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REVIEW OF EXTENSION 1

ALL TOPICS

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REVIEW OF EXTENSION ONE MATHS

does not contain curve sketching & circle geometry

1) Differentiate:

a) $\sin^{-1}\frac{x}{4}$

b) $\cos^{-1} 2x$

c) $\tan^{-1} 4x^2$

d) $x^2 \sin^{-1} x$

2) Evaluate:

a) $\int \frac{5}{4+x^2}\,dx$

b) $\int \frac{dx}{\sqrt{1-4x^2}}$

c) $\int \frac{-1}{\sqrt{4-25x^2}}\,dx$

3) Write the exact value of:

a) $\cos\left(\cos^{-1}\frac{4}{5}\right)$

b) $\tan\left(\sin^{-1}\frac{3}{5}\right)$

4) Write the domain and range of:

a) $y = 3\sin^{-1}\frac{x}{2}$

b) $y = 4\cos^{-1} 2x$

c) $y = \tan^{-1} 3x$

5) Sketch each of the functions in Q4.

6) A is the point $(6,4)$ and B is the point $(2,1)$.
Divide:

a) AB internally in the ratio $4:5$

b) BA externally in the ratio $4:5$

7) Find the acute angle (nearest degree) between the lines $x+y=6$ and $y=3x$

8) Solve for x:

$$\frac{x-7}{x^2-9} \geq 1$$

9) Evaluate:

a) $\int \frac{x^2}{\sqrt{x^3-1}}\,dx$ using the substitution $u = x^3 - 1$

b) $\int_4^5 x\sqrt{x-4}\,dx$ using $u = x-4$

10) Evaluate:

$$\int_0^1 \sqrt{1-x^2}\,dx$$

using $x = \sin\theta$

11) Consider the word

R E V I S I O N

How many ways can the letters of the word be arranged:

a) in total?

b) if the 'R' and 'V' must be next to each other?

12) 12 people attend a dinner and are seated around a circular table. What is the probability that a particular couple will be seated next to each other?

13) To crack the code of a certain alarm, two stages must be solved. The probability that Stage 1 is solved is $\frac{1}{4}$ and Stage 2 is $\frac{3}{5}$.

CONT. next column ...

a) What is the probability that the code is cracked on the first try?

b) Suppose 10 different people (all of equal code cracking ability) attempt to crack the code. What is the probability that exactly 3 crack the code?

14) Prove $(x-3)$ is a factor of $x^3 - 3x^2 - 4x + 12$

15) Write $P(x)$ in terms of its linear factors if $P(x) = x^3 + 3x^2 - 4$

16) Sketch $P(x)$ from Q 15 without the use of calculus.

17) Divide $5x^4 - 3x^3 + 1$ by $x^3 - 2$ and hence find the remainder $R(x)$.

18) Write the inverse of:

a) $y = 4x^2 - 1 \quad (x > 0)$

b) $y = e^x + 2$

c) $y = 2 + \cos 4x \; (0 \leq x \leq \pi)$

19) Use the principle of Mathematical Induction to prove that for all positive integer values of n:

a) $1^3 + 2^3 + 3^3 + \dots + n^3$

$$= \frac{n^2}{4}(n+1)^2$$

b) $5^n - 1$ is divisable by 4.

20) Use both the 'Auxillary Angle' Method and 't' Method to solve for $0 \leq \theta \leq 2\pi$:

$$\sin\theta + \cos\theta = -1$$

21) Evaluate:

$$\int_0^{\frac{\pi}{2}} 3\sin^2 x \, dx$$

22) Show that $\sin 3\theta$ equals $3\sin\theta - 4\sin^3\theta$

23) Write the exact value of:

a) $\sin 75°$

b) $\tan 15°$

c) $\tan 112\frac{1}{2}°$

24) Given $P(x) = x^3 - 2x^2 + 9x - 11$ show a root exists between $x = 1$ and $x = 2$.

a) Use the 'halving the interval' method to find the root correct to 1 dp.

b) Using $x = 1.5$ as a first approximation, use Newton's method to find a better approximation correct to 1 dp. (1 application)

25) Find the term independent of x in the expansion of

a) $\left(x^2 - \frac{4}{x}\right)^{15}$

b) $\left(x^3 - \frac{3}{2x}\right)^8$

26) Consider the expansion of $(2x+3)^{11}$.

a) Write the general term of the expansion.

b) Show $\frac{T_{k+1}}{T_k} = \frac{36-3k}{2kx}$

c) Hence find the greatest coefficient in the expansion of $(2x-3)^{11}$.

27) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are variable points on the parabola $x^2 = 4ay$.

a) Write the equation of the locus of the midpoint of PQ if PQ is a focal chord

b) If the tangents at P and Q meet at T, and PQ passes through the point $(0, 6a)$, find the locus of T.

c) If the chord PQ subtends a right angle at the origin O,

i) show $pq = -4$

ii) find the eqn of the locus of the midpoint of PQ.

28) If α, β, γ are the roots of $2x^3 - 5x^2 + x - 1 = 0$, find:

a) $(\alpha + \beta + \gamma)^2$

b) $(\alpha + 1)(\beta + 1)(\gamma + 1)$

c) $1/\alpha + 1/\beta + 1/\gamma$

d) $\alpha^2 + \beta^2 + \gamma^2$

29) Two of the roots of $x^3 + ax^2 + bx + 8 = 0$ are 4 and -2. Find the values of a and b.

30) If $x^3 - 5x^2 + 7x + k = 0$ has a double integral root, find k.

31) A particle moves in SHM according to $x = 2\sin(4t - \pi/4)$

Write down its:

a) period b) amplitude

c) centre of motion

d) initial phase

32) A particle moves according to:

$x = 2\sin 3t + 3\cos 3t$

a) Show that the particle is moving in SHM.

b) Find its period, amplitude, centre of motion

33) A particle, initially at rest 2 metres to the right of O, moves according to the rule $\ddot{x} = -4x$.

Find where it next rests and determine its maximum speed.

34) A particle moving according to $\ddot{x} = -4x$ is initially stationary at $x=6$. Find its speed and acceleration at $x = 3$.

35) A particle's motion satisfies the equation $v^2 = 10 + 3x - x^2$.

a) Show that the motion is simple harmonic.

b) Find the centre, period and amplitude of the motion.

36) A particle's motion is given by $v = \frac{1}{2}(x-1)$ ms^{-1}. If the particle is initially 2 metres to the right of O, find the exact position of the particle at 2 secs.

37) The acceleration of a particle is given by $\frac{d^2x}{dt^2} = 6x^2 - 4x - 3$. Find the exact velocity of the particle when it is 2cm to the right of O, if initially the particle is at the origin with velocity 3cms^{-1}.

38) Evaluate:

$$\int_{3/5}^{4} \frac{dx}{1+x^2}$$

39) The rate of change of volume of water in a dam is given by:

$$\frac{dv}{dt} = k(v - 5000)$$

a) Show that a solution of this differential equation is $v = 5000 + Ae^{kt}$

b) If the initial volume is 87000 KL and at 10 hours the volume is 129 000 KL, find the values of A and k.

40) A piece of metal is heated to 80°C and placed in a room where the temperature is a constant 18°C. Assuming the metal cools according to Newton's law of cooling and cools to 68°C at 15 mins, find when the metal cools to 30°C?

41) The surface area of a spherical bubble is increasing at a constant rate of $1\cdot 49\ mm^2/s$. Find the rate of increase in its volume when its radius is $0\cdot 6$ mm.

42) An observer sees a plane x km away flying at a speed of 500km/h and at a height of 2000m. At what rate is the angle of elevation from the observer to the plane increasing when the plane is 2km away?

43) A projectile is launched at an angle of θ to the horizontal at an initial velocity of V m/s.

a) Derive the horizontal and vertical components of motion.

b) Derive the cartesian equation of motion.

c) Find the time of flight for the projectile to hit the ground.

d) Find the range of the projectile.

e) Calculate the maximum height of the projectile.

44) A particle is projected at an initial velocity of $40ms^{-1}$. What is its maximum range if its angle of projection is 30°?

45) A particle is projected at $80ms^{-1}$ to strike an object 50m away and 2m above the ground. Find, nearest degree, the two angles of projection possible.

46) A particle is projected at an initial velocity of $100ms^{-1}$ and so that it just clears a wall 40metres away and 2 metres high. What is the furthest past the wall the particle can land?

Inverse functions

1999 - 5b)
2000 - 5b
2001 - 5a
2002 - 5c
2002 - 7a
2003 - 5b
2004 - 5b
1998 - 4.b

3d Trig

1999 - 6(b)
2000 - 3(c)
2003 - 7(a)
2004 - 3(d)

Newtons

2001 , 3a
2003 , 4b

Physics

1999 6a
2002 4c
1996 6
2001 7a
2003 6a
1996 5b
2003 5c - 1996 5a

1)a) Using table of standard integrals:

$\frac{d}{dx} = \frac{1}{\sqrt{16-x^2}}$

b) Let $y = \cos^{-1} 2x$ and $u = 2x$

$\therefore y = \cos^{-1} u$ $\therefore \frac{du}{dx} = 2$

$\therefore \frac{dy}{du} = \frac{-1}{\sqrt{1-u^2}}$

Now $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= \frac{-1}{\sqrt{1-u^2}} \cdot 2$

$= \frac{-2}{\sqrt{1-4x^2}}$

c) Let $y = \tan^{-1} 4x^2$ and $u = 4x^2$

$\therefore y = \tan^{-1} u$ $\therefore \frac{du}{dx} = 8x$

$\therefore \frac{dy}{du} = \frac{1}{1+u^2}$

Now $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= \frac{1}{1+u^2} \cdot 8x$

$= \frac{8x}{1+16x^4}$

d) Let $u = x^2$ and $v = \sin^{-1} x$

$\therefore \frac{du}{dx} = 2x$ and $\frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}}$

Now $\frac{d}{dx} = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$

$= x^2 \cdot \frac{1}{\sqrt{1-x^2}} + 2x \sin^{-1} x$

$= x\left(\frac{x}{\sqrt{1-x^2}} + 2\sin^{-1} x\right)$

2)a) $5\int \frac{1}{4+x^2}\, dx$

$= 5\left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right] + C$

$= \frac{5}{2}\tan^{-1}\frac{x}{2} + C$

b) $\int \frac{1}{\sqrt{1-4x^2}}\, dx$

$= \int \frac{1}{\sqrt{4(\frac{1}{4}-x^2)}}\, dx$

$= \frac{1}{2}\int \frac{1}{\sqrt{\frac{1}{4}-x^2}}\, dx$

$= \frac{1}{2}\sin^{-1}\frac{x}{\frac{1}{2}} + C$

$= \frac{1}{2}\sin^{-1} 2x + C$

2) c) $\int \frac{1}{\sqrt{4-25x^2}}\, dx$

$= \int \frac{-1}{\sqrt{25(\frac{4}{25}-x^2)}}\, dx$

$= -\frac{1}{5}\int \frac{1}{\sqrt{\frac{4}{25}-x^2}}\, dx$

$= -\frac{1}{5}\sin^{-1}\frac{x}{\frac{2}{5}} + C$

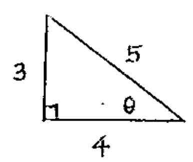
$= -\frac{1}{5}\sin^{-1}\frac{5x}{2} + C$

3)a) $\frac{4}{5}$

b) Let $\theta = \sin^{-1}\frac{3}{5}$

$\therefore \sin\theta = \frac{3}{5}$

Using Pythagoras Theorem and



, $\tan\theta = \frac{3}{4}$

$\therefore \tan(\sin^{-1}\frac{3}{5})$

$= \tan\theta$

$= \frac{3}{4}$

4)a) For $y = \sin^{-1} x$,

D: $-1 \le x \le 1$

R: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$\therefore$ for $y = 3\sin^{-1}\frac{x}{2}$,

ie. $\frac{y}{3} = \sin^{-1}\frac{x}{2}$

D: $-1 \le \frac{x}{2} \le 1$

$\therefore$ D: $-2 \le x \le 2$

R: $-\frac{\pi}{2} \le \frac{y}{3} \le \frac{\pi}{2}$

$\therefore$ R: $-\frac{3\pi}{2} \le y \le \frac{3\pi}{2}$

b) For $y = \cos^{-1} x$,

D: $-1 \le x \le 1$

R: $0 \le y \le \pi$

$\therefore$ for $y = 4\cos^{-1} 2x$,

ie. $\frac{y}{4} = \cos^{-1} 2x$

D: $-1 \le 2x \le 1$

$\therefore$ D: $-\frac{1}{2} \le x \le \frac{1}{2}$

R: $0 \le \frac{y}{4} \le \pi$

$\therefore$ R: $0 \le y \le 4\pi$

c) For $y = \tan^{-1} x$,

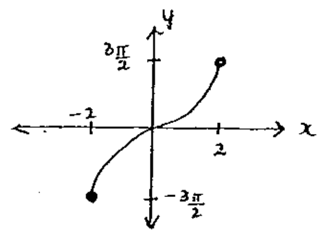
D: all real x

R: $-\frac{\pi}{2} < y < \frac{\pi}{2}$

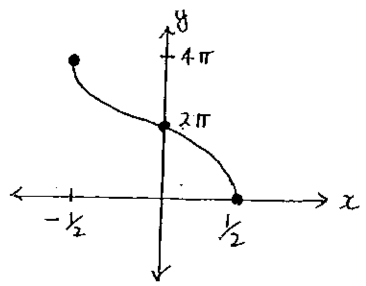
$\therefore$ for $y = \tan^{-1} 3x$,

D and R same as above

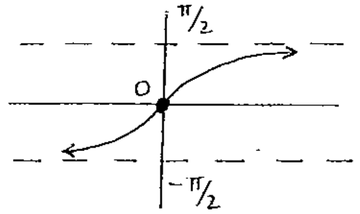
5) a)



b)



5)c)



) Using: $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

a) $(6,4)\quad(2,1)\qquad 4:5$

$(x_1,y_1)\quad(x_2,y_2)\qquad m:n$

$\left(\frac{4(2)+5(6)}{9}, \frac{4(1)+5(4)}{9}\right)$

$= \left(\frac{38}{9}, \frac{8}{3}\right)$

b) $(2,1)\quad(6,4)\qquad 4:-5$

$(x_1,y_1)\quad(x_2,y_2)\qquad m:n$

$\left(\frac{4(6)-5(2)}{4-5}, \frac{4(4)-5(1)}{4-5}\right)$

$= (-14,-11)$

) m_1 of $x+y=6$ is -1

m_2 of $y=3x$ is 3

Now $\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$= \left|\frac{-1-3}{1+(-1)(3)}\right|$

$= \left|\frac{-4}{-2}\right|$

$= 2$

$\therefore \theta = 63^\circ$

(nearest degree)

8) $\frac{x-7}{(x-3)(x+3)} \geq 1$

$\therefore x \neq \pm 3$

Let $\frac{x-7}{x^2-9} = 1$

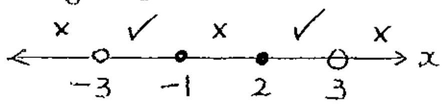
$\therefore x-7 = x^2-9$

$\therefore x^2-x-2=0$

$(x-2)(x+1)=0$

$\therefore x = 2, -1$

Testing regions:



$\therefore -3 < x \leq -1$ and $2 \leq x < 3$

9)a) $\int \frac{x^2}{\sqrt{x^3-1}}\,dx$

$u = x^3-1$

$\frac{du}{dx} = 3x^2$

$\therefore dx = \frac{du}{3x^2}$

$= \int \frac{x^2}{\sqrt{u}} \cdot \frac{du}{3x^2}$

$= \frac{1}{3}\int u^{-1/2}\,du$

$= \frac{1}{3}\left[2u^{\frac{1}{2}}\right] + C$

$= \frac{2}{3}\sqrt{x^3-1} + C$

9)b) $\int_4^5 x\sqrt{x-4}\,dx$

$u = x-4$

$\frac{du}{dx} = 1$

when $x=5$, $u=1$

when $x=4$, $u=0$

Also: $x = u+4$

$= \int_0^1 (u+4)u^{1/2}\,du$

$= \int_0^1 u^{3/2} + 4u^{1/2}\,du$

$= \left[\frac{2u^{5/2}}{5} + \frac{8u^{3/2}}{3}\right]_0^1$

$= \frac{2}{5} + \frac{8}{3}$

$= \frac{46}{15}$

10) $\int_0^1 \sqrt{1-x^2}\,dx$

$x = \sin\theta$

$\frac{dx}{d\theta} = \cos\theta$

$\therefore dx = \cos\theta\,d\theta$

when $x=1$, $\theta = \sin^{-1}1$, $\therefore \theta = \frac{\pi}{2}$

when $x=0$, $\theta = 0$

$= \int_0^{\frac{\pi}{2}} \sqrt{1-\sin^2\theta}\,.\cos\theta\,d\theta$

$= \int_0^{\frac{\pi}{2}} \sqrt{\cos^2\theta}\,.\cos\theta\,d\theta$

$= \int_0^{\frac{\pi}{2}} \cos^2\theta\,d\theta$

$= \frac{1}{2}\int_0^{\frac{\pi}{2}} 1 + \cos 2\theta\,d\theta$

since $\cos 2\theta = 2\cos^2\theta - 1$

$= \frac{1}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{2}}$

$= \frac{1}{2}\left[\frac{\pi}{2} + 0\right]$

$= \frac{\pi}{4}$

11)a) Total no. of ways $= \frac{8!}{2!}$

$= 20160$

b) Treat R and V as 1 letter

$\therefore$ No. of ways $= \frac{7!}{2!}$

$= 2520$

But R and V can be in the reverse order

$\therefore$ Total no. of ways $= 2520 \times 2$

$= 5040$

12) No. of ways of arranging 12 people $= 11!$

Treat couple as one $\therefore$ 10! ways

But couple could sit in reverse order ie. $2 \times 10!$ ways

$\therefore$ Prob(next to each other) $= \frac{2 \times 10!}{11!}$

$= \frac{2}{11}$

13)a) $P(\text{1st time}) = \frac{1}{4} \times \frac{3}{5}$

$= \frac{3}{20}$

b) $P(\text{exactly } 3) = {}^{10}C_3 \left(\frac{3}{20}\right)^3\left(\frac{17}{20}\right)^7$

14) Let $P(x) = x^3 - 3x^2 - 4x + 12$

Now $P(3) = 0$

$\therefore x - 3$ is a factor of $P(x)$

15) $P(x) = x^3 + 3x^2 - 4$

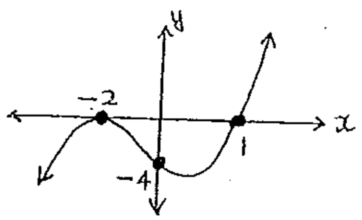
$P(1) = 0$ $\therefore x - 1$ is a factor of $P(x)$

	1	3	0	-4
1		1	4	4
	1	4	4	0

$\therefore P(x) = (x-1)(x^2 + 4x + 4)$

$= (x-1)(x+2)^2$

16)



17)
$$\begin{array}{r} 5x - 3 \\ x^3 - 2 \overline{)\, 5x^4 - 3x^3 \qquad + 1} \\ 5x^4 \qquad\quad - 10x \quad \\ \hline -3x^3 + 10x + 1 \\ -3x^3 \qquad + 6 \\ \hline 10x - 5 \end{array}$$

$\therefore R(x) = 10x - 5$

18)a) I: $x = 4y^2 - 1$

$\therefore 4y^2 = x + 1$

$\therefore y^2 = \frac{x+1}{4}$

$\therefore y^{-1} = \pm\sqrt{\frac{x+1}{4}}$

Now Domain of y is $x > 0$

$\therefore$ Range of y^{-1} is $y > 0$

$\therefore y^{-1} = \sqrt{\frac{x+1}{4}}$

b) I: $x = e^y + 2$

$\therefore e^y = x - 2$

$\therefore y^{-1} = \ln(x-2)$ $[x > 2]$

c) I: $x = 2 + \cos 4y$

$\therefore x - 2 = \cos 4y$

$\therefore 4y = \cos^{-1}(x-2)$

$\therefore y^{-1} = \frac{1}{4}\cos^{-1}(x-2)$ $[1 \le x \le 3]$

$-1 \le x - 2 \le 1$

$1 \le x \le 3$

$0 \le 4y \le \pi$

$0 \le y \le \pi/4$

19)a) Let $n = 1$

ie. $1^3 = \frac{1^2}{4}(1+1)^2$

$\therefore 1 = 1$

$\therefore$ true for $n = 1$

Assume true for $n = k$

ie. $1^3 + 2^3 + \dots + k^3 = \frac{k^2}{4}(k+1)^2$

Let $n = k+1$

$\therefore 1^3 + 2^3 + \dots + k^3 + (k+1)^3$

$= \frac{k^2}{4}(k+1)^2 + (k+1)^3$

$\therefore$ RHS $= \frac{k^2(k+1)^2 + 4(k+1)^3}{4}$

$= \frac{1}{4}(k+1)^2(k^2 + 4(k+1))$

$= \frac{1}{4}(k+1)^2(k^2 + 4k + 4)$

$= \frac{(k+1)^2(k+2)^2}{4}$

$\therefore$ true for $n = k+1$

If true for $n=k$ then true for $n=k+1$. Since true for $n=1$, then true for $n=2$, and hence true for all positive values of n

19)b) Let $n = 1$

ie. $5^1 - 1 = 4$, which is divisible by 4 $\therefore$ true for $n = 1$

Assume true for $n = k$

ie. $5^k - 1 = 4M$ (where M is a positive integer)

Let $n = k+1$

ie. $5^{k+1} - 1 = 5.5^k - 1$

$= 5.5^k - 5 + 4$

$= 5(5^k - 1) + 4$

$= 5(4M) + 4$

$= 20M + 4$

$= 4(5M + 1)$

$\therefore$ divisable by 4.

If true for $n=k$, then true for $n=k+1$. Since true for $n=1$, then true for $n=2$, and hence true for all positive values of n.

20) Auxillary Angle Method:

Let $\sin\theta + \cos\theta = R\sin(\theta + \alpha)$

$\therefore$ RHS $= R\sin\theta\cos\alpha + R\sin\alpha\cos\theta$

Equating like terms:

$R\cos\alpha = 1$ (1)

$R\sin\alpha = 1$ (2)

Squaring (1) and (2):

$R^2\cos^2\alpha = 1$ (3)

$R^2\sin^2\alpha = 1$ (4)

(3) + (4):

$R^2\sin^2\alpha + R^2\cos^2\alpha = 2$

$\therefore R^2(\sin^2\alpha + \cos^2\alpha) = 2$

$\therefore R^2 = 2$

$\therefore R = \pm\sqrt{2}$

$\therefore R = \sqrt{2}$ (since $R > 0$)

$\therefore \sin\alpha = \frac{1}{\sqrt{2}}$ and $\cos\alpha = \frac{1}{\sqrt{2}}$

$\therefore \alpha = \frac{\pi}{4}$

$\therefore \sin\theta + \cos\theta = \sqrt{2}\sin(\theta + \pi/4)$

If $\sin\theta + \cos\theta = -1$, then

$\sqrt{2}\sin(\theta + \pi/4) = -1$

$\therefore \sin(\theta + \pi/4) = -1/\sqrt{2}$

$\therefore \theta + \pi/4 = \frac{5\pi}{4}, \frac{7\pi}{4}$ for $0 \le \theta + \pi/4 \le \frac{9\pi}{4}$

$\therefore \theta = \pi, \frac{3\pi}{2}$ for $0 \le \theta \le 2\pi$

't' method:

If $\tan\frac{\theta}{2} = t$, $\sin\theta = \frac{2t}{1+t^2}$

and $\cos\theta = \frac{1-t^2}{1+t^2}$

$\therefore \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = -1$

$\therefore 2t + 1 - t^2 = -1 - t^2$

$\therefore 2t = -2$

$\therefore t = -1$

$\therefore \tan\frac{\theta}{2} = -1$

$\therefore \frac{\theta}{2} = 3\pi/4$ for $0 \le \frac{\theta}{2} \le \pi$

$\therefore \theta = 3\pi/2$ for $0 \le \theta \le 2\pi$

Cont. next page...

Test $\theta = \pi$

$\sin\pi + \cos\pi = -1$

$\therefore$ Solns are: $\pi,\ 3\pi/2$

21) $\int_0^{\frac{\pi}{2}} 3\sin^2 x \, dx$

$= 3\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$

$= 3\int_0^{\frac{\pi}{2}} \frac{1}{2}(1-\cos 2x)\, dx$

$= \frac{3}{2}\int_0^{\pi/2} 1-\cos 2x \, dx$

$= \frac{3}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\pi/2}$

$= \frac{3}{2}\left[\left(\frac{\pi}{2}\right) - 0\right]$

$= 3\pi/4$

22) $\sin 3\theta = \sin(2\theta + \theta)$

$= \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$

$= 2\sin\theta\cos\theta\cos\theta + \sin\theta(1-2\sin^2\theta)$

$= 2\sin\theta\cos^2\theta + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$

23) $\sin 75 = \sin(45+30)$

$= \sin 45\cos 30 + \sin 30\cos 45$

$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\cdot\frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

b) $\tan 15 = \tan(45-30)$

$= \frac{\tan 45 - \tan 30}{1 + \tan 45\tan 30}$

$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\left(\frac{1}{\sqrt{3}}\right)}$

$= \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$

$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$= \frac{3 - 2\sqrt{3} + 1}{2}$

$= \frac{4 - 2\sqrt{3}}{2}$

$= 2 - \sqrt{3}$

23) c) let $t = \tan\frac{\theta}{2}$ and $\theta = 225°$

Now $\tan\theta = \frac{2t}{1-t^2}$

$\therefore \tan 225 = \frac{2t}{1-t^2}$

$\therefore 1 = \frac{2t}{1-t^2}$

$\therefore 1 - t^2 = 2t$

$\therefore t^2 + 2t - 1 = 0$

$\therefore t = \frac{-2 \pm \sqrt{8}}{2}$

$\therefore t = \frac{-2 \pm 2\sqrt{2}}{2}$

$\therefore t = -1 \pm \sqrt{2}$

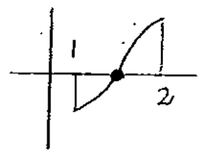
But $\tan 112\frac{1}{2}° < 0$

$\therefore t = -1 - \sqrt{2}$

$\therefore \tan 112\frac{1}{2} = -1 - \sqrt{2}$

24) $P(1) < 0$ and $P(2) > 0$

$\therefore$ a root of $P(x)$ exists between $x = 1$ and $x = 2$



$P(1.5) < 0 \ \therefore$ root lies $1.5 < x < 2$

$P(1.8) < 0 \ \therefore$ root lies $1.8 < x < 2$

$P(1.9) > 0 \ \therefore$ root lies $1.8 < x < 1.9$

$P(1.85) > 0 \ \therefore$ root is 1.8 (1dp)

b) $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_2 = 1.5 - \frac{-3.625}{9.75}$

$= 1.9$ (1dp)

25) a)

$T_{k+1} = {}^{15}C_k (x^2)^{15-k}\left(-\frac{4}{x}\right)^k$

$= {}^{15}C_k (x^2)^{15-k}(-4)^k(x^{-1})^k$

$= {}^{15}C_k\, x^{30-2k}(-4)^k x^{-k}$

$= {}^{15}C_k\, x^{30-3k}(-4)^k$

Term indep. of x when $30 - 3k = 0$

$\therefore k = 10$

$\therefore$ Term indep. of x:

$T_{11} = {}^{15}C_{10}(-4)^{10}$

b) $T_{k+1} = {}^{8}C_k (x^3)^{8-k}\left(-\frac{3}{2x}\right)^k$

$= {}^{8}C_k (x^3)^{8-k}(-3)^k(2^{-1})^k(x^{-1})^k$

$= {}^{8}C_k\, x^{24-3k}(-3)^k(2^{-k})x^{-k}$

$= {}^{8}C_k\, x^{24-4k}(-3)^k(-2)^{-k}$

Term indep. of x when $24 - 4k = 0$

$\therefore k = 6$

$\therefore$ Term indep. of x:

$T_7 = {}^{8}C_6(-3)^6(-2)^{-6}$

$= \frac{5103}{16}$

General term of $(a+b)^n$ is: $T_{k+1} = {}^{n}C_k\, a^{n-k} b^k$

26) a) General term:

$$T_{k+1} = {}^{11}C_k (2x)^{11-k} (3)^k$$
$$= {}^{11}C_k\, 2^{11-k} x^{11-k} (3)^k$$

b) $T_k = {}^{11}C_{k-1}\, 2^{12-k} x^{12-k} (3)^{k-1}$

Now: ${}^{11}C_k \div {}^{11}C_{k-1}$

$$= \frac{11!}{k!(11-k)!} \times \frac{(k-1)!(12-k)!}{11!}$$
$$= \frac{12-k}{k}$$

and $2^{11-k} x^{11-k} 3^k \div 2^{12-k} x^{12-k} 3^{k-1}$

$$= 2^{-1} x^{-1} 3$$
$$= \frac{3}{2x}$$

$$\therefore \frac{T_{k+1}}{T_k} = \frac{12-k}{k} \cdot \frac{3}{2x}$$
$$= \frac{36-3k}{2kx}$$

c) Greatest coeff when $\frac{T_{k+1}}{T_k} > 1$

$\therefore \frac{36-3k}{2k} > 1 \quad \therefore k < 7\frac{1}{5}$

∴ Greatest coeff. in expansion of $(2x+3)^{11}$ when $k = 7$

ut in expansion of $(2x-3)^{11}$,

$_8$ (ie. when $k=7$) < 0

st T_7 and T_9

$\therefore T_7 = 10\,777\,536$ and

$T_9 = 8\,660\,520$

Greatest coeff is $10\,777536$

27) a)

Mpt of $PQ = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$= \left(\frac{2ap+2aq}{2}, \frac{ap^2+aq^2}{2}\right)$$
$$= \left(a(p+q), \frac{a(p^2+q^2)}{2}\right)$$

$\therefore x = a(p+q)$ ie. $p+q = \frac{x}{a}$

and $y = \frac{a(p^2+q^2)}{2}$

$$\therefore \frac{2y}{a} = p^2+q^2$$
$$= (p+q)^2 - 2pq$$

Now, given PQ is a focal chord,

$pq = -1$

$\therefore \frac{2y}{a} = \left(\frac{x}{a}\right)^2 - 2(-1)$

$\therefore 2ay = x^2 + 2a^2$

$\therefore x^2 = 2ay - 2a^2$

$\therefore x^2 = 2a(y-a)$

b) $x^2 = 4ay$

$\therefore y = \frac{x^2}{4a}$

$\therefore y' = \frac{x}{2a}$

$\therefore y'(2ap) = p$

∴ m of T at $P = p$

∴ m of T at $Q = q$

<u>Eqn of T at P:</u>

$y - ap^2 = p(x - 2ap)$

$\therefore y - ap^2 = px - 2ap^2$

$\therefore y = px - ap^2$

∴ <u>Eqn of T at Q</u>: $y = qx - aq^2$ q.

<u>Coords of T:</u>

$px - ap^2 = qx - aq^2$

$\therefore px - qx = ap^2 - aq^2$

$\therefore (p-q)x = a(p-q)(p+q)$

$\therefore x = a(p+q)$

$\therefore y = ap(p+q) - ap^2$

$\therefore y = ap^2 + apq - ap^2$

$\therefore y = apq$

∴ Coords of T are $(a(p+q), apq)$

<u>Now $x = a(p+q)$</u>

<u>and $y = apq$</u>

<u>Also, m of PQ</u> $= \frac{ap^2 - aq^2}{2ap - 2aq}$

$$= \frac{a(p-q)(p+q)}{2a(p-q)}$$
$$= \frac{p+q}{2}$$

<u>and eqn of PQ:</u>

$y - ap^2 = \frac{p+q}{2}(x - 2ap)$

$\therefore 2y - 2ap^2 = px + qx - 2ap^2 - 2apq$

$\therefore 2y = px + qx - 2apq$

<u>If PQ passes thru $(0, 6a)$</u>

$12a = -2apq$

$\therefore pq = -6$

Since T is $(a(p+q), apq)$

then locus of T is: $y = -6a$

c) i) m of OP $= \frac{ap^2}{2ap}$

$$= \frac{p}{2}$$

and m of OQ $= \frac{q}{2}$

Since $OP \perp OQ$, $\frac{p}{2} \cdot \frac{q}{2} = -1$

$\therefore pq = -4$

ii) mpt $PQ = \left(a(p+q), \frac{a(p^2+q^2)}{2}\right)$

$\therefore x = a(p+q) \quad \therefore p+q = \frac{x}{a}$

and $y = \frac{a(p^2+q^2)}{2}$

$$\therefore \frac{2y}{a} = p^2+q^2$$
$$= (p+q)^2 - 2pq$$
$$= \left(\frac{x}{a}\right)^2 - 2(-4)$$

$\therefore \frac{2y}{a} = \frac{x^2}{a^2} + 8$

$\therefore 2ay = x^2 + 8a^2$

$\therefore x^2 = 2ay - 8a^2$

$\therefore x^2 = 2a(y - 4a)$

28) a)

$$(\alpha+\beta+\gamma)^2 = (-b/a)^2$$
$$= \left(\frac{5}{2}\right)^2$$
$$= \frac{25}{4}$$

$= \alpha\beta\gamma + \alpha\beta + \alpha\gamma + \alpha + \beta\gamma + \beta + \gamma + 1$

$= \alpha\beta\gamma + \alpha\beta + \alpha\gamma + \beta\gamma + \alpha + \beta + \gamma + 1$

$= -d/a + c/a - b/a + 1$

$= \frac{1}{2} + \frac{1}{2} + \frac{5}{2} + 1$

$= 4\frac{1}{2}$

c) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

$= \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$

$= \frac{c/a}{-d/a}$

$= \frac{1/2}{1/2}$

$= 1$

) $\alpha^2 + \beta^2 + \gamma^2$

$= (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

$= (-b/a)^2 - 2(c/a)$

$= (5/2)^2 - 2(\frac{1}{2})$

$= \frac{25}{4} - 1$

$= \frac{21}{4}$

Now $\alpha\beta\gamma = -d/a$

$\therefore -8\gamma = -8$

$\therefore \gamma = 1$

$\therefore$ 3 roots are $4, -2, 1$

ie. $(x-4)(x+2)(x-1)$

$= (x-4)(x^2+x-2)$

$= x^3 - 3x^2 - 6x + 8$

$\therefore a = -3$ and $b = -6$

30) Let $P(x) = x^3 - 5x^2 + 7x + k$

$\therefore P'(x) = 3x^2 - 10x + 7$

$= (3x-7)(x-1)$

If α is a double root of $P(x)$, then it is also a root of $P'(x)$. Because coeff. of x^3 in $P(x)$ is 1, α must be 1.

If $x = 1$ is a root of $P(x)$ then $P(1) = 0$

ie. $1 - 5 + 7 + k = 0$

$\therefore k = -3$

31) a) $P = \frac{2\pi}{4} = \pi/2$

b) amp $= 2$

c) C.O.M.: $x = 0$

d) I.P. $= \pi/4$

32) $x = 2\sin 3t + 3\cos 3t$

$\therefore v = 6\cos 3t - 9\sin 3t$

$\therefore \ddot{x} = -18\sin 3t - 27\cos 3t$

$= -9(2\sin 3t + 3\cos 3t)$

$\therefore \ddot{x} = -9x$

$\therefore$ part. is moving in SHM as acc. in form $\ddot{x} = -n^2x$

b) $P = 2\pi/3$

amp $= \sqrt{2^2 + 3^2}$

$= \sqrt{13}$

C.O.M. $x = 0$

33) $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$\therefore -4x = d/dx\left(\frac{1}{2}v^2\right)$

$\therefore \frac{1}{2}v^2 = \int -4x\, dx$

$= -2x^2 + C$

<u>when $v = 0$, $x = 2$</u>

$\therefore 0 = -8 + C$

$\therefore C = 8$

$\therefore \frac{1}{2}v^2 = -2x^2 + 8$

$\therefore v^2 = -4x^2 + 16$

Now, since particle's motion is SHM, particle must initially be at an extremity. Since C.O.M is at $x = 0$, next at rest at $x = -2$.

<u>Max. speed at C.O.M.</u>

$\therefore$ speed $= -4x^2 + 16$

<u>when $x = 0$</u>

$\therefore$ speed $= 16$

34) $\ddot{x} = -4x$

$\therefore \frac{1}{2}v^2 = -2x^2 + C$ (from Q33)

when $x = 6, v = 0$

$\therefore 0 = -72 + C$

$\therefore C = 72$

$\therefore \frac{1}{2}v^2 = -2x^2 + 72$

$\therefore v^2 = -4x^2 + 144$

<u>when $x = 3$</u>

speed $= \sqrt{-36 + 144}$

$= 6\sqrt{3}$

acc $= -12$

35) $v^2 = 10 + 3x - x^2$

a) $\frac{1}{2}v^2 = 5 + \frac{3}{2}x - \frac{x^2}{2}$

Now $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$\therefore \ddot{x} = \frac{d}{dx}\left(5 + \frac{3}{2}x - \frac{x^2}{2}\right)$

$= \frac{3}{2} - x$

$= -1(x - 3/2)$

which is in the form

$\ddot{x} = -n^2(x-k)$

$\therefore$ SHM

b) C.O.M: $x = 3/2$

Period $= 2\pi$

let $v = 0$ $\therefore x^2 - 3x - 10 = 0$

$\therefore (x-5)(x+2) = 0$

$\therefore x = 5, -2$

since a particle moving in SHM is at rest at its extremities, the extremities of this particle must be $x = 5$ and $x = -2$. Since C.O.M. is $x = 3/2$, amplitude is $3\frac{1}{2}$.

36) $v = \frac{1}{2}(x-1)$

$\therefore \frac{dx}{dt} = \frac{x-1}{2}$

$\therefore \frac{dt}{dx} = \frac{2}{x-1}$

$\therefore t = \int \frac{2}{x-1}\,dx$

$= 2\int \frac{1}{x-1}\,dx$

$\therefore t = 2\ln(x-1) + C$

when $t=0$, $x=2$ $\therefore C = 0$

$\therefore t = 2\ln(x-1)$

when $t = 2$

$2 = 2\ln(x-1)$

$\therefore \ln(x-1) = 1$

$\therefore x - 1 = e$

$\therefore x = e + 1$ metres to right of O

37) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \ddot{x}$

$= 6x^2 - 4x - 3$

$\therefore \frac{1}{2}v^2 = 2x^3 - 2x^2 - 3x + C$

when $x = 0$, $v = 3$

$\therefore \frac{9}{2} = C$

$\therefore \frac{1}{2}v^2 = 2x^3 - 2x^2 - 3x + 9/2$

$\therefore v^2 = 4x^3 - 4x^2 - 6x + 9$

when $x = 2$

$v^2 = 13$

$\therefore v = \pm\sqrt{13}$

38) $\int_{3/5}^{4} \frac{dx}{1+x^2}$

$\therefore I = \left[\tan^{-1}x\right]_{3/5}^{4}$

$\therefore I = \tan^{-1}4 - \tan^{-1}3/5$

let $A = \tan^{-1}4$ $\therefore \tan A = 4$

and $B = \tan^{-1}3/5$ $\therefore \tan B = 3/5$

Now $\tan(\tan^{-1}4 - \tan^{-1}3/5)$

$= \tan(A - B)$

$= \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

$= \dfrac{4 - 3/5}{1 + 4(3/5)}$

$= 1$

$\therefore \tan I = \tan(\tan^{-1}4 + \tan^{-1}3/5)$

$\therefore \tan I = 1$

$\therefore I = \pi/4$

39) a) $V = 5000 + Ae^{kt}$

$\therefore \frac{dV}{dt} = kAe^{kt}$

But $Ae^{kt} = V - 5000$ (from line 1)

$\therefore \frac{dV}{dt} = k(V - 5000)$

b) when $t = 0$, $V = 87\,000$

$\therefore 87000 = 5000 + Ae^0$

$\therefore A = 82\,000$

$\therefore V = 5000 + 82000e^{kt}$

when $t = 10$, $V = 129\,000$

$129000 = 5000 + 82\,000e^{10k}$

$\therefore e^{10k} = 124/82$

$= 62/41$

$\therefore 10k = \ln(62/41)$

$\therefore k = \dfrac{\ln(62/41)}{10}$

40) $T = M + Ae^{-kt}$

when $t = 0$, $M = 18$, $T = 80$

$\therefore 80 = 18 + A$

$\therefore A = 62$

when $t = 15$, $T = 68$

$\therefore 68 = 18 + 62e^{-15k}$

$\therefore 50 = 62e^{-15k}$

$\therefore 25/31 = e^{-15k}$

$\therefore k = \dfrac{\ln(25/31)}{-15}$

when $T = 30$

$30 = 18 + 62e^{-kt}$

$\therefore 12 = 62e^{-kt}$

$\therefore t = \dfrac{\ln(6/31)}{-k}$

$= 115$ min (nearest min)

41) $\frac{dA}{dt} = 1.99$ mm²/sec

$V = \frac{4}{3}\pi r^3$ $\qquad A = 4\pi r^2$

$\frac{dV}{dr} = 4\pi r^2$ $\qquad \therefore \frac{dA}{dr} = 8\pi r$

$\frac{dr}{dt} = \frac{dr}{dA} \times \frac{dA}{dt}$

$= \frac{1}{8\pi r} \times 1.99$

$= \frac{1.99}{8\pi r}$

$\therefore \frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

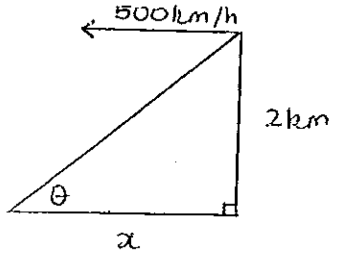
$= 4\pi r^2 \times \frac{1.99}{8\pi r}$

$= \frac{1.99r}{2}$

when $r = 0.6$

$\therefore \frac{dV}{dt} = 0.597$ mm³/s

2)



Now $\frac{dx}{dt} = -500$

and $\tan\theta = \frac{2}{x}$

$\therefore \theta = \tan^{-1}(\frac{2}{x})$

Let $u = \frac{2}{x}$ $\quad \therefore \theta = \tan^{-1} u$

$\frac{du}{dx} = -\frac{2}{x^2}$

$\therefore \frac{d\theta}{du} = \frac{1}{1+u^2}$

$= \frac{1}{1+\frac{4}{x^2}}$

$= \frac{1}{\frac{x^2+4}{x^2}}$

$= \frac{x^2}{x^2+4}$

$\therefore \frac{d\theta}{dx} = \frac{d\theta}{du} \times \frac{du}{dx}$

$= \frac{x^2}{x^2+4} \times -\frac{2}{x^2}$

$= \frac{-2}{x^2+4}$

$\therefore \frac{d\theta}{dt} = \frac{d\theta}{dx} \times \frac{dx}{dt}$

$= \frac{-2}{x^2+4} \times -500$

$= \frac{1000}{x^2+4}$

when $x = 2$

$\therefore \frac{d\theta}{dt} = 125$ radians/hour

43) a) $\ddot{x} = 0$

$\therefore \dot{x} = A$ (where A is a constant)

when $t = 0$, $\dot{x} = V\cos\theta$

$\therefore A = V\cos\theta$

$\therefore \dot{x} = V\cos\theta$

$\therefore x = Vt\cos\theta + B$ (where B is a constant)

when $t = 0$, $x = 0$

$\therefore x = Vt\cos\theta$

Now $\ddot{y} = -g$

$\therefore \dot{y} = -gt + C$ (where C is a constant)

when $t = 0$, $\dot{y} = V\sin\theta$

$\therefore C = V\sin\theta$

$\therefore \dot{y} = -gt + V\sin\theta$

$\therefore y = -\frac{gt^2}{2} + Vt\sin\theta + D$ (where D is a constant)

when $t = 0$, $y = 0$ $\quad \therefore D = 0$

$\therefore y = -\frac{gt^2}{2} + Vt\sin\theta$

b) If $x = Vt\cos\theta$, then $t = \frac{x}{V\cos\theta}$

$\therefore y = -\frac{g}{2}\left(\frac{x}{V\cos\theta}\right)^2 + V\left(\frac{x}{V\cos\theta}\right)\sin\theta$

$= -\frac{gx^2}{2V^2}\sec^2\theta + x\tan\theta$

c) let $y = 0$

$\therefore -\frac{gt^2}{2} + Vt\sin\theta = 0$

$\therefore -t\left(\frac{gt}{2} - V\sin\theta\right) = 0$

$\therefore t = 0, \frac{2V\sin\theta}{g}$

$\therefore$ Time of flight $= \frac{2V\sin\theta}{g}$

d) when $t = \frac{2V\sin\theta}{g}$

$x = V\left(\frac{2V\sin\theta}{g}\right)\cos\theta$

$= \frac{2V^2\sin\theta\cos\theta}{g}$

$\therefore x = \frac{V^2\sin 2\theta}{g}$

e) $\dot{y} = -gt + V\sin\theta$

let $\dot{y} = 0$ for maximum height

$\therefore t = \frac{V\sin\theta}{g}$

NOTE: this also proves maximum height attained at mid-flight

$\therefore$ Max. height

$= -\frac{g}{2}\left(\frac{V\sin\theta}{g}\right)^2 + V\left(\frac{V\sin\theta}{g}\right)\sin\theta$

$= \frac{-V^2\sin^2\theta}{2g} + \frac{V^2\sin^2\theta}{g}$

$= \frac{V^2\sin^2\theta}{2g}$

In Q's 44 – 46, the horizontal and vertical components of flight, and cartesian eqn of motion are assumed. In an exam however, they should be derived.

44) $y = -\frac{gt^2}{2} + Vt\sin\theta$

Let $y = 0$

$\therefore -\frac{10t^2}{2} + 40t\sin 30° = 0$

$\therefore -5t^2 + 40t\left(\frac{1}{2}\right) = 0$

$\therefore -5t^2 + 20t = 0$

$\therefore -5t(t-4) = 0$

$\therefore t = 4$ secs

$\therefore$ time of flight $= 4$ secs

$\therefore$ when $t = 4$

$\therefore x = 40(4)\cos 30$
$= 160\left(\frac{\sqrt{3}}{2}\right)$

$\therefore x = 80\sqrt{3}$ metres

45) $y = -\frac{gx^2}{2V^2}(1+\tan^2\theta) + x\tan\theta$

$\therefore 2 = \frac{-10(50)^2}{2(80)^2}(1+\tan^2\theta) + 50\tan\theta$

$\therefore 2 = -\frac{125}{64}(1+\tan^2\theta) + 50\tan\theta$

$\therefore 128 = -125 - 125\tan^2\theta + 3200\tan\theta$

$\therefore 125\tan^2\theta - 3200\tan\theta + 253 = 0$

$\therefore \tan\theta = \frac{3200 \pm \sqrt{10113500}}{250}$

$\therefore \tan\theta = 25.5207, 0.0793$

$\therefore \theta = 88°, 5°$ (nearest degree)

46) $y = -\frac{gx^2}{2V^2}(1+\tan^2\theta) + x\tan\theta$

$\therefore y = \frac{-10x^2}{2(100)^2}(1+\tan^2\theta) + x\tan\theta$

$= -\frac{x^2}{2000}(1+\tan^2\theta) + x\tan\theta$

when $x = 40$, $y = 2$

$\therefore 2 = -\frac{1600}{2000}(1+\tan^2\theta) + 40\tan\theta$

$\therefore 4000 = -1600 - 1600\tan^2\theta + 80000\tan\theta$

$\therefore 1600\tan^2\theta - 80000\tan\theta + 5600 = 0$

$\therefore 40\tan^2\theta - 2000\tan\theta + 140 = 0$

$\therefore \tan\theta = \frac{2000 \pm \sqrt{3977600}}{80}$

$\therefore \tan\theta = 49.9299,$

$\therefore \theta = 89°, 4.009°$

Now, particle will finish furthest past the wall when $\theta = 4.009°$

Now particle will hit ground when $y = 0$

ie. $-\frac{gt^2}{2} + Vt\sin\theta = 0$

$-5t^2 + 100t\sin 4.009° = 0$

$\therefore -5t(t - 20\sin 4.009) = 0$

$\therefore t = 1.3985$ secs

when $t = 1.3985$ secs

$x = 100(1.3985)\cos 4.009°$

$\therefore x = 140$ metres (nearest metre)

$\therefore$ Furthest past wall particle will land is 140 metres past wall.