PART 5

PAST EXAMINATION QUESTIONS : TRIG IDENTITIES + EQNS

1. Find the Cartesian equation of the curve which is defined parametrically by $x = 2 \sin \theta$, $y = \cos^2 \theta$. (N81/P1/16bi)

2. (a) Prove the identity $\tan A + \cot A \equiv 2 \operatorname{cosec} 2A$.
   (b) It is given that $\tan B = \frac{4}{3}$ and that B is acute. Without using tables or a calculator, find the value of (i) $\cos 2B$, (ii) $\tan \frac{B}{2}$. (J82/P2/14)

3. (a) Prove that identity $\frac{\tan 2\theta}{1 + \sec 2\theta} = \tan \theta$.
   (b) A is a obtuse angle such that $\sin A = \frac{5}{13}$. Without using tables or a calculator find the value of $\tan 2A$ and hence show that A is approximately $157\frac{1}{2}^\circ$. (N82/P1/14)

4. x and y are acute angle such that $\sin (x - y) = \frac{4}{5}$ and $\sin x \cos y = \frac{9}{10}$. Without using tables or a calculator find (i) $\sin (x + y)$, (ii) $\frac{\tan x}{\tan y}$. From your answers to (i) and (ii) deduce the value of $\tan x$. (N82/P2/14b)

5. (a) Prove the identity $\sin (\alpha - 30^\circ) + \cos (\alpha + 30^\circ) \equiv \frac{\sqrt{3} - 1}{\sqrt{2}} \sin (\alpha + 45^\circ)$.
   (b) Express $\tan 2\theta$ in terms of $\tan \theta$ and hence find, without using tables or a calculator, the value of $\tan 67\frac{1}{2}^\circ$ in surd form. (J83/P1/14)

6. Given that $\sin x = b$ and $0^\circ < x < 90^\circ$ find an expression in terms of b for (i) $\tan x$, (ii) $\cos (-x)$. (J83/P2/7b)

7. Given that $\tan x = p$ and that x is acute, find an expression for $\sin x$ in terms of p. (N83/P2/8b)

8. (a) Prove the identity $\tan 2A\,(2 \cos A - \sec A) \equiv 2 \sin A$.
   (b) α and β are acute angles such that $\sin \alpha = \frac{4}{5}$ and $\tan \beta = \frac{20}{21}$. Without using tables or a calculator find he value of $\sin (\alpha + \beta)$. Given that $\alpha + \beta = \theta - \frac{\pi}{2}$, find the value of $\cos \theta$. (N83/P2/14)

9. (a) Given that $\tan 60^\circ = \sqrt{3}$, show, without using tables or a calculator, that $\tan 15^\circ = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$.
   (b) Given that $\sin A = -\frac{4}{5}$, $\cos B = \frac{4}{5}$ and that A and B are in the same quadrant, find, without using tables or a calculator, the value of $\sin (2A + B)$. (J84/P1/14)

10. Find the cartesian equation of the curve which is defined parametrically by $x = 2 + \sin \theta$, $y = 6 \cos \theta$. (J84/P2/16aii)

11. (a) Given that $\sin (x + \alpha) + \cos (x - \alpha) = \cos x$ where α is the acute angle such that $\tan \alpha = \frac{3}{4}$, find the value of $\tan x$.
    (b) Find the value of p and of q for which $4 \cos A - \sin^2 A \equiv (\cos A + p)^2 + q$. Hence, or otherwise, find the maximum value and the minimum value of $4 \cos A - \sin^2 A$.

12. Prove the identity $\sin 2\theta - \tan \theta \cos 2\theta = \tan \theta$. Hence show that $\tan 67\frac{1}{2}^\circ = \sqrt{2} + 1$. (N84/P2/14b)

13. Find the cartesian equation of the curve which is defined parametrically by $x = \sin 2\theta$, $y = \cos^2 \theta$. (Sp2/6bii)

1. $x^2 + 4y = 4$

2. (b) (i) $-\frac{7}{25}$ (ii) $\frac{1}{2}$

3. (b) $-\frac{120}{119}$

4. (i) 1 (ii) 9; 3

5. (b) $\frac{2\tan\theta}{1-\tan^2\theta}$, $1+\sqrt{2}$

6. (i) $\frac{1}{\sqrt{1-b^2}}$ (ii) $\sqrt{1-b^2}$

7. $\frac{p}{\sqrt{1+p^2}}$

8. (b) $\frac{144}{145}$, $-\frac{144}{145}$

9. (b) $-\frac{3}{5}$

10. $(x-2)^2 + \frac{y^2}{36} = 1$

11. (a) $-\frac{2}{7}$ (b) 2, –5; 4; –4

13. $x^2 = 4y - 4y^2$