3+9n^2$

We can even use these rules in calculations using actual numbers.

a 11^2-9^2

$=(11+9)(11-9)$

$=(20)(2)$

$=40$

b 36^2

$=(30+6)^2$

$=30^2+2(30)(6)+6^2$

$=900+360+36$

$=1296$

1. Prove that $(x+y)(x-y)=x^2-y^2$.

2. Simplify these perfect squares:

a $(2p+1)^2$

b $(5x-7y)^2$

c $(-3m-2n)^2$

d $(3p-pq)^2$

e $\left(\frac{1}{x}+x\right)^2$

f $\left(2y^2-\frac{3}{y}\right)^2$

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1. Prove that $(x+y)(x-y)=x^2-y^2$.

$$(x+y)(x-y) = x^2 - xy + xy - y^2$$
$$= x^2 - y^2$$

2. Simplify these perfect squares:

a) $(2p+1)^2$

$= (2p+1)(2p+1)$

$= 4p^2 + 4p + 1$

b) $(5x-7y)^2$

$= (5x-7y)(5x-7y)$

$= 25x^2 - 35xy - 35xy + 49y^2$

$= 25x^2 - 70xy + 49y^2$

c) $(-3m-2n)^2$

$= (-3m-2n)(-3m-2n)$

$= 9m^2 + 6mn + 6mn + 4n^2$

$= 9m^2 + 12mn + 4n^2$

d) $(3p-pq)^2$

$= (3p-pq)(3p-pq)$

$= 9p^2 - 3p^2q - 3p^2q + p^2q^2$

$= 9p^2 - 6p^2q + p^2q^2$

e) $\left(\frac{1}{x}+x\right)^2$

$= \left(\frac{1}{x}+x\right)\left(\frac{1}{x}+x\right)$

$= \frac{1}{x^2} + 1 + 1 + x^2$

$= \frac{1}{x^2} + x^2 + 2$

f) $\left(2y^2 - \frac{3}{y}\right)^2$

$= \left(2y^2 - \frac{3}{y}\right)\left(2y^2 - \frac{3}{y}\right)$

$= 4y^4 - \frac{6y^2}{y} - \frac{6y^2}{y} + \frac{9}{y^2}$

$= 4y^4 - 12y + \frac{9}{y^2}$

3. Simplify these binomial products:

a) $(x+3)(x-3)$

b) $(5-m)(5+m)$

c) $(2b+3c)(2b-3c)$

d) $(7x+3y)(7x-3y)$

e) $(xy+x^2y^2)(xy-x^2y^2)$

f) $(10a^2b^3-4ab^2)(10a^2b^3+4ab^2)$

g) $\left(\frac{1}{2m}+m^2\right)\left(\frac{1}{2m}-m^2\right)$

h) $\left(\frac{2}{5x}-3y\right)\left(\frac{2}{5x}+3y\right)$

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3. Simplify these binomial products:

a $(x+3)(x-3)$
$= x^2 - 3x + 3x - 9$
$= x^2 - 9$

b $(5-m)(5+m)$
$= 25 + 5m - 5m - m^2$
$= 25 - m^2$

c $(2b+3c)(2b-3c)$
$= 4b^2 - 6bc + 6bc - 9c^2$
$= 4b^2 - 9c^2$

d $(7x+3y)(7x-3y)$
$= 49x^2 - 21xy + 21xy - 9y^2$
$= 49x^2 - 9y^2$

e $(xy + x^2y^2)(xy - x^2y^2)$
$= x^2y^2 - x^3y^3 + x^3y^3 - x^4y^4$
$= x^2y^2 - x^4y^4$

f $(10a^2b^3 - 4ab^2)(10a^2b^3 + 4ab^2)$
$= 100a^4b^6 - 40a^3b^5 + 40a^3b^5 - 16a^2b^4$
$= 100a^4b^6 - 16a^2b^4$

g $\left(\frac{1}{2m} + m^2\right)\left(\frac{1}{2m} - m^2\right)$
$= \frac{1}{4m^2} - \frac{m}{2} + \frac{m}{2} - m^4$
$= \frac{1}{4m^2} - m^4$

h $\left(\frac{2}{5x} - 3y\right)\left(\frac{2}{5x} + 3y\right)$
$= \frac{4}{25x^2} - \frac{6y}{5x} + \frac{6y}{5x} - 9y^2$
$= \frac{4}{25x^2} - 9y^2$

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4. Find △ and □ in each of the following:

a $(x+3)^2 = x^2 + \triangle x + 9$
$(x+3)^2 = x^2 + 3x + 3x + 9$
$\triangle = 6$

b $(2x+\triangle)^2 = 4x^2 + 12xy + 9y^2$
$4x^2 + 4x\triangle + \triangle^2 = 4x^2 + 12xy + 9y^2$
$\triangle^2 = 9y^2$
$\triangle = 3y$

c $(\triangle - 5q)^2 = 16p^2 - 40pq + 25q^2$
$\triangle^2 - 10q\triangle + 25q^2 = 16p^2 - 40pq + 25q^2$
$\therefore \triangle^2 = 16p^2$
$\triangle = 4p$

d $(2m+\triangle)^2 = 4m^2 + 16mn^2 + \square$
$4m^2 + 4m\triangle + \triangle^2 = 4m^2 + 16mn^2 + \square$
$\therefore 4m\triangle = 16mn^2$
$\therefore \triangle = 4n^2$
$\therefore \square = \triangle^2 = 16n^4$