

Factorisation

1 Write down the following factors by inspection.

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|----------------------|------------------------|-------------------------|
| i) $x^2 + 7x + 12$ | viii) $x^2 + 13x + 40$ | xv) $x^2 - 29x + 100$ |
| ii) $x^2 + 10x + 21$ | ix) $x^2 - 10x + 24$ | xvi) $x^2 - 4x - 5$ |
| iii) $x^2 + 3x + 2$ | x) $x^2 - 16x + 28$ | xvii) $x^2 - 22x + 120$ |
| iv) $x^2 + 8x + 7$ | xi) $x^2 - 2x - 3$ | xviii) $x^2 - x - 110$ |
| v) $x^2 + 6x + 9$ | xii) $x^2 + 2x - 15$ | xix) $x^2 - x - 42$ |
| vi) $x^2 + 10x + 25$ | xiii) $x^2 - 2x - 63$ | xx) $x^2 + 4x - 77$ |
| vii) $x^2 - 7x + 10$ | xiv) $x^2 - 2x - 35$ | xxi) $x^2 + 5x - 66$ |

2 Factorise the following

- | | | |
|-------------------------|--------------------------|---------------------------|
| i) $y^2 - 16$ | xi) $2x^2 - 11x + 12$ | xxi) $4 - 3x - x^2$ |
| ii) $25 - m^2$ | xii) $8x^2 + 26x + 15$ | xxii) $4x^2 - 12x + 9$ |
| iii) $a^2 - 9b^2$ | xiii) $9x^2 - 39x + 30$ | xxiii) $3x^2 + 10x + 7$ |
| iv) $64 - 36n^2$ | xiv) $5x^2 + 11x + 2$ | xxiv) $2x^2 + 3x - 2$ |
| v) $4x^2 - 9y^2$ | xv) $5x^2 - 16x + 3$ | xxv) $3x^2 + 7x - 6$ |
| vi) $16 - 25q^2$ | xvi) $4x^2 - 11x + 6$ | xxvi) $5x^2 - 33x + 18$ |
| vii) $1 - 36a^2b^2$ | xvii) $2x^2 - x - 1$ | xxvii) $6x^2 + 5x - 6$ |
| viii) $81x^2 - 16k^2$ | xviii) $4x^2 + 23x + 15$ | xxviii) $12x^2 + 7x - 12$ |
| ix) $(x+1)^2 - (x-2)^2$ | xix) $x^2 - 5x - 36$ | xxix) $3x^2 - 13x + 14$ |

3 Factorise

- | | | |
|----------------------------|-------------------------|--------------------------|
| i) $3x^2 - 10x + 8$ | vi) $6x^2 - 5x - 21$ | x) $12x^2 + 8xy - 15y^2$ |
| ii) $12x^2 - 7xy + y^2$ | vii) $2 + x - 10x^2$ | xii) $7x^2 - 4x - 3$ |
| iii) $4x^2 + 16xy + 15y^2$ | viii) $7x^2 - 16x - 15$ | xiii) $10x^2 - 13x - 9$ |
| iv) $1 - 12x + 35x^2$ | ix) $18x^2 - 9x - 2$ | xiv) $26x^2 - 41x + 3$ |
| v) $2x^2 - 9x - 18$ | x) $12 - 29x + 14x^2$ | xv) $5x^2 - 34xy - 7y^2$ |

4 Factorise the following - you may have to combine methods

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|--------------------------|-------------------------------|----------------------------------|
| i) $2x^2 - 18$ | viii) $16x^4 - y^4$ | xv) $45x^2y^2 - 80$ |
| ii) $7a^2 - 28$ | ix) $2x^2 + 8x - 90$ | xvi) $3a^4 - 48$ |
| iii) $4x^2 - 4x - 48$ | x) $7x - 7y + x^2 - y^2$ | xvii) $12a - 30b + 4a^2 - 25b^2$ |
| iv) $\pi R^2 - \pi r^2$ | xi) $11 - 99x^2$ | xviii) $6x^2 + 33x - 63$ |
| v) $15a - 60a^3$ | xii) $6x^2 + 30x - 36$ | xix) $x^2 - y^2 + 3x + 3y$ |
| vi) $3x^2 + 21x - 24$ | xiii) $242x^2 - 32y^2$ | xx) $3x^2 + 18x + 27$ |
| vii) $20x^2y^2 - 180z^2$ | xiv) $9x^2 - 4y^2 + 12x + 8y$ | xxi) $12x^2 - 25x + 12$ |

Sum and Difference of Two Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2) \quad a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

5 Factorise

- | | | |
|-----------------|-------------------|--------------------|
| i) $x^3 + 27$ | vi) $4a^3 + 32$ | xii) $3x^3 - 81$ |
| ii) $a^3 - 64$ | viii) $2x^3 - 54$ | xiii) $8x^3 - 125$ |
| iii) $t^3 - 1$ | vii) $5m^3 - 5$ | xiv) $4t^3 - 500$ |
| iv) $x^3 + 125$ | ix) $8x^3 + 27$ | xv) $5y^3 - 5000$ |
| v) $y^3 - 1000$ | x) $1 - 27a^3$ | xvi) $1 - 64y^6$ |

6 Miscellaneous Factors

- | | | |
|---------------------------|----------------------------|--------------------------------|
| i) $x^2 - 6x$ | ix) $2x^2 + 19x - 33$ | xvii) $42a^2 - 17a - 15$ |
| ii) $10a + 25b$ | x) $3ax - 2bx + 3ay - 2by$ | xviii) $4a^2 - b^2 + 6a + 3b$ |
| iii) $1 - 81a^2$ | xi) $9x^2 - 49y^2$ | xix) $21a^2 + 7ab - 35a$ |
| iv) $a^2 - 11a + 24$ | xii) $10x^2 - x - 3$ | xx) $14x^2 - 80x - 24$ |
| v) $3x^2 + 12xy$ | xiii) $10x^2 + 72x + 14$ | xxi) $9x^3 + 72$ |
| vi) $m^2 - 36$ | xiv) $81 - x^4$ | xxii) $6x^2 - 33x + 45$ |
| vii) $x^2 + xy + 7x + 7y$ | xv) $(x - 3y)^2 - 16y^2$ | xxiii) $2a^2 + 3ab + 8a - 12b$ |
| viii) $1 - x - 72x^2$ | xvi) $m^2 - n^2 + 5m - 5n$ | xxiv) $x^6 - 1$ |

7 Factorise completely:

- | | | |
|------------------------------|------------------------------|--------------------------|
| (a) $2 - 8x^2$ | (b) $5x^2 - 10x - 5xy + 10y$ | (c) $2a^2 - 22a + 48$ |
| (d) $3m^2 + 18m + 27$ | (e) $x^4 - 1$ | (f) $p^3 - 4p^2 - p + 4$ |
| (g) $4x^2 - 36$ | (h) $a^3 - a$ | (i) $3a^2 - 39a + 120$ |
| (j) $9 - 9p^2$ | (k) $3k^2 + 3k - 18$ | (l) $24a^2 - 42a + 9$ |
| (m) $ax^2 + axy + 3ax + 3ay$ | (n) $(x + y)^2 + 3(x + y)$ | (o) $5xy^2 - 20xz^2$ |
| (p) $6ax^2 + 5ax - 6a$ | (q) $x^2 - y^2 + 5x - 5y$ | (r) $3x^2 + 12x - 12$ |
| (s) $63x^2 - 28y^2$ | (t) $a^4 - 16$ | (u) $(a - 2)^2 - 4$ |
| (v) $1 + p + p^2 + p^3$ | (w) $8t^2 - 28t - 60$ | (x) $8 - 8x - 6x^2$ |

8 Factorise:

- | | | |
|-------------------------|-----------------------|--------------------------|
| (a) $a^3 - a^2$ | (b) $y^4 + y^2$ | (c) $x^6 + x^4 - x^3$ |
| (d) $5p^4 + 10p^5$ | (e) $6a^2b - 9a^4b^5$ | (f) $4a^4 - 8a^5 + 6a^3$ |
| (g) $(x^3)^2 - (y^3)^2$ | (h) $a^4 - b^4$ | (i) $m^{10} - n^{10}$ |
| (j) $x^4 + 7x^2 + 12$ | (k) $a^6 - 5a^3 + 6$ | (l) $y^4 + 4y^2 + 4$ |

SIMPLIFICATION OF ALGEBRAIC FRACTIONS: MULTIPLICATION AND DIVISION

1 Factorise and simplify:

- | | | |
|-------------------------|-------------------------|-------------------------|
| $\frac{5x+10}{5}$ | $\frac{4}{2x+6}$ | $\frac{12}{3x-9}$ |
| (d) $\frac{2x-10}{x-5}$ | (e) $\frac{x+7}{3x+21}$ | (f) $\frac{5a-5}{8a-8}$ |

$$(g) \frac{3a+9}{6a+18}$$

$$(h) \frac{7m-28}{3m-12}$$

$$(i) \frac{x^2+x}{x^2-x}$$

$$(j) \frac{x^2-4}{x-2}$$

$$(k) \frac{a+1}{a^2-1}$$

$$(l) \frac{4y^2-9}{4y+6}$$

$$(m) \frac{a^2-4a}{3a-a^2}$$

$$(n) \frac{2x^2-2}{2x-2}$$

$$(o) \frac{x^2-36}{3x-18}$$

$$(p) \frac{a^2-3a-4}{a+1}$$

$$(q) \frac{x^2-6x+9}{x-3}$$

$$(r) \frac{x^2-4}{x^2+3x+2}$$

$$(s) \frac{x^2+3x+2}{x^2+5x+6}$$

$$(t) \frac{m^2+5m-24}{m^2-7m+12}$$

$$(u) \frac{t^2+7t+12}{t^2-9}$$

$$(v) \frac{a^2-x^2}{a^2+3a+ax+3x}$$

$$(w) \frac{2x^2-x-1}{4x^2-1}$$

$$(x) \frac{18a^2-8}{6a^2+a-2}$$

2 Simplify the following:

$$\text{a) } \frac{2x+4}{3} \times \frac{6x}{x+2}$$

$$(b) \frac{5y-15}{2y+8} \times \frac{y+4}{10}$$

$$(c) \frac{2x-4}{3x-9} \times \frac{5x-15}{7x-14}$$

$$(d) \frac{5n+10}{n+3} \times \frac{6n+18}{4n+8}$$

$$(e) \frac{7y+28}{21} \times \frac{6}{6y+24}$$

$$(f) \frac{1+2a}{10+30a} \times \frac{6+18a}{1-2a}$$

$$(g) \frac{y^2+y}{2y+8} \times \frac{4y+6}{3y+3}$$

$$(h) \frac{x^2-3x}{x^2} \times \frac{2x^2+5x}{9x-27}$$

$$(i) \frac{x+3}{x^2-9} \times \frac{x-3}{x+1}$$

$$(j) \frac{3x+15}{x^2-25} \times \frac{x^2-49}{3x-21}$$

$$(k) \frac{a^2+5a+6}{a^2-4} \times \frac{a^2-a-2}{a^2-1}$$

$$(l) \frac{y^2+3y+2}{y^2+5y+6} \times \frac{y^2+7y+12}{y^2+5y+4}$$

$$(m) \frac{x^2+6x+5}{x^2+5x+4} \times \frac{x^2+7x+12}{x^2+12x+35}$$

$$(n) \frac{m^2-1}{m^2-6m+5} \times \frac{m^2-10m+25}{m^2-25}$$

$$(o) \frac{a^2-4}{a^2+3a-4} \times \frac{a^2-16}{a^2+2a-8}$$

$$(p) \frac{2x^2+4x+2}{x^2-1} \times \frac{x^2+3x-4}{4x+4}$$

$$(q) \frac{3x^2+5x+2}{x^2-x-2} \times \frac{x^2+x-6}{3x^2+11x+6}$$

$$(r) \frac{5a^2+16a+3}{25a^2-1} \times \frac{5a^2-a}{2a^2+5a-3}$$

$$(s) \frac{x^2-y^2+x-y}{x^2-2xy+y^2} \times \frac{10x-10y}{5x+5y+5}$$

$$(t) \frac{(a+b)^2-c^2}{a^2+ab+ac+bc} \times \frac{a^2+ab-ac-bc}{a+b+c}$$

3 Simplify:

$$(a) \frac{3a+6}{2} \div \frac{a+2}{4}$$

$$(b) \frac{x+2}{5x} \div \frac{7x+14}{10x}$$

$$(c) \frac{5m-10}{m+1} \div \frac{3m-6}{3m+3}$$

$$(d) \frac{6m+9}{2m-8} \div \frac{2m+3}{3m-12}$$

$$(e) \frac{3x}{5x+15} \div \frac{x^2+x}{x+3}$$

$$(f) \frac{24y-16}{4y+6} \div \frac{3y-2}{8y+12}$$

$$(g) \frac{5m-20}{4m+6} \div \frac{5m-20}{2m^2+3m}$$

$$(h) \frac{25k+15}{3k-3} \div \frac{5k+3}{3k}$$

$$(i) \frac{n^2-9}{2n+4} \div \frac{n+3}{2}$$

$$(j) \frac{y+7}{y-7} \div \frac{y^2-49}{y^2-7y}$$

$$(k) \frac{a^2+5a+4}{a^2-16} \div \frac{a^2-9}{a^2-a-12}$$

$$(l) \frac{x^2+6x+9}{x^2+8x+15} \div \frac{x^2+5x+6}{x^2+7x+10}$$

$$(m) \frac{x^2-4}{x^2-7x+10} \div \frac{x^2-x-6}{x^2-3x-10}$$

$$(n) \frac{p^2+7p+10}{p^2-2p-8} \div \frac{p^2+2p-15}{p^2+p-12}$$

$$(o) \frac{n^2-49}{n^2-9} \div \frac{n^2+14n+49}{n^2-6n+9}$$

$$(p) \frac{2x^2-8x-42}{x^2+6x+9} \div \frac{x^2-9x-14}{x^2+x-6}$$

$$(q) \frac{3x^2-48}{x^2-3x-4} \div \frac{x^2+4x}{x^3-x}$$

$$(r) \frac{2a^2-a-1}{a^2-1} \div \frac{6a^2+a-1}{3a^2+2a-1}$$

$$(s) \frac{x+y+x^2-y^2}{x^2+2xy+y^2} \div \frac{1+x-y}{2x+2y}$$

$$(t) \frac{p^2-(q+r)^2}{p^2+pq-pr-qr} \div \frac{p-q-r}{p^2-pq-pr+qr}$$

Factorisation - Solutions

i) $(x+3)(x+4)$	viii) $(x+8)(x+5)$	xv) $(x-4)(x-25)$
ii) $(x+7)(x+3)$	ix) $(x-4)(x-6)$	xvi) $(x-5)(x+1)$
iii) $(x+2)(x+1)$	x) $(x-14)(x-2)$	xvii) $(x-12)(x-10)$
iv) $(x+7)(x+1)$	xi) $(x-3)(x+1)$	xviii) $(x-11)(x+10)$
v) $(x+3)^2$	xii) $(x+5)(x-3)$	xix) $(x-7)(x+6)$
vi) $(x+5)^2$	xiii) $(x-9)(x+7)$	xx) $(x+11)(x-7)$
vii) $(x-5)(x-2)$	xiv) $(x-7)(x+5)$	xxi) $(x+11)(x-6)$

2 i) $(y-4)(y+4)$	xii) $(2x-3)(x-4)$	xxi) $(4+x)(1-x)$
ii) $(5-m)(5+m)$	xiii) $(4x+3)(2x+5)$	xxii) $(2x-3)^2$
iii) $(a-3b)(a+3b)$	xiv) $3(3x-10)(x-1)$	xxiii) $(3x+7)(x+1)$
iv) $4(4-3n)(4+3n)$	xv) $(5x+1)(x+2)$	xxiv) $(2x-1)(x+2)$
v) $(2x-3y)(2x+3y)$	xvi) $(5x-1)(x-3)$	xxv) $(3x-2)(x+3)$
vi) $(4-5q)(4+5q)$	xvii) $(4x-3)(x-2)$	xxvi) $(5x-3)(x-6)$
vii) $(1-6ab)(1+6ab)$	xviii) $(2x+1)(x-1)$	xxvii) $(3x-2)(2x+3)$
viii) $(9x-4k)(9x+4k)$	xix) $(4x+3)(x+5)$	xxviii) $(3x+4)(4x-3)$
ix) $3(2x-1)$	x) $(x-9)(x+4)$	xxix) $(3x-7)(x-2)$

3 i) $(3x-4)(x-2)$	vii) $(3x+7)(2x-3)$	xii) $(6x-5y)(2x+3y)$
ii) $(3x-y)(4x-y)$	viii) $(2+5x)(1-2x)$	xiii) $(7x+3)(x-1)$
iii) $(4x+5y)(x+3y)$	ix) $(7x+5)(x-3)$	xiv) $(5x+3)(2x-3)$
iv) $(1-7x)(1-5x)$	x) $(6x+1)(3x-2)$	xv) $(2x-3)(13x-1)$
v) $(2x+3)(x-6)$	xi) $(4-7x)(3-2x)$	xvi) $(5x+y)(x-7y)$

4 i) $2(x-3)(x+3)$	viii) $(2x-y)(2x+y)(4x^2+y^2)$	xv) $5(3xy-4)(3xy+4)$
ii) $7(a-2)(a+2)$	ix) $2(x+9)(x-5)$	xvi) $3(a-2)(a+2)(a^2+4)$
iii) $4(x-4)(x+3)$	x) $(x-y)(7+x+y)$	xvii) $(2a-5b)(6+2a+5b)$
iv) $\pi(R-r)(R+r)$	xi) $11(1-3x)(1+3x)$	xviii) $3(2x-3)(x+7)$
v) $15a(1-2a)(1+2a)$	xii) $6(x+6)(x-1)$	xix) $(x+y)(x-y+3)$
vi) $3(x+8)(x-1)$	xiii) $2(11x-4y)(11x+4y)$	xx) $3(x+3)^2$
vii) $20(xy-3z)(xy+3z)$	xiv) $(3x+2y)(3x-2y+4)$	xxi) $(4x-3)(3x-4)$

5 i) $(x+3)(x^2-3x+9)$	vi) $4(a+2)(a^2-2a+4)$	xii) $3(x-3)(x^2+3x+9)$
ii) $(a-4)(a^2+4a+16)$	vii) $2(x-3)(x^2+3x+9)$	xiii) $(2x-5)(4x^2+10x+25)$
iii) $(t-1)(t^2+t+1)$	viii) $5(m-1)(m^2+m+1)$	xiv) $4(t-5)(t^2+5t+25)$
iv) $(x+5)(x^2-5x+25)$	ix) $(2x+3)(4x^2-6x+9)$	xv) $5(y-10)(y^2+10y+100)$
v) $(y-10)(y^2+10y+100)$	x) $(1-3a)(1+3a+9a^2)$	xvi) $(1-4y^2)(1+4y^2+16y^4)$

- 6 i) $x(x-6)$ ix) $2x-3)(x+11)$ xvii) $(6a-5)(7a+3)$
 ii) $5(2a+5b)$ x) $(x+y)(3a-2b)$ xviii) $(2a-b+3)(2a+b)$
 iii) $(1-9a)(1+9a)$ xi) $(3x-7y)(3x+7y)$ xix) $7a(3a+b-5)$
 iv) $(a-8)(a-3)$ xii) $(5x-3)(2x+1)$ xx) $2(7x+2)(x-6)$
 v) $3x(x+4y)$ xiii) $2(5x+1)(x+7)$ xx1) $9(x+2)(x^2-2x+4)$
 vi) $(m-6)(m+6)$ xiv) $(3-x)(3+x)(9+x^2)$ xxii) $3(2x-5)(x-3)$
 vii) $(x+y)(x+7)$ xv) $(x-7y)(x+y)$ xxiii) $(a+4)(2a-3b)$
 viii) $(1-9x)(1+8x)$ xvi) $(m-n)(m+n+5)$ xxiv) $(x-1)(x+1)(x^4+x^2+1)$ or
 (x-1)(x^2+x+1)(x+1)(x^2-x+1)

7. (a) $2(1-2x)(1+2x)$ (b) $5(x-y)(x-2)$
 (c) $2(a-8)(a-3)$ (d) $3(m-3)(m-3)$ (e) $(x-1)(x+1)(x^2+1)$ (f) $(p-4)(p-1)(p+1)$
 (g) $4(x-3)(x+3)$ (h) $a(a-1)(a+1)$ (i) $3(a-8)(a-5)$ (j) $9(1-p)(1+p)$
 (k) $3(k+3)(k-2)$ (l) $3(2a-3)(4a-1)$ (m) $a(x+y)(x+3)$ (n) $(x+y)(x+y+3)$
 (o) $5x(y-2z)(y+2z)$ (p) $a(3x-2)(2x+3)$ (q) $(x-y)(x+y+5)$ (r) $3(x-2)(x-2)$
 (s) $7(3x-2y)(3x+2y)$ (t) $(a^2+4)(a-2)(a+2)$ (u) $a(a-4)$ (v) $(1+p)(1+p^2)$
 (w) $4(2t+3)(t-5)$ (x) $2(2+x)(2-3x)$
8. (a) $a^2(a-1)$ (b) $y^2(y^2+1)$ (c) $x^3(x^3+x-1)$
 (d) $5p^4(1+2p)$ (e) $3a^2b(2-3a^2b^4)$ (f) $2a^3(2a-4a^2+3)$ (g) $(x^3-y^3)(x^3+y^3)$
 (h) $(a^2+b^2)(a-b)(a+b)$ (i) $(m^5+n^5)(m^5-n^5)$ (j) $(x^2+3)(x^2+4)$ (k) $(a^3-2)(a^3-3)$
 (l) $(y^2+2)^2$

1. (a) $x+2$ (b) $\frac{2}{x+3}$ (c) $\frac{4}{x-3}$ (d) 2 (e) $\frac{1}{3}$ (f) $\frac{5}{8}$ (g) $\frac{1}{2}$ (h) $\frac{7}{3}$ (i) $\frac{x+1}{x-1}$ (j) $x+2$ (k) $\frac{1}{a-1}$
 (l) $\frac{2y-3}{2}$ (m) $\frac{a-4}{3-a}$ (n) $x+1$ (o) $\frac{x+6}{3}$ (p) $a-4$ (q) $x-3$ (r) $\frac{x-2}{x+1}$ (s) $\frac{x+1}{x+3}$ (t) $\frac{m+8}{m-4}$
 (u) $\frac{t+4}{t-3}$ (v) $\frac{a-x}{a+3}$ (w) $\frac{x+1}{2x+1}$ (x) $\frac{2(3a-2)}{2a-1}$

2. (a) $4x$ (b) $\frac{y-3}{4}$ (c) $\frac{10}{21}$ (d) $\frac{15}{2}$ (e) $\frac{1}{3}$

(f) $\frac{3(1+2a)}{5(1-2a)}$ (g) $\frac{y(2y+3)}{3(y+4)}$ (h) $\frac{2x+5}{9}$ (i) $\frac{1}{x+1}$ (j) $\frac{x+7}{x-5}$ (k) $\frac{a+3}{a-1}$ (l) 1 (m) $\frac{x+3}{x+7}$

(n) $\frac{m+1}{m+5}$ (o) $\frac{(a+2)(a-4)}{(a+4)(a-1)}$ (p) $\frac{x+4}{2}$ (q) 1 (r) $\frac{a}{2a-1}$ (s) 2 (t) $\frac{(a+b-c)(a-c)}{a+c}$

3. (a) 6 (b) $\frac{2}{7}$ (c) 5 (d) $\frac{9}{2}$ (e) $\frac{3}{5(x+1)}$ (f) 16 (g) $\frac{m}{2}$ (h) $\frac{5k}{k-1}$ (i) $\frac{n-3}{n+2}$ (j) $\frac{y}{y-7}$

(k) $\frac{a+1}{a-3}$ (l) 1 (m) $\frac{x+2}{x-3}$ (n) $\frac{p+4}{p-4}$ (o) $\frac{(n-7)(n-3)}{(n+1)(n+7)}$ (p) 2 (q) $3(x-1)$ (r) 1 (s) 2

(t) $\frac{(p+q+r)(p-q)}{(p+q)}$