PART I

## PAST EXAMINATION QUESTIONS : TRIG IDENTITIES + EQNS

1. Calculate the maximum and minimum values of $3 \sin x + \cos x$. (J60/P2/2)
2. (i) If $A$ is an acute angle whose tangent is $\frac{15}{8}$ and $B$ is an obtuse angle whose sine is $\frac{12}{13}$, find, without using tables or calculators, the values of (a) $\tan 2A$, (b) $\cos 2B$; (c) $\sin (B - A)$; (d) $\tan (A + B)$.
   (ii) Prove that $\frac{1 + \sin 2x + \cos 2x}{\cos x + \sin x} = 2 \cos x$. (J60/P2/5)
3. (i) If $A$ is an acute angle whose cosine is $\frac{3}{5}$ and $B$ is an obtuse angle whose sine is $\frac{7}{25}$, find, without using tables or calculators, the values of (a) $\cos (A + B)$; (b) $\cos 2A$; (c) $\tan (B - A)$.
   (ii) If $\cos 2x = \tan^2 y$, prove that $\cos 2y = \tan^2 x$. (N60/P2/5)
4. (a) Write down a formula for $\cos 2\theta$ in terms of $\sin \theta$.
   (b) Without using tables or calculators, obtain an expression for $\sin 15°$. (N61/P1/7ii)
5. Write down the expansions of $\sin (A + B)$ and $\cos (A + B)$. Hence show that $\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$. Deduce an expression for $\tan 2A$ in terms of $\tan A$. Hence, assuming that $\tan 30° = \frac{1}{\sqrt{3}}$, obtain a quadratic equation for $\tan 15°$. Find the value of $\tan 15°$ in surd form. (J62/P1/6)
6. $ABC$ is an acute-angled triangle inscribed in a circle of radius 5 cm and centre $O$. The sine of the angle $A$ is equal to $\frac{3}{5}$. Calculate, without using tables or calculators, (a) the length of $BC$; (b) $\sin OBC$; (c) $\sin BOC$; (d) $\cos BOC$. (N62/P1B/8)
7. Write down the expansion of $\cos (A + B)$. Write $90 - C$ for $A$ in this expansion and thus find the corresponding expansion for $\sin (B - C)$. If $A$ is an acute angle such that $\sin A = \frac{3}{5}$ and if $B$ is an obtuse angle such that $\cos B = -\frac{5}{13}$, calculate, without using tables or calculators, the values of $\sin B + \sin A \cos (A + B) + \cos A \sin (A - B)$. (N62/P2/8)
8. Given that $\tan (A + B) = 1$ and that $\tan (A - B) = \frac{1}{7}$, find, without using tables or calculators, the values of $\tan A$ and $\tan B$. (J63/P1/6)
9. Tables and calculators may NOT be used in this question.
   (i) Given that $\tan A = \frac{1}{5}$, find the values of $\tan 2A$, $\tan 4A$ and $\tan (45° - 4A)$.
   (ii) Find a formula for $\cos 3\alpha$ in terms of $\cos \alpha$. (Formulae for $\cos 2\alpha$ and $\sin 2\alpha$ may be used without proof.) Use your result to find the exact value of $8 \cos^3 20° - 6 \cos 20°$. (N63/P1/6)
10. Given that $\tan 2A = \frac{3}{4}$ and that the angle $A$ is acute, calculate, without using tables or calculators, the vales of (a) $\cos 2A$; (b) $\sin A$; (c) $\tan A$; (d) $\tan 3A$. (J64/P1/6)
11. (i) Given that $A = B + C$, prove that $\tan A - \tan B - \tan C = \tan A \tan B \tan C$.
    (ii) Prove that $\sin (\alpha + 30°) = \cos \alpha + \sin (\alpha - 30°)$. (N64/P1/7)
12. It is given that $\tan A$ and $\tan B$ are the roots of the equation $t^2 - pt + q = 0$. Find, in terms of $p$ and $q$, expressions for (a) $\tan (A + B)$; (b) $\sin^2 (A + B)$; (c) $\cos 2(A + B)$. (J65/P1/6)
13. If $A + B = 45°$, show that $\tan A + \tan B + \tan A \tan B = 1$. Hence, or otherwise, express $\tan 22{\cdot}5°$ in surd form. (J66/P1/6)

1. Maximum value = 3·16
   Minimum value = −3·16
2. (i) (a) $-\frac{240}{161}$
   (b) $-\frac{119}{169}$
   (c) $\frac{171}{221}$
   (d) $-\frac{21}{220}$
3. (a) $-\frac{4}{5}$
   (b) $-\frac{7}{25}$
   (c) $-\frac{117}{44}$
4. (a) $1 - 2\sin^2\theta$
   (b) $\frac{\sqrt{3}-1}{2\sqrt{2}}$
5. $\sin A \cos B + \cos A \sin B$,
   $\cos A \cos B - \sin A \sin B$;
   $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$,
   $\tan^2 15 + 2\sqrt{3}\ \tan 15 - 1 = 0,\ 2 - \sqrt{3}$
6. (a) 6 cm
   (b) $\frac{4}{5}$
   (c) $\frac{24}{25}$
   (d) $\frac{7}{25}$
7. $\cos A \cos B - \sin A \sin B$,
   $\sin B \cos C - \cos B \sin C$; $-\frac{24}{65}$
8. $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$; or
   $\tan A = -2$, $\tan B = -3$
9. (i) $\frac{5}{12}, \frac{120}{119}, -\frac{1}{239}$
   (ii) $4\cos^2\alpha - 3\cos\alpha$, 1
10. (a) $\frac{4}{5}$
    (b) $\sqrt{0 \cdot 1}$
    (c) $\frac{1}{3}$
    (d) $\frac{13}{9}$
12. (a) $\frac{p}{1-q}$
    (b) $\frac{p^2}{p^2+(1-q)^2}$
    (c) $\frac{(1-q)^2-p^2}{(1-q)^2+p^2}$
13. $\sqrt{2} - 1$