PAST EXAMINATION QUESTIONS: TRIG IDENTITIES + EQNS

1. (i) Show that $\tan A\,(\operatorname{cosec} 2A + \cot 2A) = 1$.

   (ii) If A is an obtuse angle whose sine is $\frac{5}{13}$ and B is an acute angle whose tangent is $\frac{3}{4}$, without using tables or calculators, find the values of (a) $\sin 2B$; (b) $\tan (A - B)$. (N66/P1/8)

2. (i) If $\cos y = 4 \cos (y + 60°)$, calculate $\tan y$.

   (ii) Eliminate θ from the equations $\tan \theta = a$, $\cos 2\theta = b$. (N66/P2/8)

3. Write down expressions for $\sin 2A$ and $\cos 2A$ in terms of $\sin A$ and $\cos A$. Hence, or otherwise, obtain an expression for $\sin 3A$ in terms of $\sin A$. Given that $\cos 3A = 4 \cos^3 A - 3 \cos A$, show that $\frac{\sin 3A - \sin A}{\cos A + \cos 3A} = \tan A$. (N67/P1/7)

4. If A, B, C are the angles of a triangle and $\tan A = 1$ and $\tan B = 2$, prove without using tables or calculators that $\tan C = 3$. If a, b, c are the corresponding sides of the triangle, prove that $\frac{a}{\sqrt{5}} = \frac{b}{2\sqrt{2}} = \frac{c}{3}$. (N67/P2/8)

5. Given that y is acute and $\tan y = \frac{1}{a}$ find, in terms of a, the value of $a \cos 2y + \sin 2y$.

6. Prove that $\cot A + \tan 2A = \cot A \sec 2A$, where A is any angle. (N69/P2/6i)

7. Given that $\sin X = -\frac{3}{5}$, $\cos Y = -\frac{5}{13}$ and X is in the same quadrant as Y, evaluate, without using tables or calculators, (a) $\cos (X - Y)$, (b) $\tan (X + Y)$, (c) $\cos \frac{1}{2}X$. (J70/P1/6)

8. By expressing each side of the equation in terms of $\tan A$, or otherwise, show that $\frac{\sin 2A + \cos 2A + 1}{\sin 2A + \cos 2A - 1} = \frac{\tan (45° + A)}{\tan A}$. Hence show that $\tan 15° = 2 - \sqrt{3}$. (N70/P1/5)

9. If $\sin \theta = a$ and $\sin 2\theta = b$, obtain an expression for b in terms of a (J71/P1/6ii)

10. (i) Given that the angle A is acute, prove that $\sin^2 A + \cos^2 A = 1$. If the magnitude of the angle A is y, calculate x and y given that $x + \frac{1}{3} = \sin y°$ and $x - \frac{1}{3} = \cos y°$.

    (ii) Given that $\cos A = \frac{1}{8}$ and that $270° < A < 360°$ find, without using tables or calculators, the value of $\cos \frac{A}{2}$. (J71/P2/6)

11. (i) Use the expansion of $\cos (A + B)$ to find $\cos 75°$ without using tables or calculators, leaving your answer in surd form with an integral denominator.

12. (ii) Prove that $\cot A - \tan A = 2 \cot 2A$. (N71/P2/6)

    Given that $\tan A = \frac{1}{2}$ calculate, without using tables or calculators, the values of $\tan 2A$ and $\tan 3A$. (J72/P2/7i)

13. Given that angle $2A$ is acute and that $\sin 2A = \frac{\sqrt{5}}{3}$ calculate, without using tables or calculators, the values of (a) $\cos 2A$, (b) $\sin A$, (c) $\tan A$, (d) $\tan \frac{1}{2}A$. (Answers may be left in surd form.) (J73/P1/6)

1. (ii) (a) $\frac{24}{25}$ (b) $-\frac{56}{33}$

2. (i) $\frac{1}{6}\sqrt{3}$ (ii) $b = \frac{1-a^2}{1+a^2}$

3. $2 \sin A \cos A$, $\cos^2 A - \sin^2 A$
$= 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$;
$3 \sin A - 4 \sin^3 A$

5. a

7. (a) $\frac{56}{65}$
(b) $-\frac{63}{16}$
(c) $\pm\frac{1}{\sqrt{10}}$

9. $b^2 = 4a^2(1 - a^2)$

10. (i) $x = 0{\cdot}624$, $y = 73{\cdot}1$
(ii) $-\frac{3}{4}$

11. (i) $\frac{\sqrt{6}-\sqrt{2}}{4}$

12. $1\frac{1}{3}, 5\frac{1}{2}$

13. (a) $\frac{2}{3}$
(b) $\frac{\sqrt{6}}{6}$
(c) $\frac{\sqrt{5}}{5}$
(d) $\sqrt{6} - \sqrt{5}$