PAST EXAMINATION QUESTIONS: TRIG IDENTITIES + EQNS.

1. (a) Prove the identity $\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$.
   (b) If A, B and C are the angles of a triangle and $\tan A = 1$ and $\tan B = 2$ show, without using tables or calculators, that $\tan C = 3$. (J78/P2/12)

2. Given that $\tan\theta = \frac{3}{4}$ and that $180° < \theta < 270°$, find, without using tables or calculators, value of (i) $\tan 2\theta$, (ii) $\tan 3\theta$, (iii) $\tan\frac{1}{2}\theta$. For this particular value of θ, verify that

   $\sin\theta = \dfrac{2\tan\frac{1}{2}\theta}{1+\tan^2\frac{1}{2}\theta}$. (N78/P1/16)

3. The equation of a curve is given, in terms of a parameter θ, by $x = \cos\theta + \sin\theta$, $y = \cos\theta + 2\sin\theta$. Express $\sin\theta$ and $\cos\theta$ in terms of x and y and hence obtain the cartesian equation of the curve. (J79/P1/14b)

4. By squaring $\sin^2 A + \cos^2 A$, or otherwise, show that $\sin^4 A + \cos^4 A = \frac{1}{4}(3 + \cos 4A)$. (J79/P1/15a)

5. Given that $\sin(x+\theta) = 2\cos(x-\theta)$, express $\tan x$ in terms of $\sin\theta$ and $\cos\theta$. (J79/P2/12a)

6. Prove that the maximum value of $6\sin x + 8\cos x$ is 10. (N79/P1/14a)

7. A curve is defined parametrically by $x = 3\sin t$, $y = \cos 2t$. Find the cartesian equation of the curve. (N79/P2/12b)

8. Prove that (i) $2\sin(A+45°)\cos(A+45°) = \cos 2A$. (ii) $2\cos(B+45°)\cos(B-45°) = \cos 2B$. (N79/P2/14a)

9. (a) Given that $\tan 2A = 1$ and that A is acute show, without using tables or calculators, that $\tan A = \sqrt{2} - 1$.
   (b) Given that $\sin\theta = s$, and θ is acute, express in terms of s, (i) $\cos 2B$, (ii) $\sin 2B$ (iii) $\tan 2B$. (J80/P2/14)

10. (a) Given that $\cos A = \frac{3}{4}$ find, without using tables or calculators, the value of (i) $\cos 2A$, (ii) $\cos 4A$.
    (b) If $\tan B = 3\tan C$ prove that $\tan(B - C) = \frac{\sin 2C}{2 - \cos 2C}$. (N80/P2/14)

11. (a) A, B, C are the angles of a triangle such that $\cos A = \frac{3}{5}$ and $\cos B = \frac{5}{13}$. Without using tables or a calculator find the value of (i) $\tan 2A$, (ii) $\cos(A+B)$, (iii) $\cos C$.
    (b) Prove the identity $\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$. (J81/P1/14)

12. Find the value of x between 0° and 360° for which $(\cos x - \sin x)$ has a maximum value. (J81/p1/15b)

13. (a) Given that $\sin A = \frac{\sqrt{5}}{3}$ and that A is obtuse find, without using tables or calculators, the value of (i) $\cos 2A$, (ii) $\sin 2A$, (iii) $\sin 4A$. Answers may be left in surd form.
    (b) Prove the identity $\tan(A+45°) + \tan(A-45°) = 2\tan 2A$. (N81/P1/14)

2. (i) $3\frac{3}{7}$

(ii) $-2\frac{29}{44}$

(iii) -3

3. $y-x,\ 2x-y,\ 5x^2-6xy+2y^2=1$

5. $\frac{\sin\theta-2\cos\theta}{2\sin\theta-\cos\theta}$

7. $y=1-\frac{2}{9}x^2$

9. (b) (i) $1-2s^2$

(ii) $2s\sqrt{1-s^2}$

(iii) $\frac{2s\sqrt{1-s^2}}{1-2s^2}$

10. (a) (i) $\frac{1}{8}$ (ii) $-\frac{31}{32}$

11. (a) (i) $-\frac{24}{7}$

(ii) $-\frac{33}{65}$

(iii) $\frac{33}{65}$

12. 315°

13. (a) (i) $-\frac{1}{9}$

(ii) $-\frac{4\sqrt{5}}{9}$

(iii) $\frac{8\sqrt{5}}{81}$