

PAST EXAMINATION QUESTIONS: TRIG IDENTITIES & EQNS

- Given that $\sin \beta = p$ where β is an acute angle measured in degrees, obtain an expression, in terms of p , for (i) $\tan \beta$, (ii) $\sin(90^\circ - \beta)$, (iii) $\sin(180^\circ + \beta)$. (N98/P1/8)
- Prove the identity $(1 + \operatorname{cosec} \theta)(1 - \sin \theta) \equiv \cos \theta \cot \theta$. (N98/P1/9)
- Given that $\frac{\cos(A-B)}{\cos(A+B)} = -\frac{9}{7}$, find the value of $\tan A$ and $\tan B$. (N98/P2/4c)
- (a) The parametric equations of a curve are $x = 3 \sin \alpha + \cos \alpha$, $y = \sin \alpha - 2 \cos \alpha$. Express each of $\sin \alpha$ and $\cos \alpha$ in terms of x and y . Hence obtain the cartesian equation of the curve.
(b) The cartesian equation of a curve is $(y - 3)^2 = 1 + x^2$. Given that x may be defined parametrically by $x = \cot \theta$, and that $y = 1$ when $\theta = \frac{\pi}{6}$, express y in terms of $\operatorname{cosec} \theta$.

5. Show that $\frac{1}{\sec \theta + 1} + \frac{1}{\sec \theta - 1} \equiv 2 \operatorname{cosec} \theta \cot \theta$. (J99/P1/7)

6. Find, in terms of h , an expression for

(a) $\tan A$,

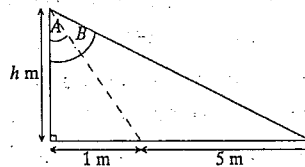
(b) $\tan B$,

where A , B and h are as shown in the diagram.

Hence obtain, in terms of h , an expression for

$\tan(B - A)$.

Given that $B - A = 45^\circ$, find the two possible values of h . (J99/P2/4c)



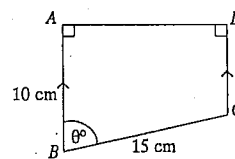
7. Show that $(\tan \theta + \sin \theta)(\tan \theta - \sin \theta) \equiv \tan^2 \theta \sin^2 \theta$. (N99/P1/6)

8. (a) Show that $4 \sin A \cos^3 A - 4 \cos A \sin^3 A \equiv \sin 4A$.

- (b) The trapezium $ABCD$ is right angled at A and at D , and AB is parallel to DC . The acute angle ABC is θ° , $AB = 10$ cm and $BC = 15$ cm.

(i) Express AD and DC in terms of θ .

(ii) Find the value of θ for which the perimeter is 45 cm.



(c) Given that $\frac{\sin(A-B)}{\sin(A+B)} = 2$, find the value of $\tan A \cot B$. (N99/P2/5)

9. Show that $(1 - \sin A + \cos A)^2 \equiv 2(1 - \sin A)(1 + \cos A)$. (J2000/P1/9)

10. Find the value of each of the constants a and b for which $\sin x \cos x (5 \tan x + 2 \cot x) \equiv a + b \sin^2 x$. (N2000/P1/9)

11. Given that $2 \sin A \cos A + (\cos A + \sin A)^2 - (2 \cos A + \sin A)^2 \equiv p \sin^2 A + q$, find the value of the constant p and of the constant q . (J01/P1/3)

12. Prove that $\cot A + \tan A \equiv \sec A \operatorname{cosec} A$. (N01/P1/4)

13. Given that $\sin(A + B) = 2 \sin(A - B)$, express $\tan A$ in terms of $\tan B$. (N01/P2/5c)

1. (i) $\frac{p}{\sqrt{1-p^2}}$

(ii) $\sqrt{1-p^2}$

(iii) $-p$

3. $\tan A \tan B = 8$

4. (a) $\sin a = \frac{2x+y}{7}$, $\cos a = \frac{x-3y}{7}$

$5x^2 - 2xy + 10y^2 = 49$

(b) $y = 3 - \operatorname{cosec} \theta$

6. (a) $\tan A = \frac{1}{h}$

(b) $\tan B = \frac{6}{h}$; $\tan(B-A) = \frac{5h}{h^2+6}$;

$h = 2$ or 3

8. (b) (i) $AD = 15 \sin \theta$,

$DC = 10 - 15 \cos \theta$

(ii) 73.1°

(c) -3

10. $a = 2$, $b = 3$

11. $p = 3$, $q = -3$

13. $\tan A = 3 \tan B$