PAST EXAMINATION QUESTIONS : BINOMIAL THEOREM

1. Find and simplify the coefficient of x^7 in the binomial expansion of $\left(x^2+\frac{2}{x}\right)^8$. (J67/P2/4i)

2. Expand in ascending powers of x, up to and including the term in x^2, (a) $\left(1+\frac{x}{3}\right)^9$, (b) $(2-x)^6$. Hence find the coefficient of x^2 in the expansion of $\left(1+\frac{x}{3}\right)^9(2-x)^6$. (N67/P1/1)

3. Expand $(1+2x)^5$ and $(1-2x)^5$. Use your expansions to calculate the difference between $(1{\cdot}002)^5$ and $(0{\cdot}998)^5$ correct to eight places of decimals. (J68/P2/1)

4. Write down, in terms of x and n, the term containing x^3 in the expansion of $[1-(\frac{x}{n})]^n$ by the binomial theorem. If this term equals $\frac{7}{8}$ when $x=-2$, and n is a positive integer, calculate the value of n. (N68/P2/2)

5. Expand $\left\{1+\left(x+x^2\right)\right\}^{10}$ as a series in ascending powers of x up to and including the term in x^3. find the value of $(1{\cdot}0101)^{10}$ correct to three places of decimals. (J69/P1/1)

6. Write down and simplify the first four terms in the binomial expansion of $(a+b)^7$. Given that the third and fourth terms are equal, and that $a + b = 1$, calculate the numerical values of a and b. (N69/P2/2)

7. Write down and simplify the first 4 terms in the expansion by the binomial theorem of $\left(1-\frac{1}{2}x\right)^{10}$. Find the coefficient of x^2 in the expansion of $(5+4x)\left(1-\frac{1}{2}x\right)^{10}$. (J70/P2/4ii)

8. Write down the expansion in ascending powers of x of $(1+2x)^7$, simplifying each term. Use your expansion to calculate the value of $1{\cdot}02^7$ correct to 5 places of decimals. (N70/P2/1)

9. Write down the binomial expansions of $(2+x)^5$ and $(2-x)^5$. Use your results to express $(2+x)^5-(2-x)^5$ in powers of x. Calculate the exact value of $(2{\cdot}1)^5-(1{\cdot}9)^5$. (N71/P2/3)

10. Expand $\left(1+\frac{x}{100}\right)^{10}$ in a series of ascending powers of x up to and including x^4. A sum of money, £y, increases in such a way that after one year it amounts to £$y\times\frac{102}{100}$, after two years to £$y\times\frac{102}{100}\times\frac{102}{100}$, and so on. Use your expansion in order to find, to the nearest pound, the sum obtained at the end of 10 years from an initial sum of £1 000. (J72/P1/2)

11. Write down the fourth term in the binomial expansion of the function $\left(px+\frac{q}{x}\right)^n$. (a) If this term is independent of x find the value of n. (b) With this value of n calculate the values of p and q given that the fourth term is equal to 160, both p and q are positive and $p-q=1$. (N72/P2/3)

1. 448

2. (a) $1 + 3x + 4x^2$

   (b) $64 - 192x + 240x^2$; -80

3. $1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5$,
   $1 - 10x + 40x^2 - 80x^3 - 80x^4 - 32x^5$;
   0·02000016

4. $-\frac{(n-1)(n-2)}{6n^2}x^3$; 8

5. $1 + 10x + 55x^2 + 210x^3$; 1·106

6. $a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3$,
   $a = \frac{5}{8}$, $b = \frac{3}{8}$

7. $1 - 5x + \frac{45}{4}x^2 - 15x^3$, $36\frac{1}{4}$

8. $1 + 14x + 84x^2 + 280x^3 + 560x^4 + 672x^5 + 448x^6 + 128x^7$, 1·14869

9. $32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5$,
   $32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5$
   $160x + 80x^3 + 2x^5$; 16·08002

10. $1 + \frac{x}{10} + \frac{45x^2}{10000} + \frac{12x^3}{100000} + \frac{21x^4}{10000000}$; £1219

11. $\frac{n(n-1)(n-2)p^{n-3}q^3}{6}x^{n-6}$

    (a) 6 (b) 2, 1