CO-ORDINATE GEOMETRY

1. For the diagram where P=(0,3), Q=(-5,0) and R=(3,-2)

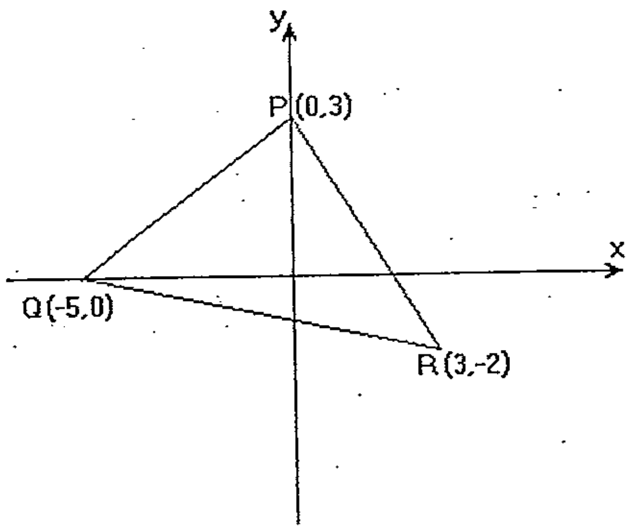
   (a) Find the equation of the line PQ

   (b) Show that the Δ PQR is a right angled isosceles triangle

   (c) (i) Find M the midpoint of QR

   (ii) Find the perpendicular bisector, ℓ, of QR.

   (iii) Prove this line ℓ passes through the point P.



2. The line through AP meets the line $y=\sqrt{3}\,x$ at the point C.

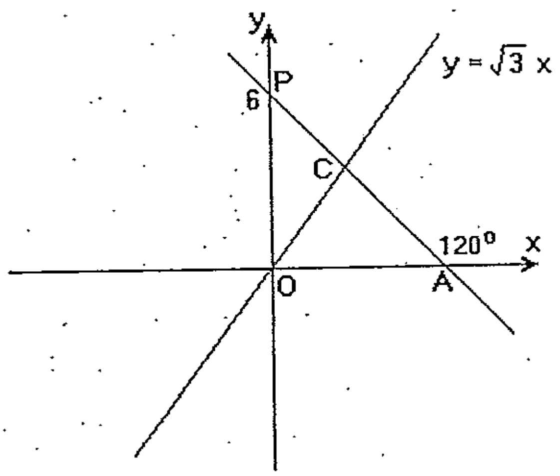
   If AP makes an angle of 120° with the positive x-axis, find:

   (a) the equation of AP

   (b) the co-ordinates of A (in exact form)

   (c) the co-ordinates of C

   (d) the area of the triangle Δ AOC



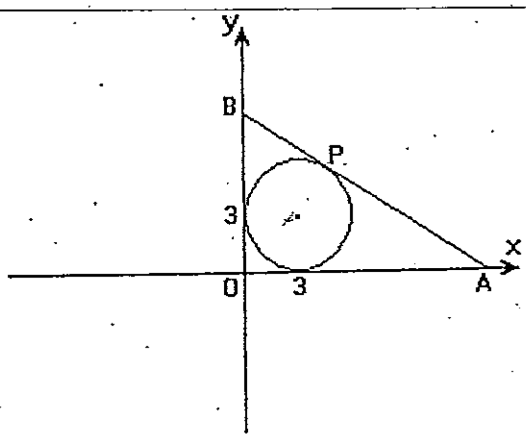
3. For the diagram opposite AB has equation $3x+4y-36=0$.

   (a) Show that the line $3x+4y-36=0$ is a tangent to the circle $(x-3)^2+(y-3)^2=9$

   (b) If this tangent cuts the x-axis at A, and the y-axis at B, find the co-ordinates of A & B.

   (c) Show that the point of contact of the tangent is P = (4·8 , 5·4)

   (d) If this point, P, divides AB in the ratio K : 1 find K



Answers – Co-ord. Geometry

Qu 1 (a) $m_{PQ} = \frac{(3)-(0)}{(0)-(-5)} = 3/5$ $\therefore$ Equ: $y = \frac{3}{5}x + 3$ (or $3x - 5y + 15 = 0$)

(b) $m_{PR} = \frac{(3)-(-2)}{(0)-(3)} = \frac{5}{-3}$ $\therefore$ Since $m_{PQ} \times m_{PR} = 3/5 \times -5/3 = -1 \Rightarrow PR \perp PQ$

Also $^*PQ = \sqrt{5^2+3^2} = \sqrt{34}$ and $^*PR = \sqrt{5^2+3^2} = \sqrt{34}$ $\therefore$ $^*PR = {}^*PQ$

Hence since $PQ \perp PR$ and $^*PQ = {}^*PR$ then ΔPQR is right-isosceles triangle

(c)(i) $M = \left(\frac{-5+3}{2}, \frac{0-2}{2}\right) = (-1,-1)$

(ii) $m_{QR} = \frac{(0)-(-2)}{(-5)-(3)} = \frac{2}{-8} = -1/4$ $\therefore$ $m_l = +4$

Since "l" passes through $M=(-1,-1)$ equ of "l" is $y+1 = 4(x+1)$

ie $y = 4x+3$

(iii) Substitute $(0,3)$ into equ of "l": L.H.S. = 3 R.H.S. = $4(0)+3$ = LHS.

$\therefore$ "l" passes through $(0,3)$

Qu 2

(a) gradient of AP $m_{AP} = \text{Tan}\,120° = -\sqrt{3}$ (& y-intercept is $b=6$)

$\therefore$ Equ. of AP is $y = -\sqrt{3}x + 6$

(b) AP cuts x-axis at A when $y=0$ $\therefore$ $0 = -\sqrt{3}.x + 6 \Rightarrow x = 6/\sqrt{3}$

$= 2\sqrt{3}$

$\therefore$ $A = (2\sqrt{3}, 0)$

(c) Solving $\left.\begin{array}{l} y = -\sqrt{3}x + 6 \\ y = \sqrt{3}.x \end{array}\right\} \Rightarrow \left.\begin{array}{l}\sqrt{3}.x = -\sqrt{3}x + 6 \\ \therefore 2\sqrt{3}x = 6\end{array}\right\}$ $\therefore$ $x = \sqrt{3}$, $y = 3$ ie $C = (\sqrt{3}, 3)$

(d) Area of ΔOCA is $\frac{1}{2}$ base $\times$ height (of C) $= \frac{1}{2} \times 2\sqrt{3} \times 3 = 3\sqrt{3}$ units^2

Qu ③

(a) We need to show that the centre of the circle $(3,3)$ has a perpendicular distance from $3x+4y-36=0$ of 3 units (radius of circle)

$$p = \left|\frac{3(3)+4(3)-36}{\sqrt{3^2+4^2}}\right| = \left|\frac{9+12-36}{5}\right| = \left|\frac{-15}{5}\right| = 3 \text{ units} \quad // \quad \therefore 3x+4y-36=0 \text{ is a tangent.}$$

(b) AB cuts the x-axis when $y=0$ $\therefore x=12$ ie at $A=(12,0)$
and " " y-axis " $x=0$ $\therefore y=9$ ie at $B=(0,9)$.

(c) We could solve simultaneously - but this is algebraically tedius. OR
Prove that P is 3 units from centre $(3,3)$ AND that P is on the tangent $3x+4y-36=0$ (substitute it in) } OR
We could find the equ. of the line through circle centre $(3,3)$ while perpendicular to AB and solve it simultaneously with AB}.

(d)

A $(12,0)$ B $(0,9)$

$k:1$

$$P = \left(\frac{12+0}{k+1}, \frac{0+9k}{k+1}\right) = (4\cdot8, 5\cdot4) \quad !$$

$\therefore \dfrac{12}{k+1} = 4\cdot8$ $\therefore$ $12 = 4\cdot8k + 4\cdot8$
$120 = 48k + 48$
$1\cdot5 = k$

$\therefore$ P divides AB in the ratio $1\cdot5 : 1$ OR $3 : 2$

Check answer: $\dfrac{9k}{k+1} = 5\cdot4 \Rightarrow 9k = 5\cdot4k + 5\cdot4$
$90k - 54k = 54$
$36k = 54$
$\Rightarrow k = 1\cdot5$ ✓