YEAR 10 Advanced - Quadratic Equation Test

QUESTION 1

Solve for x:
(leave any non rational answers in <u>surd</u> form)

(a) $6x - x^2 = 0$

(b) $(x-3)^2 = 16$

(c) $x^3 = 9x$

(d) $3x^2 - 24x = 60$

(e) $5x^2 - 8x + 2 = 0$

(f) $\dfrac{10}{x} + 1 = x - 2$

QUESTION 2

Solve the following by using the "*completing the square*" method.

(a) $x^2 - 10x = -7$

(b) $2x^2 - 10x + 5 = 0$

QUESTION 3

(a) A rectangular garden 12m x 7m is surrounded by a path of uniform width. Find the width of the path to the nearest cm(ie 2 d.p.) if the area of the path is 24m².

12 m

7m

(b) A watch making company with manufacturing plants in Australia and Japan can produce 1000 watches per day. If it costs $45 to produce a watch in Australia and $30 in Japan, how many watches can be made in each country to have an average cost of $34.50 per watch?

(c) The height of a ball above the ground, thrown vertically upwards is given by:
$$h = 30t - 5t^2$$

(i) Find the time, t, elapsed before the ball reaches a height of 40 metres

(ii) Find how long before the ball falls back to the ground.
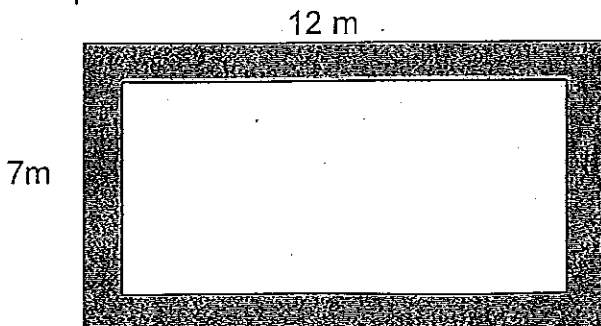
(iii) Find the maximum height

QUESTION 4

a) Find the value of k which would give only one solution.
$2kx^2 + 15x + 20 = 0$

b) Find the values of k for which $x^2 + 6x + k = 0$ has no solutions.

c) Write down the quadratic equation whose roots are . $\dfrac{2}{3}$, & $-1\dfrac{3}{4}$ Your answer should be in general form with no fractions

QUESTION 5

(a) If $a^4 - a^2b^2 + ab^2 - a^3 = 0$ find the values of a that will satisfy this equation

(b) By substituting $x^2 + x = v$, find all the solutions to the equation
$(x^2 + x)^2 - 8(x^2 + x) + 12 = 0$

Q1. (a) $x = 0$ or $x = 6$

(b) $x = 7$ or $x = -1$

(c) $x = 3, -3, 0$

(d) $x = 10$ or $x = -2$

(e) $x = \dfrac{4+\sqrt{6}}{5}$ or $x = \dfrac{4-\sqrt{6}}{5}$

(f) $x = 5$ or $x = -2$

Q2.

(a) $x^2 - 10x + 25 = -7 + 25$

$(x-5)^2 = 18$

$x = 5 \pm 3\sqrt{2}$

(b) $2\left(x^2 - 5x + \dfrac{25}{4}\right) = -5 + \dfrac{25}{2}$

$2\left(x - \dfrac{5}{2}\right)^2 = \dfrac{15}{2}$

$x - \dfrac{5}{2} = \pm\sqrt{\dfrac{15}{4}}$

$x = \dfrac{5}{2} \pm \dfrac{\sqrt{15}}{2}$

Q3 (a) Let width $= x$

$\therefore$ Area of path is :-

$A = 12 \times 7 - (7-2x)(12-2x)$

$= 84 - \{84 - 38x + 4x^2\}$

$\therefore 24 = 38x - 4x^2 \quad (\div 2)$

$\therefore x = \dfrac{19 \pm \sqrt{265}}{4} = 8.82 \, m^2$

(b) Let $x =$ no. made in Aust.

$1000 - x =$ No. made in Japan

$\text{Aver. Cost} = \dfrac{45x + (1000-x) \times 30}{1000}$

$\therefore 34.5 = \dfrac{15x + 30000}{1000}$

$\therefore x = 300 \text{ (Aust)} \quad y = 700 \text{ (Japan)}$

(c) Solve $40 = 30t - 5t^2 \rightarrow t = 2 \; (t=4)$

(ii) Solve $0 = 30t - 5t^2 \rightarrow t = 6 \; (t=0)$

(iii) $h = ?$ when $t = 3 \rightarrow h = 45m$.

Q4 (a) One solution $\Rightarrow \Delta = 0$

$\Delta = 225 - 160k = 0$ when $k = \dfrac{45}{32}$

(b) No solutions $\Rightarrow \Delta < 0$

$\Delta = 36 - 4k < 0 \rightarrow k > 9$

(c) roots are $\dfrac{2}{3}$ & $-\dfrac{7}{4}$ then

factors are $\left(x - \dfrac{2}{3}\right)$ & $\left(x + \dfrac{7}{4}\right)$

ie quadratic equ^n is :

$(3x - 2)(4x + 7) = 0$

ie $12x^2 + 13x - 14 = 0$

Q5.

(a) $a^2(a^2 - b^2) + a(b^2 - a^2) = 0$

$\therefore (a^2 - b^2)(a^2 - a) = 0$

$\therefore (a - b)(a + b) a (a - 1) = 0$

$\therefore a = 0$ or $a = 1$ or ...

$a = b$ or $a = -b$

(b) $(V - 6)(V - 2) = 0$

$\therefore V = 2$ or $V = 6$

ie $x^2 + x = 2 \qquad x^2 + x = 6$

$(x + 2)(x - 1) = 0 \qquad (x + 3)(x - 2) = 0$

$\therefore x = -2, 1, -3,$ or 2.