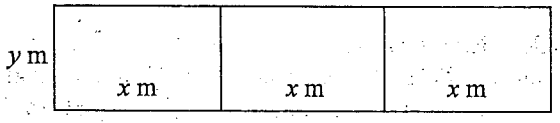
PART 2

# PAST EXAMINATION QUESTIONS : QUADRATIC INEQUALITIES

1. A farmer has 600 m of fencing and wishes to make an enclosure consisting of three equal rectangles, as in the figure.

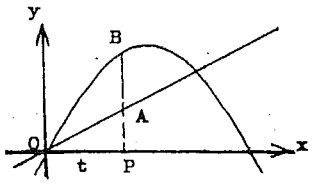



Express $y$, and the total area $A$ m$^2$, in terms of $x$, and find the value of $x$ for which $A$ is a maximum. (J74/P2/3)

2. Given that $x^2 - 3x + 5 = (x-p)^2 + q$ for all values of $x$, calculate the numerical values of $p$ and $q$. Hence state (i) the minimum value of $x^2 - 3x + 5$, (ii) the value of $x$ at which the minimum value occurs. Sketch the curve $y = x^2 - 3x + 5$. (J74/P2/14)

3. Determine in each of the following cases the range(s) of values of $x$ for which (i) $(x-3)(x+5) < 0$, (ii) $6 + x < 2x^2$. Illustrate your answers by sketching graphs of the curves $y = (x-3)(x+5)$ and $y = 6 + x - 2x^2$. (N74/P2/14)

4. The figure shows the curve $y = x(4 - x)$ and the line $y = \frac{1}{2}x$. $P$ is the point $(t, 0)$, where $0 < t < 3\frac{1}{2}$, and $PAB$ is parallel to the $y$-axis. Express the length of $AB$ in terms of $t$ and find the value of $t$ for which this length is a maximum. (J76/P1/6)




5. Find the values of $x$ for which $3x + 4 < x^2$. (J76/P2/12b)

6. Given that $y = 7 - px - x^2 = 16 - (q+x)^2$, for all values of $x$, where $p$ and $q$ are both positive, (i) calculate the values of $p$ and $q$, (ii) state the maximum value of $y$, and the value of $x$ at which it occurs (iii) find the range of values of $x$ for which $y$ is positive. (J80/P1/13)

7. Write $x^2 + 6kx + 144$ in the form $(x+p)^2 + q$ and thus obtain expressions for $p$ and $q$ in terms of $k$. Hence find the range of values of $k^2$ such that $x^2 + 6kx + 144$ is positive for all values of $x$, and deduce the corresponding range of values of $k$. (J81/P1/12b)

8. The curve $y = ax^2 + bx + c$ has a maximum point at (2, 18) and passes through the point (0, 10). Evaluate $a$, $b$ and $c$. (N81/P2/1)

9. Given that the curve whose equation is $y = p - (x-q)^2$ crosses the $x$-axis at the points (1, 0) and (3, 0), find (i) the value of $p$ and of $q$, (ii) the maximum value of $y$. (N82/P2/13a)

10. Express $y = \frac{1}{2}\{(x+5)^2 + (x-7)^2\}$ in the form $y = (x+q)^2 + r$. Hence find the least value of $y$ and the corresponding value of $x$. (J83/P2/13b)

11. A rectangular sheep pen has one fixed sided which is part of a long straight stone wall. The remainder is to be made by using 80 m of fencing. Find the dimensions of the rectangle with the greatest possible area. (N83/P1/3)

12. Find the range of values of $k$ for which $8 - 3x - x^2 \leq k$ for all real values of $x$. (J84/P1/13a)

1. $150 - \frac{3}{2}x$, $450 - \frac{9}{2}x^2$; 50

2. $1\frac{1}{2}, 2\frac{3}{4}$

   (i) $2\frac{3}{4}$

   (ii) $1\frac{1}{2}$

3. (i) $-5 < x < 3$

   (ii) $x > 2$ or $x < -1\frac{1}{2}$

4. $\frac{1}{2}(7t - 2t^2)$; 1·75

5. $x > 4$ or $x < -1$

6. (i) 6, 3

   (ii) 16, –3

   (iii) $-7 < x < 1$

7. $3k$, $144 - 9k^2$; $k^2 < 16$; $-4 < k < 4$

8. $a = -2$, $b = 8$, $c = 10$

9. (i) $p = 1$, $q = 2$

   (ii) 1

10. $(x - 1)^2 + 36$, $y = 36$ where $x = 1$

11. 20 m × 40 m

12. $k \geq 10\frac{1}{4}$