C.E.M. TUITION

Name :________________________________

Topic : Quadratic Functions

(Lesson Notes)

Year 10 - Advanced

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23. Quadratic functions I :

A quadratic function is a function defined by the rule : $f(x) = ax^2 + bx + c$ e.g. $f(x) = x^2 + 2x - 1$ or $y = 2x^2 + 5$. Such a function can be sketched as a parabola on the number plane.

A quadratic expression is one that is in the form : $ax^2 + bx + c$ e.g. $x^2 + 2x - 3$. Such an expression cannot be solved but can only be factorised.

A quadratic equation such as : $ax^2 + bx + c = 0$ e.g. $x^2 + 2x - 3 = 0$ may be factorised and solved.

23.1 Graphs of quadratic functions :

(a) Coefficient of the term in x^2

If $a > 0$ then the parabola is concave upwards.

If $a < 0$ then the parabola is concave downwards.

(b) The x intercepts of the parabola

Find the x intercepts of the parabola, if any, by letting $y = 0$.

(c) The axis and the vertex of the parabola

Now, find the mid-point of these intercepts which gives the equation of the axis of symmetry and its vertex. Otherwise use the equation of the axis of symmetry i.e. $x = -\dfrac{b}{2a}$.

Examples :

Sketch the following parabolas :

(1) $y = x^2 + 5x + 4$

(2) $y = x^2 + 5x + 6$

(3) $y = x^2 + 2x - 3$

(4) $y = x^2 - 5x - 6$

(5) $y = 1 - 2x - x^2$

(6) $y=-3x^2+7x-2$

(7) $y = x^2 - 4x + 6$

(8) $y = 2x^2 - 8x + 5$

23.2 Maximum or minimum values :

The turning point of the parabola determines the maximum (i.e. $a < 0$) or minimum (i.e. $a > 0$) of a quadratic function.

Examples :

(1) Find the minimum value of the function :

$$y = 2x^2 + 6x - 5$$

$y = -\frac{19}{2}$

(2) Find the maximum value of the function :

$$f(x) = 20 + 8x - x^2$$

$\boxed{f(x) = 36}$

(3) A farmer wishes to enclose a vegetable patch using 20 metres of mesh-wire netting.

(a) Find the area of the largest rectangular patch he can enclose.

$25\ \text{m}^2, x = 5$

(b) What are the dimensions of this patch ?

5m by 5m

(4) From a market survey, a manufacturer knows that the cost $C(x)$ in dollars of producing a unit number of goods is given by : $C(x) = x^2 - 24x + 150$, where x is the number of plants operating. Find the number of plants he should operate to minimise the unit cost.

Min. cost = \$6, with 12 plants

23.3 Quadratic equations :

A quadratic function such as $ax^2+bx+c=0$ may be solved by factoring or the quadratic formula which is given by :

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Examples :

Solve the following by factorising:

(1) $x^2-2x-15=0$

$x=-3$ or 5

(2) $2x^2-5x+2=0$

$x=\frac{1}{2}$ or 2

Use the quadratic formula to :

(3) evaluate x if $x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{5}}{2}$

(4) find x to 1 decimal place

$x^2 + 2x - 5 = 0$

$x = -3.4$ or 1.4

23.4 Quadratic inequalities :

Solve the quadratic inequation such as $ax^2 + bx + c > 0$ by testing suitable values of x to make the statement true.

Examples :

Solve this inequation :

(1) $x^2 - 4x + 3 < 0$

$1 < x < 3$

(2) $x^2 - 3x \geq 10$

$x \leq -2$ or $x \geq 5$

(3) $48 - 2x - x^2 \geq 0$

$-8 \leq x \leq 6$

(4) $2x^2 - x - 3 > 0$

$x < -1$ or $x > 1\frac{1}{2}$

(2) Solve this inequation and graph the solution on a number line.

$12x^2 - 7x - 10 \leq 0$

$-\frac{2}{3} \leq x \leq \frac{5}{4}$