TEST 7 Expansion and Factorisation of Algebraic Expressions

Marks: /80

Time: 1 hour 30 minutes

Name: ..... Date: .....

INSTRUCTIONS TO CANDIDATES

Section A (40 marks) **Time: 45 minutes**

1. Answer all the questions in this section.
2. Calculators may **not** be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [ ] at the end of each question.

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1 Expand

(a) $(5x + 3)^2$,

(b) $(2x + 5y)(2x - 5y)$.

Answer (a) ..... [2]

(b) ..... [2]

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2 Factorise

(a) $8x^2 - 6xy$,

(b) $1 - 121x^2$,

(c) $x^2 + 12x + 36$.

Answer (a) ..... [2]

(b) ..... [2]

(c) ..... [2]

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3 Evaluate the following using algebraic rules.

(a) $35^2 - 280 + 16$

(b) $\dfrac{884^2 - 116^2}{219 \times 24 - 24 \times 211}$

Answer (a) ..... [2]

(b) ..... [3]

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4 Simplify $(2x + 5)(x - 2) + 3(x + 4)$.

Answer ..... [2]

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5 (a) Factorise $xt - yt$.
(b) Hence evaluate $1896 \times 0.05489 - 896 \times 0.05489$.

Answer (a) ..... [1]

(b) ..... [1]

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6 Factorise
(a) $63y^2 - 7x^2$,
(b) $3px - 9x + py - 3y$,
(c) $7(x + y)^2 - 4(x + y)(x - y)$.

Answer (a) ..... [2]

(b) ..... [2]

(c) ..... [2]

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7 (a) Expand $(2x + 3)(5 - 3x)$.
(b) Factorise completely $5x^3 - 2x^2 - 5x + 2$.

Answer (a) ..... [2]

(b) ..... [2]

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8 Factorise completely
(a) $6x^2 - 4x - 3xy + 2y$,
(b) $4x^3 - 100x$.

Answer (a) ..... [2]

(b) ..... [2]

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9 (a) Simplify

(i) $x^2 - (x + 6)(x - 6)$,

(ii) $x^2 - (x + y)(x - y)$.

(b) Using your answers to part (a), write down the value of $29\,642^2 - 29\,650 \times 29\,634$.

Answer *(a)* (i) ..... [2]

(ii) ..... [1]

(b) ..... [1]

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10 If $x^2 + y^2 = 188$ and $xy = 74$, find the value of $\frac{1}{8}(x + y)^2$.

Answer ..... [3]

INSTRUCTIONS TO CANDIDATES

Section B (40 marks) **Time: 45 minutes**

1. Answer all the questions in this section.
2. Calculators may be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [ ] at the end of each question.

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11 (a) Expand and simplify $(2x + 5)(5 - 2x)^2$. [2]

(b) Factorise completely

(i) $4x^2 - (x + y)^2$, [1]

(ii) $p(2x - 3y) - q(12y - 8x)$. [1]

Answer (a) ..... [3]

(b) (i) ..... [2]

(ii) ..... [2]

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12 (a) If $89^2 - 11^2 = 39k$, find the value of k.

(b) (i) Factorise $x^2 - 16$.

(ii) Use your answer to part (b) (i) to find two factors of 2484 other than 1 and 2484.

(c) Given that $(a + b)^2 = 36$ and $a^2 + b^2 = 17$, calculate the value of $4ab$.

Answer (a) $k =$ ..... [3]

(b) (i) ..... [2]

(ii) ....., ...

..... [2]

(c) ..... [3]

13 (a) Simplify $3x(1 - 2x) + (2x - 3)^2$.

(b) Factorise

(i) $64a^2 - 4d^2e^2$,

(ii) $8px^2 - 3q + 12qx^2 - 2p$.

Answer (a) ..... [3]

(b) (i) ..... [2]

(ii) ..... [2]

14 (a) Expand and simplify

(i) $(3x - 2)(3x + 2)(x - 4)$,

(ii) $(2x - 1)(x^2 + 3x - 1)$.

(b) Factorise completely $8x^3y - 50xy^3$.

Answer *(a)* (i) ..... [3]

(ii) ..... [2]

(b) ..... [3]

15 (a) Factorise

(i) $p(q^2 - r^2) - qr(q - r)$,

(ii) $s(2 - s)(1 - s) + s^2(s + 3)$.

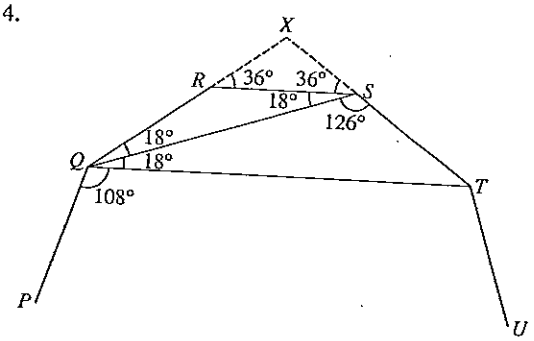
(b) Given that $x + y = 12$ and $x^2 - y^2 = 36$, find the value of $7x - 7y$.

Answer *(a)* (i) ..... [2]

(ii) ..... [3]

(b) ..... [3]

14.



(a) $X\hat{R}S = \frac{360°}{10}$
$= 36°$

Teacher's Tip
$X\hat{R}S$ is the exterior angle of a regular polygon with 10 sides.

(b) $\triangle RQS$ is isosceles.
$R\hat{Q}S + R\hat{S}Q = X\hat{R}S$ ext. $\angle$ = sum of int. opp. $\angle$s
$2R\hat{S}Q = 36°$ $R\hat{Q}S = R\hat{S}Q$
$R\hat{S}Q = \frac{36°}{2}$
$= 18°$

(c) $Q\hat{S}T = 180° - X\hat{S}R - R\hat{S}Q$ adj. $\angle$s on a str. line
$= 180° - 36° - 18°$
$= 126°$

(d) $R\hat{Q}S = R\hat{S}Q = 18°$ $\triangle QRS$ is isosceles.
$S\hat{Q}T = R\hat{S}Q$ alt. $\angle$s, $RS \parallel QT$
$= 18°$
Interior angle of polygon $= 180° - 36°$
$= 144°$
$\therefore P\hat{Q}T = 144° - 18° - 18°$
$= 108°$

15. (a) Each interior angle of the hexagonal tile
$= \frac{(6-2) \times 180°}{6}$
$= 120°$

(b) **Angles at any one point is 360°. No combination of squares and hexagons i.e. 90° and 120° gives 360°.**

(c) (i) Size of each angle
$= 360° - 90° - 120°$
$= 150°$

(ii) Exterior angle $= 180° - 150° = 30°$
No. of sides $= \frac{360°}{30°} = 12$

Test 7: Expansion and Factorisation of Algebraic Expressions

Section A

1. (a) $(5x + 3)^2$
$= (5x)^2 + 2(5x)(3) + 3^2$ Use $(a + b)^2$
$= 25x^2 + 30x + 9$ $= a^2 + 2ab + b^2$.

(b) $(2x + 5y)(2x - 5y)$
$= (2x)^2 - (5y)^2$ Use $(a + b)(a - b) = a^2 - b^2$.
$= 4x^2 - 25y^2$

2. (a) $8x^2 - 6xy$
$= 2x(4x - 3y)$

Teacher's Tip
Factorise by extracting common factor.
$2x$ is the common factor.

(b) $1 - 121x^2$
$= 1^2 - (11x)^2$ Use $a^2 - b^2 = (a + b)(a - b)$.
$= (1 + 11x)(1 - 11x)$

(c) $x^2 + 12x + 36$
$= x^2 + 2(6)x + 6^2$ Use $a^2 + 2ab + b^2 = (a + b)^2$.
$= (x + 6)^2$

3. (a) $35^2 - 280 + 16$
$= 35^2 - 2(35)(4) + 4^2$ Use $a^2 - 2ab + b^2 = (a - b)^2$.
$= (35 - 4)^2$
$= 31^2$
$= 961$

(b) $\frac{884^2 - 116^2}{219 \times 24 - 24 \times 211}$
$= \frac{(884 + 116)(884 - 116)}{24(219 - 211)}$ ← Use $a^2 - b^2 = (a + b)(a - b)$.
$= \frac{1000 \times 768}{24 \times 8}$ ← Use $a \times b - a \times c = a(b - c)$.
$= 4000$

4. $(2x + 5)(x - 2) + 3(x + 4)$ Expand the expression and collect like terms.
$= 2x^2 - 4x + 5x - 10 + 3x + 12$
$= 2x^2 + 4x + 2$

5. (a) $xt - yt$
$= t(x - y)$ Extract common factor, t.

(b) $1896 \times 0.05489 - 896 \times 0.05489$
$= 0.05489(1896 - 896)$ Extract common factor, 0.05489.
$= 0.05489(1000)$
$= 54.89$

6. (a) $63y^2 - 7x^2$
$= 7(9y^2 - x^2)$ Extract common factor, 7.
$= 7[(3y)^2 - x^2]$ Use $a^2 - b^2 = (a + b)(a - b)$.
$= 7(3y + x)(3y - x)$

(b) $3px - 9x + py - 3y$
$= 3x(p - 3) + y(p - 3)$ $(p - 3)$ is common to both terms. Factor it out.
$= (p - 3)(3x + y)$

Teacher's Tip

1) To factorise the expression, find the common factor of the first two terms and the common factor of the last two terms.

2) Since $(p - 3)$ is common to both terms, factor it out.

(c) $7(x + y)^2 - 4(x + y)(x - y)$
$= (x + y)[7(x + y) - 4(x - y)]$ Extract $(x + y)$ as common factor.
$= (x + y)(7x + 7y - 4x + 4y)$
$= (x + y)(3x + 11y)$

7. (a) $(2x + 3)(5 - 3x)$ Expand the expression and collect like terms.
$= 10x - 6x^2 + 15 - 9x$
$= -6x^2 + x + 15$

(b) $5x^3 - 2x^2 - 5x + 2$
$= x^2(5x - 2) - (5x - 2)$ Extract $(5x - 2)$ as common factor.
$= (5x - 2)(x^2 - 1)$
$= (5x - 2)(x^2 - 1^2)$
$= (5x - 2)(x + 1)(x - 1)$

Teacher's Tip

Remember to continue to factorise $(x^2 - 1^2)$ using $a^2 - b^2 = (a + b)(a - b)$.

8. (a) $6x^2 - 4x - 3xy + 2y$
$= 2x(3x - 2) - y(3x - 2)$ Extract $(3x - 2)$ as common factor.
$= (3x - 2)(2x - y)$

(b) $4x^3 - 100x$
$= 4x(x^2 - 25)$ Extract common factor, $4x$.
$= 4x(x^2 - 5^2)$ Use $a^2 - b^2 = (a + b)(a - b)$ on $x^2 - 5^2$.
$= 4x(x + 5)(x - 5)$

9. (a) (i) $x^2 - (x + 6)(x - 6)$
$= x^2 - (x^2 - 36)$ Use $(a + b)(a - b) = a^2 - b^2$.
$= x^2 - x^2 + 36$
$= 36$

(ii) $x^2 - (x + y)(x - y)$
$= x^2 - (x^2 - y^2)$ Use $(a + b)(a - b) = a^2 - b^2$.
$= x^2 - x^2 + y^2$
$= y^2$

(b) $29\,642^2 - 29\,650 \times 29\,634$
$= 29\,642^2 - (29\,642 + 8)(29\,642 - 8)$
$x^2 - (x + y)(x - y)$ From part (a) (ii).
$= 8^2$ $x^2 - (x + y)(x - y) = y^2$ [From part (a) (ii)]
$= 64$

10. $x^2 + y^2 = 188$
$xy = 74$ } Given
$(x + y)^2 = x^2 + 2xy + y^2$
$= (x^2 + y^2) + 2xy$
$= 188 + 2(74)$
$= 336$

$\frac{1}{8}(x + y)^2 = \frac{1}{8} \times 336$
$= 42$

Section B

11. (a) $(2x + 5)(5 - 2x)^2$
$(2x + 5)(25 - 20x + 4x^2)$ Expand $(5 - 2x)^2$ using $(a - b)^2 = a^2 - 2ab + b^2$.
$= 50x - 40x^2 + 8x^3$
$+ 125 - 100x + 20x^2$ Expand term by term.
$= 8x^3 - 20x^2 - 50x + 125$ Collect like terms together.

(b) (i) $4x^2 - (x + y)^2$
$= (2x)^2 - (x + y)^2$ Use $a^2 - b^2 = (a + b)(a - b)$.
$= [2x + (x + y)][2x - (x - y)]$
$= (3x + y)(x + y)$

(ii) $p(2x - 3y) - q(12y - 8x)$ Factorise $(12y - 8x)$ first by extracting common factor, -4.
$= p(2x - 3y) - q[-4(2x - 3y)]$
$= p(2x - 3y) + 4q(2x - 3y)$ Extract $(2x - 3y)$ as common factor.
$= (2x - 3y)(p + 4q)$

12. (a) $89^2 - 11^2 = 39k$ Factorise left-hand side using $a^2 - b^2 = (a + b)(a - b)$.
$(89 + 11)(89 - 11) = 39k$
$100 \times 78 = 39k$
$k = \frac{100 \times \cancel{78}^{2}}{\cancel{39}_{1}}$
$= 200$

(b) (i) $x^2 - 16 = x^2 - 4^2$ Use $a^2 - b^2 = (a + b)(a - b)$.
$= (x + 4)(x - 4)$

(ii) $x^2 - 16 = 2484$ To find the two factors of 2484, let $x^2 - 16 = 2484$.
$x^2 = 2500$
$x = \sqrt{2500} = 50$
$\therefore 2484 = (50 + 4)(50 - 4)$
$= (54)(46)$
$\therefore$ the two factors of 2484 are 54 and 46.

(c) $(a+b)^2 = 36$
$a^2 + b^2 = 17$ } Given

$(a+b)^2 = a^2 + 2ab + b^2$
$(a+b)^2 = (a^2 + b^2) + 2ab$
$36 = 17 + 2ab$
$2ab = 36 - 17$
$= 19$
$4ab = 2 \times 19 = 38$

To find the value of ab, use $(a+b)^2 = a^2 + 2ab + b^2$.

13. (a) $3x(1-2x) + (2x-3)^2$
$= 3x - 6x^2 + 4x^2 - 12x + 9$
$= \mathbf{-2x^2 - 9x + 9}$

Teacher's Tip

Expand $3x(1-2x)$ using $a(b+c) = ab + ac$.
Use $(a-b)^2 = a^2 - 2ab + b^2$ to expand $(2x-3)^2$.
Then collect like terms together.

(b) (i) $64a^2 - 4d^2e^2$
$= 4(16a^2 - d^2e^2)$
$= 4[(4a)^2 - (de)^2]$ Use $a^2 - b^2 = (a+b)(a-b)$.
$= \mathbf{4(4a + de)(4a - de)}$

(ii) $8px^2 - 3q + 12qx^2 - 2p$
$= 8px^2 - 2p + 12qx^2 - 3q$ Regroup first.
$= 2p(4x^2 - 1) + 3q(4x^2 - 1)$ Factorise by taking out common terms.
$= (4x^2 - 1)(2p + 3q)$
$= \mathbf{(2x + 1)(2x - 1)(2p + 3q)}$ Continue to factorise $4x^2 - 1 = (2x)^2 - 1^2$ using $a^2 - b^2 = (a+b)(a-b)$.

14. (a) (i) $(3x-2)(3x+2)(x-4)$
$= [(3x)^2 - 2^2)](x-4)$ Use $(a+b)(a-b) = a^2 - b^2$.
$= (9x^2 - 4)(x-4)$
$= \mathbf{9x^3 - 36x^2 - 4x + 16}$

(ii) $(2x-1)(x^2 + 3x - 1)$ Expand term by term.
$= 2x^3 + 6x^2 - 2x - x^2 - 3x + 1$ Collect like terms together.
$= \mathbf{2x^3 + 5x^2 - 5x + 1}$

(b) $8x^3y - 50xy^3$ Factorise by taking out common terms.
$= 2xy(4x^2 - 25y^2)$
$= 2xy[(2x)^2 - (5y)^2]$ Use $a^2 - b^2 = (a+b)(a-b)$.
$= \mathbf{2xy(2x + 5y)(2x - 5y)}$

15. (a) (i) $p(q^2 - r^2) - qr(q-r)$ Use $a^2 - b^2 = (a+b)(a-b)$.
$= p(q+r)(q-r) - qr(q-r)$
$= (q-r)[p(q+r) - qr]$ Factorise the expression. $(q-r)$ is the common factor.
$= \mathbf{(q-r)(pq + pr - qr)}$

(ii) $s(2-s)(1-s) + s^2(s+3)$
$= s[(2-s)(1-s) + s(s+3)]$ Factorise by taking out s, the common factor.
$= s[(2 - 2s - s + s^2) + (s^2 + 3s)]$ Simplify the quadratic equation.
$= s(2s^2 + 2)$
$= \mathbf{2s(s^2 + 1)}$

(b) $x + y = 12$
$x^2 - y^2 = 36$ } Given

$x^2 - y^2 = (x+y)(x-y)$ Use $a^2 - b^2 = (a+b)(a-b)$.
$36 = (12)(x-y)$
$x - y = \frac{36}{12} = 3$
$7(x-y) = 7 \times 3$
$7x - 7y = 21$

Test 8: Solving Quadratic Equations by Factorisation

Section A

1. (a) $(x+3)(2x-1) = 0$
$\therefore x + 3 = 0$ or $2x - 1 = 0$
$x = -3$ or $x = \frac{1}{2}$

(b) $(4x-1)^2 = 25$
$4x - 1 = \pm\sqrt{25}$ Square root both sides.
$= \pm 5$ Note that there are two possible roots.
$\therefore 4x - 1 = 5$ or $4x - 1 = -5$
$4x = 6$ or $4x = -4$
$x = 1\frac{1}{2}$ or $x = -1$

2. (a) $12x^2 + 17x + 6$
$= (4x + 3)(3x + 2)$

$4x$	$+3$	$+9x$
$3x$	$+2$	$+8x$
$12x^2$	$+6$	$+17x$

(b) $1376 = 12(10)^2 + 17(10) + 6$ Rewrite 1376 in the form $12x^2 + 17x + 6$.
$= [4(10) + 3][3(10) + 2]$
$= (43)(32)$
$\therefore$ the two factors of 1376 are **43** and **32**.

3. (a) $12x = 5x^2$
$5x^2 - 12x = 0$
$x(5x - 12) = 0$
$\therefore x = 0$ or $5x - 12 = 0$
$5x = 12$
$x = 2\frac{2}{5}$