TEST 8 **Solving Quadratic Equations by Factorisation**

Marks: /80

Time: 1 hour 30 minutes

Name:	Date: ....

INSTRUCTIONS TO CANDIDATES

Section A (40 marks) **Time: 45 minutes**

1. Answer **all** the questions in this section.
2. Calculators may **not** be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [ ] at the end of each question.

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1 Solve the following quadratic equations.

(a) $(x + 3)(2x - 1) = 0$

(b) $(4x - 1)^2 = 25$

Answer (a) .... [2]

(b) .... [3]

2 **(a)** Factorise $12x^2 + 17x + 6$ completely.
(b) Hence or otherwise, find the two factors of 1376.

Answer (a) .... [2]

(b) .... [2]

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3 Solve the equations
(a) $12x = 5x^2$,
(b) $(2x - 3)(x - 2) = 21$.

Answer (a) .... [2]

(b) .... [3]

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4 **(a)** Factorise $8y^2 - 10y - 25$.
(b) Solve the equation $2x^2 = 162$.

Answer (a) .... [2]

(b) .... [2]

5 Solve the equation $(4x - 7)^2 = 5(4x - 7)$.

Answer .... [3]

6 **(a)** Factorise $12 + 2x - 4x^2$.

(b) Solve the equation $3x(x + 4) = 0$.

Answer *(a)* .... [2]

(b) .... [2]

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7 Solve the equation $(3x + 1)(4x + 3) = 8x + 6$.

Answer .... [3]

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8 (a) Factorise $x^2 - 49$.
(b) Hence find the prime factors of 851.

Answer (a) .... [1]

(b) .... [3]

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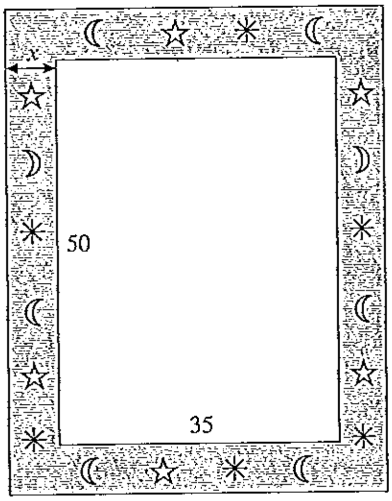
9 Mr Lee is 5 times as old as his son. 3 years ago, the product of their ages was 128. Find their present age.

Answer Mr Lee: .... years old

Son: .... years old [4]

10 The diagram shows a rectangular mirror measuring 50 cm by 35 cm. It is bordered by a uniform frame of width x cm. If the area of the border is 744 cm^2, find the value of x.



Answer x = ... [4]

INSTRUCTIONS TO CANDIDATES

Section B (40 marks)

Time: 45 minutes

1. Answer **all** the questions in this section.
2. Calculators may be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [ ] at the end of each question.

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11 (a) (i) Solve the equation $a^2 - 2a - 15 = 0$.

(ii) Hence, find all the solutions for the equation $(b - 1)^2 - 2(b - 1) - 15 = 0$.

(b) The sum of the squares of three consecutive positive odd numbers is 286 less than the square of their sum. Find the numbers.

Answer (a) (i) .... [2]

(ii) .... [2]

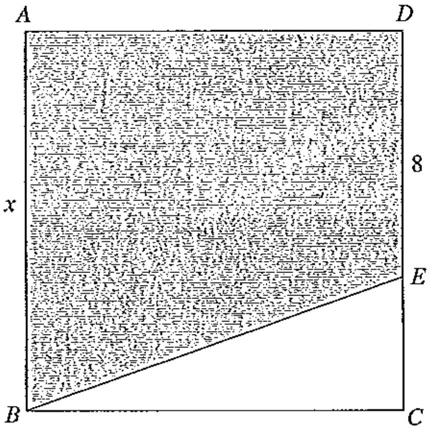
(b) .... [5]

12 In the diagram, $ABCD$ is a square of side x cm and $DE = 8$ cm. The area of the shaded part is 5 times the area of the unshaded part.

(a) Write down but do not simplify, an expression in terms of x, for the area of the shaded part.

(b) Form an equation in x and solve it.

(c) Hence find the area of the quadrilateral $ABED$.



Answer *(a)* .... cm^2 [2]

(b) .... [4]

(c) .... cm^2 [1]

13 A shopkeeper paid \$1800 for x jackets. He sold all but 5 of the jackets at \$20 more per jacket than he paid.

(a) Write down, an expression, in terms of x

(i) for the cost of each jacket,

(ii) for the selling price of each jacket.

He made a profit of \$200 from the transactions.

(b) Form an equation in x and show that it reduces to $x^2 - 15x - 450 = 0$. [3]

(c) Solve the equation $x^2 - 15x - 450 = 0$ to find the number of jackets the shopkeeper bought.

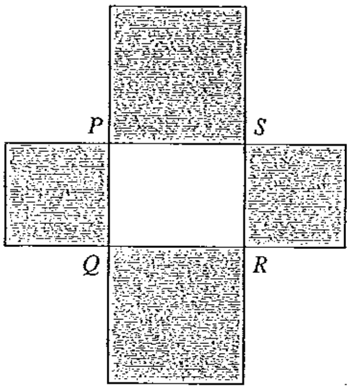
Answer (a) (i) \$ .... [1]

(ii) \$ .... [1]

(c) .... jackets [2]

14 (a) Given that $px^2 + 5x - 6p - 10 = 0$, calculate the values of x when $p = 3$.

(b) In the diagram, $PQRS$ is a rectangle. The shaded areas are four squares. Given that the perimeter of $PQRS$ is 26 cm and the area of the four squares is 178 cm^2, find the area of $PQRS$.



Answer (a) .... [3]

(b) .... cm^2 [5]

15 A motorist left Town A at 07 30 and travels to Town B. After travelling for 80 km at an average speed of x km/h, he stops to rest for one hour.

(a) Write down, an expression in terms of x, for the time in hours, that he took to travel before stopping for a rest.

He then continues the remaining journey of 30 km at an average speed which was 15 km/h slower than the first part of the journey.

(b) Write down, an expression in terms of x, for the time in hours, that he took to travel for the second part of the journey.

(c) Given that he reached Town B at 10 30, form an equation in x and show that it reduces to $x^2 - 70x + 600 = 0$. [3]

(d) Hence solve the equation $x^2 - 70x + 600 = 0$ to find the values of x.

(e) What time will the motorist reach Town B if he had completed the whole journey without stopping at a constant speed of x km/h?

Answer (a) .... h [1]

(b) .... h [1]

(d) .... [2]

(e) .... [2]

(c) $(a + b)^2 = 36$
$a^2 + b^2 = 17$ } Given

$(a + b)^2 = a^2 + 2ab + b^2$
$(a + b)^2 = (a^2 + b^2) + 2ab$
$36 = 17 + 2ab$
$2ab = 36 - 17$
$= 19$
$4ab = 2 \times 19 = \mathbf{38}$

To find the value of ab, use $(a + b)^2 = a^2 + 2ab + b^2$.

13. (a) $3x(1 - 2x) + (2x - 3)^2$
$= 3x - 6x^2 + 4x^2 - 12x + 9$
$= \mathbf{-2x^2 - 9x + 9}$

Teacher's Tip

Expand $3x(1 - 2x)$ using $a(b + c) = ab + ac$.
Use $(a - b)^2 = a^2 - 2ab + b^2$ to expand $(2x - 3)^2$.
Then collect like terms together.

(b) (i) $64a^2 - 4d^2e^2$
$= 4(16a^2 - d^2e^2)$
$= 4[(4a)^2 - (de)^2]$ Use $a^2 - b^2 = (a + b)(a - b)$.
$= \mathbf{4(4a + de)(4a - de)}$

(ii) $8px^2 - 3q + 12qx^2 - 2p$
$= 8px^2 - 2p + 12qx^2 - 3q$ Regroup first.
$= 2p(4x^2 - 1) + 3q(4x^2 - 1)$ Factorise by taking out common terms.
$= (4x^2 - 1)(2p + 3q)$
$= \mathbf{(2x + 1)(2x - 1)(2p + 3q)}$ Continue to factorise $4x^2 - 1 = (2x)^2 - 1^2$ using $a^2 - b^2 = (a + b)(a - b)$.

14. (a) (i) $(3x - 2)(3x + 2)(x - 4)$
$= [(3x)^2 - 2^2](x - 4)$ Use $(a + b)(a - b) = a^2 - b^2$.
$= (9x^2 - 4)(x - 4)$
$= \mathbf{9x^3 - 36x^2 - 4x + 16}$

(ii) $(2x - 1)(x^2 + 3x - 1)$ Expand term by term.
$= 2x^3 + 6x^2 - 2x - x^2 - 3x + 1$ Collect like terms together.
$= \mathbf{2x^3 + 5x^2 - 5x + 1}$

(b) $8x^3y - 50xy^3$
$= 2xy(4x^2 - 25y^2)$ Factorise by taking out common terms.
$= 2xy[(2x)^2 - (5y)^2]$
$= \mathbf{2xy(2x + 5y)(2x - 5y)}$ Use $a^2 - b^2 = (a + b)(a - b)$.

15. (a) (i) $p(q^2 - r^2) - qr(q - r)$
$= p(q + r)(q - r) - qr(q - r)$ Use $a^2 - b^2 = (a + b)(a - b)$.
$= (q - r)[p(q + r) - qr]$
$= \mathbf{(q - r)(pq + pr - qr)}$ Factorise the expression. $(q - r)$ is the common factor.

(ii) $s(2 - s)(1 - s) + s^2(s + 3)$
$= s[(2 - s)(1 - s) + s(s + 3)]$ Factorise by taking out s, the common factor.
$= s[(2 - 2s - s + s^2) + (s^2 + 3s)]$ Simplify the quadratic equation.
$= s(2s^2 + 2)$
$= \mathbf{2s(s^2 + 1)}$

(b) $x + y = 12$
$x^2 - y^2 = 36$ } Given

$x^2 - y^2 = (x + y)(x - y)$ Use $a^2 - b^2 = (a + b)(a - b)$.
$36 = (12)(x - y)$
$x - y = \frac{36}{12} = 3$
$7(x - y) = 7 \times 3$
$7x - 7y = \mathbf{21}$

Test 8: Solving Quadratic Equations by Factorisation

Section A

1. (a) $(x + 3)(2x - 1) = 0$
$\therefore x + 3 = 0$ or $2x - 1 = 0$
$x = \mathbf{-3}$ or $x = \mathbf{\frac{1}{2}}$

(b) $(4x - 1)^2 = 25$
$4x - 1 = \pm\sqrt{25}$ Square root both sides.
$= \pm 5$ Note that there are two possible roots.
$\therefore 4x - 1 = 5$ or $4x - 1 = -5$
$4x = 6$ or $4x = -4$
$x = \mathbf{1\frac{1}{2}}$ or $x = \mathbf{-1}$

2. (a) $12x^2 + 17x + 6$
$= \mathbf{(4x + 3)(3x + 2)}$

$4x$	$+3$	$+9x$
$3x$	$+2$	$+8x$
$12x^2$	$+6$	$+17x$

(b) $1376 = 12(10)^2 + 17(10) + 6$ Rewrite 1376 in the form $12x^2 + 17x + 6$.
$= [4(10) + 3][3(10) + 2]$
$= (43)(32)$
$\therefore$ the two factors of 1376 are **43** and **32**.

3. (a) $12x = 5x^2$
$5x^2 - 12x = 0$
$x(5x - 12) = 0$
$\therefore x = \mathbf{0}$ or $5x - 12 = 0$
$5x = 12$
$x = \mathbf{2\frac{2}{5}}$

(b) $(2x-3)(x-2)=21$

$$2x^2-4x-3x+6-21=0$$
$$2x^2-7x-15=0$$
$$(2x+3)(x-5)=0$$

$2x$	$+3$	$+3x$
x	-5	$-10x$
$2x^2$	-15	$-7x$

$\therefore 2x+3=0$ or $x-5=0$

$x=-1\frac{1}{2}$ or $x=5$

Teacher's Tip

Expand and write in the general form $ax^2+bx+c=0$ before solving the quadratic equation.

4. (a) $8y^2-10y-25$
$=(2y-5)(4y+5)$

$2y$	-5	$-20y$
$4y$	$+5$	$+10y$
$8y^2$	-25	$-10y$

(b) $2x^2=162$

$x^2=\frac{162}{2}=81$

$x=\pm\sqrt{81}$ Take square root on both sides.
$=9$ or -9

5. $(4x-7)^2=5(4x-7)$
$(4x-7)^2-5(4x-7)=0$ Factor out common factor, $(4x-7)$.
$(4x-7)[(4x-7)-5]=0$
$(4x-7)(4x-12)=0$
$\therefore 4x-7=0$ or $4x-12=0$
$x=1\frac{3}{4}$ or $x=3$

6. (a) $12+2x-4x^2$
$=(4-2x)(3+2x)$

(b) $3x(x+4)=0$
$\therefore 3x=0$ or $x+4=0$
$x=0$ or $x=-4$

7. $(3x+1)(4x+3)=8x+6$ Expand and write in the general form $ax^2+bx+c=0$ before solving.
$12x^2+9x+4x+3=8x+6$
$12x^2+5x-3=0$
$(3x-1)(4x+3)=0$

$3x$	-1	$-4x$
$4x$	$+3$	$+9x$
$12x^2$	-3	$+5x$

$\therefore 3x-1=0$ or $4x+3=0$
$x=\frac{1}{3}$ or $x=-\frac{3}{4}$

8. (a) $x^2-49=x^2-7^2$
$=(x+7)(x-7)$

(b) $851=900-49$
$=30^2-7^2$
$=(30+7)(30-7)$
$=(37)(23)$
$\therefore$ the prime factors of 851 are **23** and **37**.

9. Let the son's present age be x years old.
Then Mr Lee's present age will be $5x$ years old.
3 years ago, their ages were $(x-3)$ and $(5x-3)$ respectively.

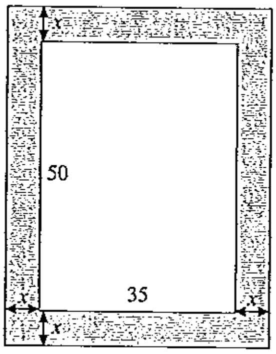
$\therefore (x-3)(5x-3)=128$ Given the product of their ages 3 years ago was 128.
$5x^2-3x-15x+9=128$
$5x^2-18x-119=0$
$(5x+17)(x-7)=0$

$5x$	$+17$	$+17x$
x	-7	$-35x$
$5x^2$	-119	$-18x$

$\therefore 5x+17=0$ or $x-7=0$
$x=-3\frac{2}{5}$ (rejected) or $x=7$

$\therefore$ the son is **7 years old** while Mr Lee is **35 years old** at present.

Check answer: 3 years ago, the son was 4 years old and Mr Lee was 32 years old.
$4\times 32=128$

10. 

$(50+2x)(35+2x)=(50\times 35)+744$

$1750+100x+70x+4x^2=1750+744$
$4x^2+170x-744=0$ Divide both sides by 2.
$2x^2+85x-372=0$
$(2x+93)(x-4)=0$

$2x$	$+93$	$+93x$
x	-4	$-8x$
$2x^2$	-372	$+85x$

$\therefore 2x+93=0$ or $x-4=0$
$x=-46\frac{1}{2}$ (rejected) or $x=4$
$\therefore x=4$

Reject $x=-46\frac{1}{2}$ since length cannot be negative.

Section B

11. (a) (i) $a^2 - 2a - 15 = 0$

$(a+3)(a-5) = 0$

$\therefore a+3 = 0$ or $a-5=0$

$a = -3$ or $a = 5$

(ii) $(b-1)^2 - 2(b-1) - 15 = 0$ Compare to $a^2 - 2a - 15$.

$\therefore a = b - 1$

$b - 1 = -3$ or $b - 1 = 5$ From part (b) (i) $a = -3$ or 5.

$b = -2$ or $b = 6$

(b) Let the first positive odd number be x. Then the next two consecutive positive odd numbers will be $(x+2)$ and $(x+4)$ respectively.

$[x^2 + (x+2)^2 + (x+4)^2] + 286$

$= [x + (x+2) + (x+4)]^2$

$x^2 + (x^2 + 4x + 4) + (x^2 + 8x + 16) + 286 = (3x+6)^2$

$3x^2 + 12x + 306 = 9x^2 + 36x + 36$

$6x^2 + 24x - 270 = 0$ Divide both sides by 6.

$x^2 + 4x - 45 = 0$

$(x-5)(x+9) = 0$

$\therefore x - 5 = 0$ or $x + 9 = 0$

$x = 5$ or $x = -9$ (rejected)

When $x = 5$,

$x + 2 = 5 + 2 = 7$,

$x + 4 = 5 + 4 = 9$.

$\therefore$ the three numbers are **5, 7 and 9.**

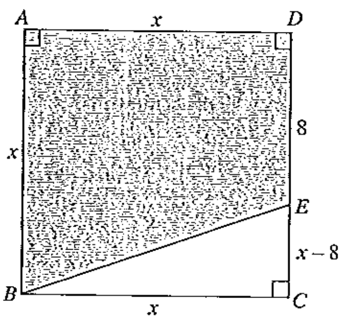
Check answer: $5^2 + 7^2 + 9^2 + 286 = 441$

$(5+7+9)^2 = 21^2 = 441$

Teacher's Tip

$x = -9$ is rejected since it is not a positive odd number.

12.



(a) Area of shaded part

$= \frac{1}{2}(x+8)(x)$ cm^2

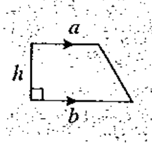
Teacher Tip

Area of trapezium

$= \frac{1}{2}(a+b)h$

a, b = length of parallel sides

h = height



(b) $\frac{1}{2}(x+8)(x)$ Area of trapezium $ABED$ is 5 times the area of $\triangle BCE$. (Given)

$= 5\left[\frac{1}{2}(x)(x-8)\right]$

$x(x+8) = 5x(x-8)$ Multiply both sides by 2.

$x^2 + 8x = 5x^2 - 40x$

$4x^2 - 48x = 0$

$4x(x-12) = 0$

$\therefore 4x = 0$ or $x - 12 = 0$

$x = 0$ or $x = 12$

(c) $x = 12$ Reject $x = 0$.

Area of $ABED = \frac{1}{2}(12+8)(12)$

$= 120$ cm^2

Check answer: Area of $\triangle BCE = \frac{1}{2} \times 12 \times 4$

$= 24$ cm^2

Area of $ABED = 5 \times$ Area of $\triangle BCE$

$= 5 \times 4 = 120$ cm^2

13. (a) (i) The cost of each jacket

$= \$\left(\frac{1800}{x}\right)$

(ii) Selling price of each jacket

$= \$\left(\frac{1800}{x} + 20\right)$

(b) $(x-5)\left(\frac{1800}{x} + 20\right) = 1800 + 200$

$(x-5)\left(\frac{1800 + 20x}{x}\right) = 2000$

$(x-5)(1800 + 20x) = 2000x$ Multiply both sides by x.

$1800x + 20x^2 - 9000 - 100x = 2000x$

$20x^2 - 300x - 9000 = 0$ Divide both sides by 20.

$x^2 - 15x - 450 = 0$ Shown

(c) $x^2 - 15x - 450 = 0$

$(x-30)(x+15) = 0$

$\therefore x - 30 = 0$ or $x + 15 = 0$

$x = 30$ or $x = -15$ (rejected)

$\therefore$ the shopkeeper bought **30 jackets.**

Check answer: The shopkeeper bought 30 jackets at \$60 $\left(\frac{\$1800}{30} = \$60\right)$ each. He sold 25 jackets at \$80 each, thus taking in \$2000. ($80 \times \$25$)

14. (a) $px^2 + 5x - 6p - 10 = 0$

When $p = 3$,

$3x^2 + 5x - 6(3) - 10 = 0$

$3x^2 + 5x - 28 = 0$

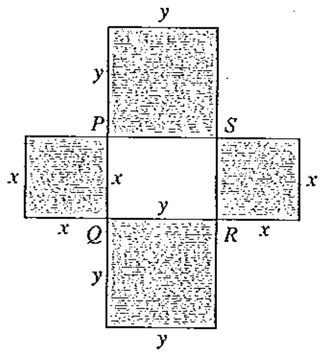
$(3x-7)(x+4) = 0$

$\therefore 3x - 7 = 0$ or $x - 4 = 0$

$3x = 7$

$x = 2\frac{1}{3}$ or $x = 4$

(b)



Let x cm and y cm be the lengths of PQ and QR respectively.

Perimeter of $PQRS$ = 26 cm (Given)

$2(x + y) = 26$ Divide both sides by 2.

$x + y = 13$

Area of the 4 squares = 178 cm^2 (Given)

$x^2 + x^2 + y^2 + y^2 = 178$

$2x^2 + 2y^2 = 178$ Divide both sides by 2.

$x^2 + y^2 = 89$

$(x + y)^2 = x^2 + 2xy + y^2$ Use $(a + b)^2 = a^2 + 2ab + b^2$.

$(x + y)^2 = (x^2 + y^2) + 2xy$ Substitute $x + y = 13$ and $x^2 + y^2 = 89$.

$(13)^2 = 89 + 2xy$

$2xy = 169 - 89 = 80$

$xy = \frac{80}{2} = 40$

Area of $PQRS = xy$

$= \mathbf{40\ cm^2}$

15. (a) Time taken $= \frac{80 \text{ km}}{x \text{ km/h}}$ Use Total time taken $= \frac{\text{Total distance travelled}}{\text{Average speed}}$

$= \left(\frac{80}{x}\right)$h

(b) Time taken $= \frac{30 \text{ km}}{(x - 15) \text{ km/h}}$ Average speed is 15 km/h slower. ∴ average speed $= (x - 15)$ km/h

$= \left(\frac{30}{x - 15}\right)$h

(c) Total time taken = 10 30 − 07 30 = 3 h

Rest ↓

$\frac{80}{x} + 1 + \frac{30}{x - 15} = 3$

$\frac{80}{x} + \frac{30}{x - 15} = 2$

$\frac{80(x - 15) + 30x}{x(x - 15)} = 2$

$80(x - 15) + 30x = 2x(x - 15)$ Multiply both sides by $x(x - 15)$.

$80x - 1200 + 30x = 2x^2 - 30x$

$2x^2 - 140x + 1200 = 0$ Divide both sides by 2.

$\mathbf{x^2 - 70x + 600 = 0}$ Shown.

(d) $x^2 - 70x + 600 = 0$

$(x - 60)(x - 10) = 0$

$\therefore x - 60 = 0$ or $x - 10 = 0$

$\mathbf{x = 60}$ or $\mathbf{x = 10}$

(e) $x = 60$ Reject $x = 10$.

Time taken $= \frac{80 + 30}{60}$ Total time taken $= \frac{\text{Total distance travelled}}{\text{Average speed}}$

$= 1\frac{5}{6}$ h

= 1 h 50 min

07 30 + 1 h 50 min = 09 20

∴ the motorist will reach Town B at **09 20.**

Test 9: Algebraic Manipulation

Section A

1. $\frac{x}{a} + \frac{x}{3b} = c$

$x\left(\frac{1}{a} + \frac{1}{3b}\right) = c$ Extract common factor, x.

$x\left(\frac{3b + a}{3ab}\right) = c$ The LCM of a and $3b$ is $3ab$.

$x = \frac{3abc}{3b + a}$ Cross multiply and express x in terms of the other variables.

2. (a) $\frac{3u - 2v}{u - 4v} = \frac{4}{5}$ Cross-multiply.

$5(3u - 2v) = 4(u - 4v)$

$15u - 10v = 4u - 16v$

$11u = -6v$ Divide both sides by $11v$.

$\frac{u}{v} = -\frac{6}{11}$

(b) (i) $20a^2b^3 = 2^2 \times 5 \times a^2 \times b^3$

$5a^5b^2 = 5 \times a^5 \times b^2$

$\therefore$ HCF $= 5 \times a^2 \times b^2$

$= \mathbf{5a^2b^2}$

Teacher's Tip

To obtain the **Highest Common Factor (HCF)**, find the largest number of factors which are common to both expressions.

(ii) $12pq^2r^3 = 2^2 \times 3 \times p \times q^2 \times r^3$

$21p^2qr^4 = 3 \times 7 \times p^2 \times q \times r^4$

$\therefore$ LCM $= 2^2 \times 3 \times 7 \times p^2 \times q^2 \times r^4$

$= \mathbf{84p^2q^2r^4}$

Teacher's Tip

To obtain the **Least Common Multiple (LCM)**, find the smallest group of factors which contain all the factors of the two expressions.