TUTORIAL PROBLEMS (2) – WEEK 1

For the exercises below, do only every second question (the even numbered questions). The answers to all these questions are at the end of this handout – so if you are having difficulty getting correct answers for a particular exercise, go on and do all the questions in that exercise.

1. Basic Algebra

Algebraic Factors

1. Write down the expansion of:

(a) $(x+5)(x+1)$	(b) $(x-2)(x-3)$	(c) $(a-3)(a+4)$
(d) $(p-7)(p-3)$	(e) $(2x+3)(x+5)$	(f) $(3x-4)(x-2)$
(g) $(3y-2)(4y+3)$	(h) $(x-14)(6x-1)$	(i) $(3x+4)^2$
(j) $(4p-5)^2$	(k) $(3x^2-5x+2)(2x-4)$	(l) $(2a+b)^3$
(m) $(x^2+5)(x^2-2x-3)$	(n) $(x-2)(x+2)(x+2)$	(o) $(5m-2n)^3$

2. Factorise

(a) $bx+by$	(b) $3n+3m$	(c) $6y+12$
(d) $18a-6b$	(e) y^2-4y	(f) a^2-ax
(g) $12xy+16yz$	(h) $17a^2x^5+2a^4x^3$	(i) $bx+by-bz$
(j) $3x^3-x^2+x$	(k) $am^2+2am+3m$	(l) $3xy-3yz+21y$
(m) $a(x+y)-az$	(n) $10a^2+a(c+b)$	(o) $a(x+y)+b(x+y)$

3. Factorise

(a) $xy+xz+yw+zw$	(b) $5a+5b+xa+xb$	(c) $3a+12c+ab+4bc$
(d) $2xy-2xz+7y-7z$	(e) $a^2+ab+ac+bc$	(f) $5a+ab+5b+b^2$
(g) $3ax-bx-3ay+by$	(h) $15ac-10ad+3bc-2bd$	(i) ab^2+ac-b^2d-cd
(j) $a(x-y)+b(y-x)$	(k) $2a-2b+ax-xb$	(l) x^4+x^3+2x+2
(m) $x^2(x+2y)-y(x+2y)$	(n) $xz+z^2-x-z$	(o) $2x^3-x^2-12x+6$

4. Factorise

(a) x^2-y^2	(b) a^2-36	(c) x^2-48
(d) $25-n^2$	(e) x^2-1	(f) $9-4a^2$
(g) x^2-16y^2	(h) $49a^2-121b^2$	(i) $25a^2b^2-4c^2$
(j) $a^2x^2-16b^2y^2$	(k) $1-(a-b)^2$	(l) $49x^2-64y^2z^2$
(m) $100-x^4$	(n) a^4-y^4	(o) $(x+y)^2-x^2$

5. Factorise

(a) x^2+3x+2	(b) $x^2+7x+12$	(c) $x^2+13x+40$
(d) $x^2-7x+12$	(e) $x^2-13x+36$	(f) $x^2-18x+56$
(g) x^2-2x-3	(h) $x^2-10x-11$	(i) $x^2+15x+36$
(j) $x^2-10x+25$	(k) x^2+6x+9	(l) x^2+2x+1
(m) x^2+6x+5	(n) $x^2-22x+120$	(o) x^2-x-42

(i) $\frac{x^2}{x^2+3x+2}-\frac{2x}{x+2}$

(j) $\frac{m}{m^2+mn}+\frac{n}{n^2+mn}$

(k) $\frac{x-1}{2}+\frac{x+3}{5}+\frac{x+7}{10}$

(l) $\frac{2a^2-b^2}{a^2}-\frac{b^2-c^2}{b^2}-\frac{c^2-a^2}{c^2}$

(m) $\frac{1}{x+2}+\frac{1}{x+3}$

(n) $\frac{1}{x^2-9x+20}+\frac{1}{x^2-11x+30}$

(o) $\frac{1}{2x^2-x-1}-\frac{3}{6x^2-x-2}$

(p) $\frac{x-3}{x+2}-\frac{x-2}{x+3}+\frac{1}{x-1}$

Surds

1. Express each of the surds in the simplest form.

(a) $\sqrt{8}$ (b) $\sqrt{12}$ (c) $\sqrt{32}$ (d) $3\sqrt{63}$

(e) $\sqrt{72}$ (f) $\sqrt{96}$ (g) $\sqrt{288}$ (h) $\frac{1}{6}\sqrt{18}$

(i) $\frac{3}{2}\sqrt{192}$ (j) $\sqrt{320}$ (k) $\sqrt{\frac{10}{25}}$ (l) $\sqrt{\frac{72}{25}}$

2. Simplify each of the following surds.

(a) $4\sqrt{3}+3\sqrt{3}-2\sqrt{3}$ (b) $3\sqrt{5}+5\sqrt{5}-\sqrt{5}$ (c) $3\sqrt{45}-\sqrt{20}+7\sqrt{5}$

(d) $\sqrt{12}+\sqrt{27}$ (e) $\sqrt{18}+\sqrt{50}$ (f) $4\sqrt{48}-5\sqrt{27}$

(g) $\sqrt{128}+3\sqrt{18}-\sqrt{162}$ (h) $3\sqrt{32}+2\sqrt{75}-5\sqrt{162}$ (i) $2\sqrt{363}-5\sqrt{320}-\sqrt{192}$

(j) $-2\sqrt{a}-4\sqrt{a}+3\sqrt{a}$ (k) $5\sqrt{y}-\sqrt{x}-2\sqrt{y}$ (l) $2\sqrt{x}+2\sqrt{y}-3\sqrt{x}$

3. Express each of the surds in the simplest form.

(a) $\sqrt{3}\times\sqrt{3}$ (b) $4\sqrt{5}\times2\sqrt{2}$ (c) $(\sqrt{5})^2$

(d) $(2\sqrt{7})^2$ (e) $\sqrt{8}\times\sqrt{5}\times\sqrt{125}$ (f) $\sqrt{2}(\sqrt{3}+2\sqrt{2})$

(g) $(2\sqrt{7}-1)^2$ (h) $2\sqrt{5}(3\sqrt{3}-4\sqrt{2})$ (i) $\sqrt{2}(\sqrt{18}+\sqrt{8}-4\sqrt{2})$

(j) $(\sqrt{3}+4)(\sqrt{3}-2)$ (k) $(2\sqrt{5}-3)(2\sqrt{5}+3)$ (l) $(3\sqrt{5}-2\sqrt{2})(2\sqrt{5}+3\sqrt{2})$

(m) $(2\sqrt{x}+\sqrt{y})(\sqrt{x}-3\sqrt{y})$ (n) $(3\sqrt{a}-\sqrt{b})^2$ (o) $\sqrt{x}(6\sqrt{y}-3\sqrt{x})$

4. Multiply each surd by its conjugate and write down the answer.

(a) $\sqrt{2}-1$ (b) $\sqrt{11}-\sqrt{5}$ (c) $2\sqrt{5}+\sqrt{3}$ (d) $2\sqrt{11}-3\sqrt{5}$

5. Express each of the surds in the simplest form with a rational denominator.

(a) $\frac{1}{\sqrt{7}}$ (b) $\frac{3}{\sqrt{5}}$ (c) $\frac{4}{3\sqrt{7}}$ (d) $\frac{3\sqrt{2}}{\sqrt{8}}$

(e) $\frac{\sqrt{12}}{5\sqrt{3}}$ (f) $\frac{4\sqrt{2}}{3\sqrt{8}}$ (g) $\frac{\sqrt{2}+3}{\sqrt{5}}$ (h) $\frac{1-\sqrt{3}}{\sqrt{3}}$

(i) $\frac{\sqrt{5}-\sqrt{2}}{\sqrt{3}}$ (j) $\frac{4\sqrt{3}+2\sqrt{2}}{3\sqrt{2}}$ (k) $\frac{\sqrt{x}}{\sqrt{y}}$ (l) $\frac{2\sqrt{a}}{\sqrt{ab}}$

6. Simplify the following surds by rationalizing the denominator.

(a) $\frac{1}{\sqrt{3}+1}$ (b) $\frac{1}{\sqrt{3}+\sqrt{7}}$ (c) $\frac{1}{5+3\sqrt{2}}$ (d) $\frac{2}{2\sqrt{2}-1}$

(e) $\frac{3}{\sqrt{5}-2}$ (f) $\frac{\sqrt{3}}{4\sqrt{3}+5}$ (g) $\frac{6}{7+2\sqrt{5}}$ (h) $\frac{\sqrt{5}}{\sqrt{5}-1}$

(i) $\frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}$ (j) $\frac{2\sqrt{3}-1}{2\sqrt{3}+1}$ (k) $\frac{\sqrt{5}+\sqrt{3}}{2\sqrt{10}-\sqrt{6}}$ (l) $\frac{5\sqrt{2}-1}{5\sqrt{2}+1}$

(j) $(x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2)$

(k) $(2x+3)(4x^2-6x+9)$

(l) $x^4y^4(x-y)(x^2+xy+y^2)$

(m) $5(y-10)(y^2+10y+100)$

(n) $(a+b)(a^2-ab+b^2+1)$

(o) $(2x-3)(x^2+3x+39)$

Algebraic Fractions

1. (a) $\frac{2b}{3c}$ (b) $\frac{a}{3b}$ (c) $\frac{2}{3}$ (d) $\frac{a-1}{a+1}$

   (e) $x+2$ (f) $\frac{x+1}{x+2}$ (g) $\frac{3(x-2)}{2(x-2y)}$ (h) $\frac{a}{a+1}$

   (i) $\frac{-3(x+2)}{5}$ (j) $2(x-5)$ (k) $\frac{2(a-1)}{a+1}$ (l) $\frac{a^2+3a+9}{a+3}$

   (m) $\frac{a+b+c}{3}$ (n) $\frac{x^2-y^2}{x^2+y^2}$ (o) $-\frac{(x+1)}{x^2(3+x)}$ (p) $-\frac{x^2(2x+3)}{2+x}$

2. (a) $\frac{75}{xy}$ (b) $\frac{1}{10}$ (c) $\frac{4}{mn}$

   (d) 8 (e) $\frac{a^2}{2}$ (f) $\frac{5}{24}$

   (g) 6 (h) 2 (i) $\frac{2}{x+y}$

   (j) $\frac{m}{n}$ (k) $\frac{1-b}{b(1+a)}$ (l) 1

   (m) $\frac{a+2}{a+5}$ (n) $-\frac{y^2}{x^2}$ (o) 1

3 (a) $\frac{8x}{15}$ (b) $\frac{7m}{6}$

   (c) $\frac{8x+11}{6}$ (d) $\frac{5x-4}{6}$

   (e) $\frac{3x+y}{x(x+y)}$ (f) $\frac{3a-5}{(a+1)(a-3)}$

   (g) $\frac{2a}{5(b-c)}$ (h) $\frac{c(5-a)}{a(a+b)}$

   (i) $\frac{-x}{x+1}$ (j) $\frac{2}{m+n}$

   (k) $\frac{4(x+1)}{5}$ (l) $\frac{a^4b^2-b^4c^2+a^2c^4}{a^2b^2c^2}$

   (m) $\frac{2x+5}{(x+2)(x+3)}$ (n) $\frac{2}{(x-4)(x-6)}$

   (o) $\frac{1}{(x-1)(2x+1)(3x-2)}$ (p) $\frac{x^2+11}{(x-1)(x+2)(x+3)}$