PART 7

PAST EXAMINATION QUESTIONS : TRIG IDENTITIES + EQNS

1. Prove the identity $\frac{(\sin x + \cos x)^2}{\sin x \cos x} \equiv 2 + \sec x \operatorname{cosec} x$. (N88/P1/3)

2. Two acute angles, α and β, are such that $\tan \alpha = \frac{4}{3}$ and $\tan(\alpha + \beta) = -1$. Without evaluating α or β, (i) show that $\tan \beta = 7$, (ii) evaluate $\sin \alpha$ and $\sin \beta$, (iii) evaluate $\sin^2 2\alpha + \sin^2 2\beta$. (N88/P2/3b)

3. The cartesian equation of a certain curve can be written in the form $(x - 1)^2 + (y - 3)^2 = 1$. Given that x is defined parametrically by $x = 1 + \cos \theta$, and that $y = 2$ when $\theta = \frac{\pi}{2}$, express y in terms of θ. (N88/P2/8b)

4. Prove the identity $\cos x \operatorname{cosec} x + \sin x \sec x \equiv \operatorname{cosec} x \sec x$. (J89/P1/6)

5. Find the cartesian equation of the curve which is defined parametrically by $x = 3 \sin^2 t$, $y = \cos t$. (J89/P2/8bii)

6. Prove the identity $(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin^3 x + \cos^3 x$. (N88/P1/6)

7. Given that $\sin A = \frac{1}{3}$, find, without using tables or a calculator, the values of (i) $\cos 2A$, (ii) $\cos 4A$. (N88/P2/6a)

8. Prove the identity $(\sec x - \tan x)(\operatorname{cosec} x + 1) \equiv \cot x$. (J90/P1/4)

9. The Cartesian equation of a curve can be written in the form $(y - 1)^2 - (x + 2)^2 = 1$. Given that x is defined parametrically by $x = \tan \theta - 2$, and that $y = 0$ when $\theta = 0$, express y in terms of θ. (J90/P2/8b)

10. Given that $\tan A = p$ and that A is acute, obtain an expression in terms of p, for (i) $\sec A$, (ii) $\operatorname{cosec} A$. (N90/P1/2)

11. Given that $\frac{\sin(A - B)}{\sin(A + B)} = \frac{5}{7}$, show that $\tan A = k \tan B$ and state the value of k. (N90/P2/4a)

12. The parametric equations of a curve are $x = \operatorname{cosec} \theta - \cot \theta$, $y = \operatorname{cosec} \theta - 2 \cot \theta$. Express $\operatorname{cosec} \theta$ and $\cot \theta$ in terms of x and y. Hence obtain the cartesian equation of the curve. (N90/P2/8c)

13. Prove the identity $\cot^2 \theta - \cos^2 \theta \equiv \cot^2 \theta \cos^2 \theta$. (J91/P1/3)

14. Given that $p = \cos A + \sin A$ and $q = \cos A - \sin A$, (i) show that $p^2 - q^2 = 2 \sin 2A$, (ii) find the numerical value of $p^2 + q^2$, (iii) express $\frac{p}{q}$ in terms of $\tan A$. (J91/P2/3a)

2. (ii) $\sin\alpha = \frac{4}{5}$, $\sin\beta = \frac{7}{\sqrt{50}}$

(iii) 1

3. $y = 3 - \sin\theta$

5. $x = 3(1 - y^2)$

7. (i) $\frac{7}{9}$ (ii) $\frac{17}{81}$

9. $y = 1 - \sec\theta$

10. (i) $\sqrt{1+p^2}$ (ii) $\frac{\sqrt{1+p^2}}{p}$

11. 6

12. $2x - y, x - y, 3x^2 - 2xy = 1$

14. (ii) 2 (iii) $\frac{1+\tan A}{1-\tan A}$