PAST EXAMINATION QUESTIONS: BINOMIAL THEOREM

1. Write down and simplify the first four terms in the expansion of $\left(x+\frac{1}{200x^2}\right)^{10}$ in descending powers of x. Use your expansion to evaluate $(1 \cdot 005)^{10}$ correct to five significant figures. (J73/P1/2)

2. Find the coefficient of x^{-16} in the expansion of $\left(x^2-\frac{1}{x}\right)^{25}$. (sp1/7)

3. The expansion of $(1+px)^n$, where $n > 0$, by the binomial theorem is $1 + 20x + 45p^2x^2 + kx^3 + \ldots$ Calculate n, p and k. (J74/P2/5)

4. The expansion by the binomial theorem of $\left(2x+\frac{1}{4}\right)^9$ is $512x^9 + 576x^8 + ax^7 + bx^6 + \ldots$ Calculate (i) the numerical values of a and b, (ii) the coefficient of x^7 when the expansion of $\left(2x-\frac{1}{4}\right)^9$ is multiplied by $4x - 1$. (N74/P1/3)

5. Write down the first four terms in the binomial expansion of $(1+x)^7$. Find the values of x for which the sum of the 2nd and 4th terms is equal to twice the 3rd term. (J75/P2/4)

6. Write down the first three terms, in ascending powers of x, in the binomial expansion of $(1-x)^{20}$, and hence find the value of $(0 \cdot 996)^{20}$, correct to 4 significant figures. (N75/P1/4)

7. In the binomial expansion of $\left(1+\frac{1}{5}\right)^n$, the second and third terms are equal. Calculate the value of n. (J76/P1/3)

8. Obtain the binomial expansion of $\left(\sqrt{2}+1\right)^5$ in the form $a\sqrt{2}+b$, where a and b are integers. State the corresponding result for the expansion of $\left(\sqrt{2}-1\right)^5$ and show that $\left(\sqrt{2}-1\right)^5$ is the reciprocal of $\left(\sqrt{2}+1\right)^5$. (N76/P2/7)

9. (a) Write down and evaluate the middle term of the binomial expansion of $\left(3+\frac{1}{30}\right)^6$.

   (b) The third term of the binomial expansion of $\left(1+\frac{3}{5}\right)^n$ is 6 times the second term. Calculate the value of n. (J77/P2/7)

10. Use the binomial theorem to evaluate $\left(1-\frac{1}{200}\right)^{16}$ correct to 5 decimal places. (N77/P2/3)

1. $x^{10} + \frac{x^7}{20} + \frac{9x^4}{8000} + \frac{3x}{200000}$; $1{\cdot}0511$

2. 2300

3. $10, 2, 960$

4. (i) $288, 84$ (ii) 48

5. $1 + 7x + 21x^2 + 35x^3$; 0; 1 or $\frac{1}{5}$

6. $1 - 20x + 190x^2$; $0{\cdot}9230$

7. 11

8. $29\sqrt{2} + 41, 29\sqrt{2} - 41$

9. (a) $0{\cdot}02$ (b) 21

10. $0{\cdot}92293$