Parabolas

Quadratic functions have graphs that are parabolas.
The general equation of a quadratic function can be given in two forms:

$$y = ax^2 + bx + c \textbf{ OR } y = a(x - h)^2 + k$$

In both forms the highest exponent of x is 2.

The vertex, stationary point or turning point gives either the maximum or minimum value of the function.

Factorising Transforming

Sketching Parabolas

Method 1: Factorisation.

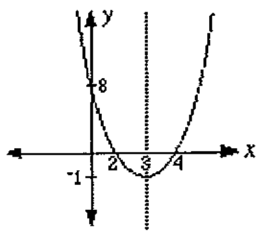
If the equation is in the form $y = ax^2 + bx + c$ the following method should be used:

Step (a) Factorise the function.

Step (b) Find the x- and y-intercepts by putting y = 0 and x = 0.

Step (c) Find the axis of symmetry - always midway between the two x-intercepts.

Step (d) Find the coordinates of the vertex or turning point.

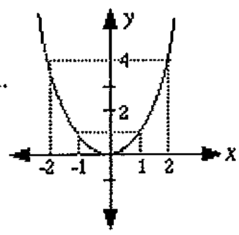
Example	Answer	Graph
Sketch the graph of the function $y = x^2 - 6x + 8$	**Step (a)** $y = x^2 - 6x + 8 = (x - 4)(x - 2)$ **Step (b)** Put x = 0 into the equation. $y = 0^2 - 6 \times 0 + 8$ $y = 8$ **The y-intercept is 8** Put y = 0 $0 = (x - 4)(x - 2)$ $x = 4$ or $x = 2$	Now sketch the graph 

	The x-intercepts are 4 and 2 **Step (c)** Axis of symmetry is always midway between the x-intercepts. **Axis of symmetry is the line x = 3** **Step (d)** The vertex occurs at x = 3 When x = 3 $y = (3 - 4)(3 - 2)$ $y = -1$ **The vertex is at the point (3, -1)**	

Method 2: Transformation.

This method finds the coordinates of the vertex.
The y-intercept is found by putting x = 0.

The graph of the basic parabola $y = x^2$ is shown in the diagram.

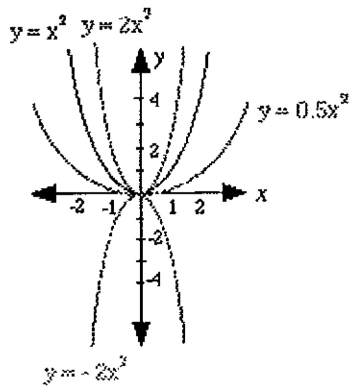


This basic parabola is moved or transformed as follows.

(a) $y = \mathbf{a}x^2$ The **a** has the effect of changing the parabola in the y- direction.

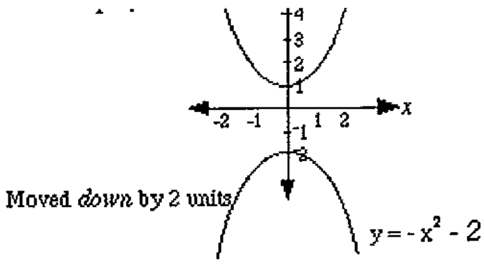
It affects the steepness of the graph.

- If **a** is large, the parabola is **steeper**.
- If **a** is small, the parabola is **flatter**.
- If **a** is negative, the parabola is **inverted**.



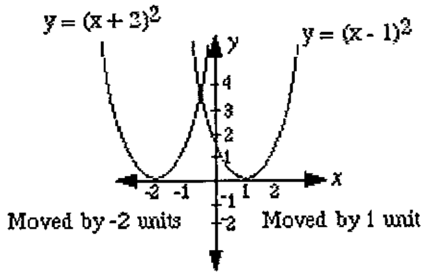
(b) $y = x^2 + k$

The **k** has the effect of moving the parabola along the y-axis by **k** units.



(c) $y = (x - h)^2$

The **h** has the effect of moving the basic parabola along the x-axis by **h** units.



Parabolas

1. Match up each of the graphs below with the following functions:

(a) $y = x^2 - 2$

(b) $y - 2 = (x + 1)^2$

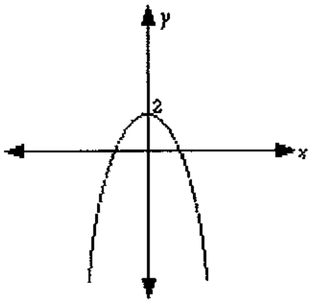
(c) $y = (x + 2)(x - 1)$

(d) $y = (x - 2)(x + 1)$

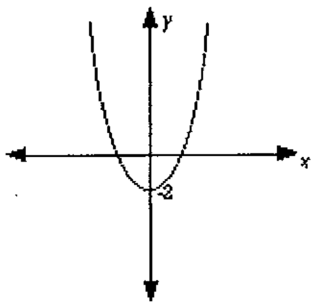
(e) $y = -x^2 + 2$

(f) $y = x^2 + 2$

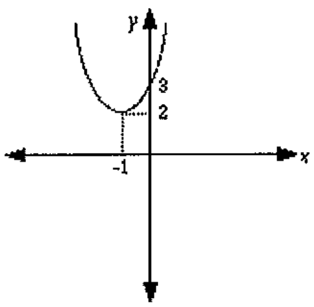
(i)



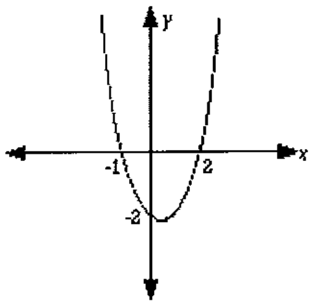
(ii)



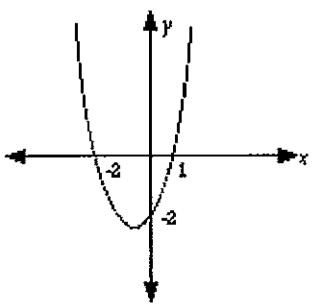
(iii)



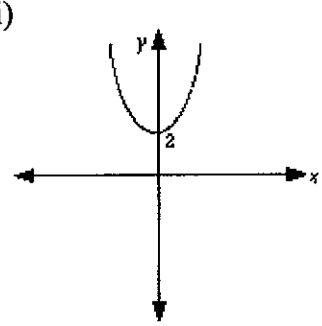
(iv)



(v)



(vi)



2. The sketch shows the function

y = x (x - 2)

(a) What are the coordinates of A?

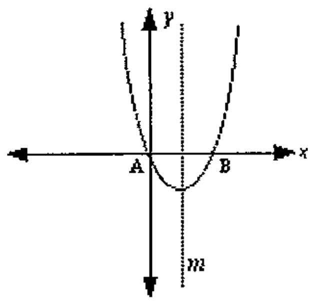
(b) What are the coordinates of B?

(c) What is the equation of m?

(d) What are the coordinates of the turning

point of the curve?

(e) What is the minimum value of the function?



3. The sketch below is of the function
y = (x - 2)(x + 3)

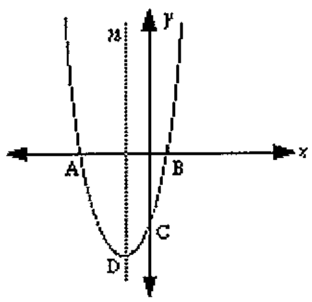
(a) What are the coordinates of A?

(b) What are the coordinates of B?

(c) What are the coordinates of C?

(d) What is the equation of the axis of symmetry n ?

(e) What are the coordinates of D, the turning
point of the curve?



4. Sketch the graphs of the following functions, clearly marking all intercepts, the axis of symmetry, and the vertex of the curve.

(a) y = (x - 2)(x + 4)

(b) $y = x^2 - 7x + 6$

(c) y = (x + 3)(x + 4)

(d) $y = x^2 - 2x - 35$

(e) $y = (2x - 1)(x + 3)$

(f) $y = (3 - x)(x - 2)$

5. Sketch the graphs of the following functions, clearly marking the vertex and the y-intercept.

(a) $y = x^2 + 3$

(b) $y = (x + 2)^2 - 4$

(c) $y - 1 = x^2$

(d) $y + 3 = 2x^2$

(e) $y = -(x - 1)^2$

(f) $y = (x - 3)^2 + 1$