PART 2

PAST EXAMINATION QUESTIONS : BINOMIAL THEOREM

1. Use the binomial theorem to find the exact value of $\left(\sqrt{3}+1\right)^5-\left(\sqrt{3}-1\right)^5$. Hence show without using tables or calculators, that the value of $\left(\sqrt{3}+1\right)^5$ lie between 152 and 153. (N62/P1B/2)

2. (i) Expand $(1-2x)^4$ and $(1-x)^8$, simplifying the coefficients. For what values of x is the sum of the first four terms of the first expansion equal to the sum of the first three terms of the second? (ii) Use the binomial theorem to express $(16\cdot 32)^3$ in the form $a(2^n)$. (N62/P2/1)

3. (i) Use the binomial theorem to find the exact value of $(10\cdot 1)^5$. (ii) Expand $\left(1+2x+3x^2\right)^n$ in a series of ascending powers of x up to and including the term in x^2. (J63/P1/1)

4. Use the binomial theorem to find the exact numerical value of $\left(2+\sqrt{5}\right)^5+\left(2-\sqrt{5}\right)^5$. (N63/P2/4ii)

5. Use the binomial theorem to evaluate $\left(2+\sqrt{2}\right)^4+\left(2-\sqrt{2}\right)^4$. Hence, without using tables, show that $\left(2+\sqrt{2}\right)^4$ lies between 135 and 136. (J64/P1/1)

6. Write down the first four terms in the binomial expansion of $(1+2x)^7$, and simplify the coefficients. (N64/P2/4i)

7. Use the binomial theorem to find the value of $(1\cdot 02)^{10}$ correct to five places of decimals. (J65/P1/1i)

8. Using the formula $a\frac{r^n-1}{r-1}$ for the sum of the first n terms of a geometrical progression, express in its simplest form the sum of the series $1+(1+x)+(1+x)^2+\ldots+(1+x)^{20}$. Use the binomial theorem to verify that the coefficients of x, in the given expression and in your answer, are equal. (N65/P2/6)

9. Without using tables or calculators, evaluate $\left(\sqrt{3}+1\right)^6+\left(\sqrt{3}-1\right)^6$. (J66/P1/1i)

10. In the binomial expansion of $(1+x)^n$ the first three terms are $1+3+4+\ldots$. Calculate the numerical values of n and x, and the value of the fourth term of the expansion. (J66/P2/4ii)

11. Write down the coefficients of x^4 and of x^5 in the expansion by the binomial theorem of $(2+x)^7$. Calculate the value of x if the 5th and 6th terms, in the expansion of $(2+x)^7$ in ascending powers of x, are equal. (N66/P2/2)

1. 152

2. (i) $1 - 8x + 24x^2 - 32x^3 + 16x^4$,
$1 - 8x + 28x^2 - 56x^3 + 70x^4 - 56x^5 +$
$28x^6 - 8x^7 + x^8$; 0, $-\frac{1}{8}$

(ii) 0·132651 (2^{15})

3. (i) 105, 101·00501

(ii) $1 + 2nx + (2n^2 + n)x^2$

4. 1364

5. 136

6. $1 + 14x + 84x^2 + 280x^3$

7. 1·21899

8. $\frac{1}{x}[(1+x)^{21} - 1]$; 210 each

9. 416

10. $x = \frac{1}{3}$, $n = 9$, $\frac{28}{9}$

11. 280, 84; $3\frac{1}{3}$