PAST EXAMINATION QUESTIONS: TRIG IDENTITIES + EQNS

1. Given that $\tan X = \frac{15}{8}$, $\cos Y = -\frac{3}{5}$ and X and Y are in the same quadrant, calculate, without using tables or calculators, the values of (a) $\sin(X+Y)$, (b) $\cos\frac{1}{2}Y$. (N73/P1/6)

2. Given that $\sin\frac{1}{2}A = \frac{4}{5}$ and that A is obtuse calculate, without the use of tables or calculators, the values of $\sin A$ and $\cos A$. (N73/P2/7i)

3. Given that $\tan 2A = \frac{3}{4}$ and that angle A is acute, calculate, without using tables or calculators, the values of (a) $\cos 2A$, (b) $\sin A$, (c) $\tan 3A$. (sp1/16)

4. Prove that $\frac{\sin(A+B)-\sin(A-B)}{\cos(A+B)+\cos(A-B)} = \tan B$. It is given that $\tan(A+B) = -\frac{5}{12}$ and $\tan(A-B) = \frac{3}{4}$ where $0° < A+B < 180°$ and $0° < A-B < 90°$. Write down, without the use of tables or calculators, the values of $\sin(A+B)$, $\cos(A+B)$, $\sin(A-B)$ and $\cos(A-B)$. Hence calculate the value of $\tan B$. (J74/P1/15)

5. Find the co-ordinates of the turning points on the curve $y = \sin x + \cos x$ for $0 < x < 2\pi$. (J74/P1/16b)

6. Given that $\frac{\sin(A-B)}{\sin(A+B)} = \frac{3}{5}$ show that $\sin A \cos B = 4 \cos A \sin B$. If $\tan B = \frac{1}{2}$ calculate, without using tables or calculators, the values of $\tan A$ and $\tan 2A$. (N74/P1/16)

7. Given that $270° < 4x < 360°$ and $\cos 4x = \frac{17}{32}$ calculate, without the use of tables or calculators, the values of $\cos 2x$, $\cos x$ and $\cos 3x$. (J75/P2/15)

8. (a) Given that $\sin x \cos y = p$ and $\cos x \sin y = q$ find, in terms of p and q, the values of the following: (i) $\sin(x+y)\sin(x-y)$, (ii) $\frac{\tan x}{\tan y}$, (iii) $\sin 2x \sin 2y$.

   (b) Prove that $\frac{\cos A}{1+\cos 2A} + \frac{\sin A}{1-\cos 2A} = \frac{\sin A + \cos A}{\sin 2A}$. (N75/P1/16)

9. Given that $90° < y < 360°$ and that $\tan y = \frac{5}{12}$, calculate, without using tables or a calculator, the value of $\cos y$. (J76/P1/2b)

10. If $c = \cos\theta$, express, in terms of c, (i) $\tan^2\theta$, (ii) $\sin 2\theta$, (iii) $\sin 4\theta$, (iv) $\cos\frac{1}{2}\theta$. (J76/P1/15a)

11. If $\sin\theta$, $\tan\theta$ and $\sin 2\theta$ are in geometric progression $(0° < \theta < 90°)$ show that $\cos^3\theta = \frac{1}{2}$. (J76/P2/13b)

12. Given that $\tan A = \frac{4}{3}$ and $0° < A < 90°$, calculate, without using tables or calculators, the values of $\cos 2A$ and $\cos 4A$. (N76/P1/5a)

13. Given that $\cos x = -\frac{2}{3}$ and $\sin y = \frac{1}{\sqrt{6}}$, that $0° \le x \le 360°$ and that x and y are in the same quadrant, find, in surd form, without using tables or calculators, the value of (i) $\tan 2x$, (ii) $\cos(x+y)$, (iii) $\cos\frac{1}{2}x$. (J77/P1/14)

1. (a) $\frac{77}{85}$ (b) $\pm\frac{1}{\sqrt{5}}$

2. 0·96, −0·28

3. (a) 0·8
   (b) $\sqrt{0\cdot1}$
   (c) $\frac{13}{9}$

4. $\frac{5}{13}, -\frac{12}{13}, \frac{3}{5}, \frac{4}{5}$; 1·75

5. $(\frac{\pi}{4}, \sqrt{2}), (\frac{5\pi}{4}, -\sqrt{2})$

6. $2, -\frac{4}{3}$

7. $-\frac{7}{8}, \frac{1}{4}, -\frac{11}{16}$

8. (a) (i) $p^2 - q^2$
   (ii) $\frac{p}{q}$
   (iii) $4pq$

9. $-\frac{12}{13}$

10. (i) $\frac{1}{c^2} - 1$
   (ii) $\pm 2c\sqrt{1-c^2}$
   (iii) $\pm 4c\,(2c^2-1)\sqrt{1-c^2}$
   (iv) $\pm\sqrt{\frac{1}{2}(1+c)}$

12. −0·28, −0·8432

13. (i) $4\sqrt{5}$
   (ii) $\frac{\sqrt{5}}{3\sqrt{6}}$
   (iii) $\frac{1}{\sqrt{6}}$