PART 8

PAST EXAMINATION QUESTIONS: TRIG IDENTITIES + EQNS

1. Find the cartesian equation of the curve defined parametrically by $x = 2\sec\theta$, $y = 3 + 2\tan\theta$. (J91/P2/8c)

2. Prove the identity $(\operatorname{cosec}\theta - \cot\theta)^2 \equiv \frac{1-\cos\theta}{1+\cos\theta}$. (N91/P1/5a)

3. Given that $\tan\alpha = p$ and $\tan(\alpha - \beta) = q$, (i) express $\tan\beta$ in terms of p and q, (ii) calculate the value of $\tan(\alpha + \beta)$ when $p = 1$ and $q = \frac{1}{2}$. (N91/P2/3a)

4. The parametric equations of a curve are $x = 2\cos\theta + \sin\theta$, $y = \cos\theta - 2\sin\theta$. Write down expressions for $2x + y$ and $x - 2y$ in terms of θ. Hence determine the cartesian equation of the curve in its simplest form. (N91/P2/8b)

5. Prove the identity $\operatorname{cosec} A - \frac{\sin A}{1+\cos A} \equiv \cot A$. (J92/P1/5)

6. (a) Given that $\tan A = t$, find an expression, in terms of t, for (i) $\cos A$, (ii) $\cos 2A$.
   (b) Give that, for all values of x, $5\cos(x + 45°) + P\sin(x + 45°) = R\cos x$, evaluate P and R. (J92/P2/3b, c)

7. (a) For the curve $(x + 1)^2 = 2y - y^2 + 3$, a parametric form of y is $y = 1 + 2\sin\theta$. Find a corresponding parametric expression for x, in the form $x = a + b\cos\theta$.
   (b) The parametric equations of a curve are $x = 3\cos\theta$, $y = 2 - \sin\theta$. Given that the curve cuts the y-axis at A and B, find the distance AB. (J92/P2/8b, c)

8. Prove the identity $\frac{\cot A - \tan A}{\cot A + \tan A} \equiv \cos^2 A - \sin^2 A$. (N92/P1/4)

9. Given that $\cot A = p$, find an expression, in terms of p, for (i) $\sin 2A$, (ii) $\sec 2A$. (N92/P2/4a)

10. Prove the identity $(\sin A - \cos A)(\tan A + \cot A) \equiv \sec A - \operatorname{cosec} A$. (J93/P1/5)

11. Find the cartesian equation of the curve defined by the parametric equations $x = 2 - \sin\theta$, $y = 3 + \cos\theta$. (J93/P2/8bii)

12. Prove the identity $(\cot A - \tan A)\sin A \equiv 2\cos A - \sec A$. (N93/P1/3)

13. Given that $\tan(45° + \alpha) = p$, express $\tan\alpha$ in terms of p. (J94/P2/4a)

1. $x^2 - y^2 + 6y - 13 = 0$

3. (i) $\frac{p-q}{1+pq}$ (ii) 2

4. $5\cos\theta$, $5\sin\theta$, $x^2 + y^2 = 5$

6. (a) (i) $\pm\frac{1}{\sqrt{1+t^2}}$ (ii) $\frac{1-t^2}{1+t^2}$

(b) 5, $5\sqrt{2}$ or 7·07

7. (a) $-1 \pm 2\cos\theta$ (b) 2

9. (i) $\frac{2p}{1+p^2}$ (ii) $\pm\frac{1+p^2}{1-p^2}$

11. $x^2 + y^2 - 4x - 6y + 12 = 0$

13. $\tan\alpha = \frac{p-1}{p+1}$