PAST EXAMINATION QUESTIONS : BINOMIAL THEOREM

1. Find, in ascending powers of x, the first four terms in the expansion of (i) $(1-3x)^5$, (ii) $(1+5x)^7$. Hence find the coefficient of x^2 in the expansion of $(1-3x)^5(1+5x)^7$. (N83/P1/6)

2. (a) Find the coefficient of a^3 in the expansion of $(2+3a)^5$.
   (b) (i) Obtain the expansion of $(1+x)^4+(1-x)^4$ in ascending powers of x. (ii) Use this expansion to find the arithmetic mean of $(1\cdot1)^4$ and $(0\cdot9)^4$. (J84/P1/6)

3. Write down and simplify the expansion $(1-p)^5$. Use this result to find the expansion of $\left(1-x-x^2\right)^5$ in ascending powers of x as far as the term in x^3. Find the value of x which would enable you to estimate $(0\cdot9899)^5$ from this expansion. (N84/P1/6)

4. Expand $\left(2-\frac{x}{2}\right)^5$ by the binomial theorem and reduce the terms to their simplest form. Given that the first three terms in the expansion of $\left(2-\frac{x}{2}\right)^5(1+ax)$ are $32-16x+bx^2$, find the value of a and b. (Sp1/4)

5. Write down and simplify the expansion of $(2-p)^5$. Use this result to find the expansion of $\left(2-2x+\frac{x^2}{2}\right)^5$ in ascending powers of x as far as the term in x^2. (J85/P2/6)

6. Find, in ascending powers of x, the first three terms in the expansion of $(1+ax)^6$. Given that the first two non-zero terms in the expansion of $(1+bx)(1+ax)^6$ are 1 and $\frac{-21x^2}{4}$, find the possible values of a and of b. (N85/P2/9)

7. Find the first three terms in the expansion, in ascending powers of x, of (i) $(1-3x)^5$, (ii) $(2+x)^4$. Hence find the coefficient of x^2 in the expansion of $(1-3x)^5(2+x)^4$. (J86/P1/7)

8. Obtain and simplify (i) the first four terms on the expansion of $\left(2+x^2\right)^6$ in ascending powers of x, (ii) the coefficient of x^4 in the expansion of $\left(1-x^2\right)\left(2+x^2\right)^6$. (N86/P1/5)

9. Evaluate the coefficient of x^5 and x^4 in the binomial expansion of $\left(\frac{x}{3}-3\right)^7$. Hence evaluate the coefficient of x^5 in the expansion of $\left(\frac{x}{3}-3\right)^7(x+6)$. (J87/P1/6)

10. Find the first three terms in the expansion of $(1-2x)^5$ in ascending powers of x, simplifying the coefficients. Given that the first three terms in the expansion of $(a+bx)(1-2x)^5$ are $2+cx+10x^2$, state the value of a and hence find the value of b and of c. (N87/P1/7)

11. Find the first three terms of the expansion, in ascending powers of x, of (i) $(1+2x)^6$, (ii) $(1-3x)^6$. Hence obtain the coefficient of x^2 in the expansion of $\left(1-x-6x^2\right)^6$.

1. (i) $1 - 15x + 90x^2 - 270x^3$
   (ii) $1 + 35x + 525x^2 + 4375x^3$; 90
2. (a) 1080
   (b) (i) $2 + 12x^2 + 2x^4$
   (ii) 1.0601
3. $1 - 5p + 10p^2 - 10p^3 + 5p^4 - p^5$;
   $1 - 5x + 5x^2 + 10x^3$; 0.01 or -1.01
4. $32 - 40x + 20x^2 - 5x^3 + \frac{5}{8}x^4 - \frac{x^5}{32}$;
   $\frac{3}{4}, -10$
5. $32 - 80p + 80p^2 - 40p^3 + 10p^4 - p^5$;
   $32 - 160x + 360x^2$
6. $1 + 6ax + 15a^2x^2$; $a = \frac{1}{2}, b = -3$;
   $a = -\frac{1}{2}, b = 3$
7. (i) $1 - 15x + 90x^2$
   (ii) $16 + 32x + 24x^2$; 984
8. (i) $64 + 192x^2 + 240x^4 + 160x^6$
   (ii) 48
9. $\frac{7}{9}, -\frac{35}{3}$; -7
10. $1 - 10x + 40x^2$; 2, 7, -13
11. (i) $1 + 12x + 60x^2$
    (ii) $1 - 18x + 135x^2$; -21