A Selection of Australian Mathematics Competition Publications

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Australian Mathematics Competition

for the Westpac Awards

1991 JUNIOR DIVISION COMPETITION PAPER (School Years 7 and 8)

TUESDAY 30 JULY 1991

INSTRUCTIONS AND INFORMATION

General

1. Do not open this booklet until told to do so by your teacher.
2. Calculators are not permitted. Scribbling paper, graph paper, ruler and compasses are permitted.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. Avoid random guessing as one quarter of the marks assigned for that question will be deducted for an incorrect response.
5. Read carefully the instructions on the answer sheet. It is the student's responsibility that the answer sheet is correctly coded.
6. When your teacher gives the signal, begin working on the problems. You have $1\frac{1}{4}$ hours working time.

Integrity of the Competition

To ensure the integrity of the Competition and to identify the outstanding students the AMC reserves the right to re-examine students before deciding whether to grant official status to their score.

Answers on the Answer Sheet

1. All answers should be recorded on the answer sheet.
2. Use only B or 2B lead pencil.
3. If a coding error is made use only a plastic eraser to ensure that all lead marks and smudges are **completely** removed.

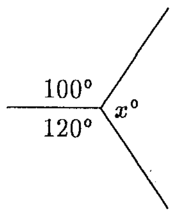
Questions 1-10, 3 marks each

1. $9.2 + 2.9$ equals

(A) 11.11 (B) 12.1 (C) 18.4 (D) 9.49 (E) 11.1

2. In the diagram x equals

(A) 100 (B) 110 (C) 120 (D) 130 (E) 140



3. $\frac{3}{5} \times \frac{10}{21}$ equals

(A) $\frac{1}{2}$ (B) $\frac{2}{7}$ (C) $\frac{13}{105}$ (D) $\frac{15}{13}$ (E) $\frac{103}{105}$

4. 0.5×0.03 equals

(A) 0.15 (B) 0.53 (C) 0.053 (D) 0.015 (E) 1.5

5. The number which is 55 less than 10 010 is

(A) 9 955 (B) 9 965 (C) 9 555 (D) 99 955 (E) 99 965

6. If we multiply 24 by $\frac{1}{12}$ and then add 12, we get

(A) $36\frac{1}{12}$ (B) 24 (C) 14 (D) 36 (E) 300

7. The area of a square is 25 square centimetres. Its perimeter, in centimetres, is

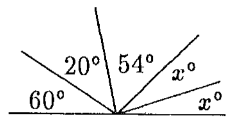
(A) 16 (B) 15 (C) 20 (D) 10 (E) 25

8. The sum of the greatest and the least of the numbers 0.32, 0.302, 0.7, 0.688 and 0.649 is

(A) 1.008 (B) 1.002 (C) 1.388 (D) 1.02 (E) 0.969

9. In the diagram x equals

(A) 34 (B) 33 (C) 46 (D) 67 (E) 23



10. The number 1991 is a palindrome because it is unchanged if its digits are written in the reverse order. How many years are there from 1991 until the next palindromic year?

(A) 9 (B) 11 (C) 121 (D) 231 (E) 1001

Questions 11-20, 4 marks each

11. If $x > 7$, which of the following is smallest?

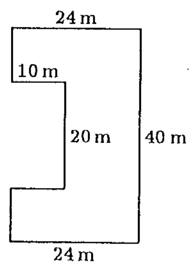
(A) $\frac{x}{7}$ (B) $\frac{7}{x}$ (C) $\frac{7}{x+1}$ (D) $\frac{x+1}{7}$ (E) $\frac{7}{x-1}$

12. A traveller receives \$1.25 in New Zealand currency for each of his Australian dollars. To receive 1000 New Zealand dollars, the amount in Australian dollars he would need to change is

(A) 750 (B) 800 (C) 875 (D) 1200 (E) 1250

13. Lawn food is to be applied to a lawn at the rate of 2.5 kg per $100\,m^2$. The lawn has dimensions as shown in the diagram, and all angles are right angles. The amount of lawn food needed, in kilograms, is

(A) 18 (B) 19 (C) 20 (D) 22 (E) 24



14. If your heart pumps about 80 millilitres of blood each second, then the volume of blood, in litres, it pumps in one day, is about

(A) 7 (B) 70 (C) 500 (D) 5000 (E) 7000

15. The average of four numbers is 48. If 8 is subtracted from each number the average of the four new numbers is

(A) 16 (B) 40 (C) 46 (D) 44 (E) 6

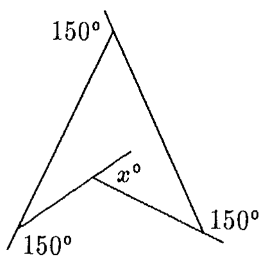
16. The lengths, in centimetres, of the sides of a triangle are $2x$, $3x$ and $4x$. If the perimeter of this triangle is 45 cm, then the difference, in centimetres, between the lengths of the longest and shortest sides is

(A) 5 (B) 10 (C) 15 (D) 20 (E) 25

17. If $P\uparrow$ means $P+1$ and $P\downarrow$ means $P-1$ then $(4\uparrow)\times(3\downarrow)$ is equal to

(A) $9\downarrow$ (B) $10\uparrow$ (C) $11\downarrow$ (D) $12\uparrow$ (E) $13\downarrow$

18. Three external angles of a quadrilateral are each 150°. Another angle is marked $x°$, as shown. The value of x is

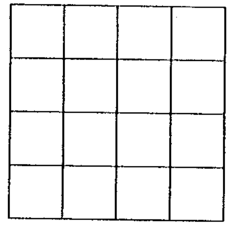


(A) 90 (B) 30 (C) 60 (D) 45 (E) 120

19. One of the elephants in the zoo is on a special diet and eats every day a portion of carrots which is equal to what one of the rabbits eats in one year (365 days). Together, in one day, the elephant and the rabbit eat 111 kilograms of carrots. How many kilograms of carrots does the rabbit eat in one day?

(A) $\frac{1}{2}$ (B) $\frac{111}{165}$ (C) $\frac{37}{122}$ (D) $\frac{19}{61}$ (E) $\frac{22}{73}$

20. Piran has a 4×4 grid of squares on which he is trying to place as many counters as possible. No more than one counter may be placed on any square and no more than three on any row, column or diagonal. What is the maximum number of counters he can place in this way?



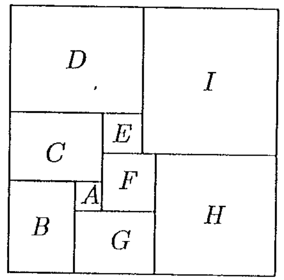
(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

Questions 21-30, 5 marks each

21. Suppose that the mosquito population of the world is 36 000 000 000 and that these mosquitoes can be packed into a cubic box with no space wasted. If the average volume of a mosquito is 6 mm^3, how long would the edge of the box have to be?

(A) 6 m (B) 36 cm (C) 6 cm (D) 36 m (E) 6 km

22. Nine squares are arranged as shown. If square A has area 1 cm^2 and square B has area 81 cm^2 then the area, in square centimetres, of square I is

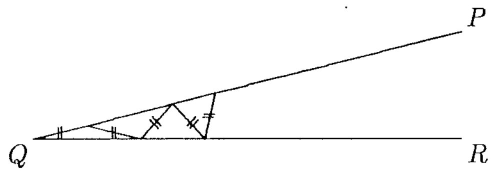


(A) 196 (B) 256 (C) 289 (D) 324 (E) 361

23. How many positive integers less than 1000 have the sum of their digits equal to 6?

(A) 28 (B) 19 (C) 111 (D) 18 (E) 27

24. In the diagram $\angle PQR = 12°$, and a sequence of isosceles triangles is drawn as shown.



What is the largest number of such triangles that can be drawn?

(A) 8 (B) 4 (C) 5 (D) 7 (E) 6

25. Maria is about to travel on a bus, and she knows she must tender the exact fare. She is not sure what this is, but she knows it is greater than \$1.00 and less than \$3.00. What is the minimum number of coins she must carry to be sure of carrying the correct fare? (Assume that the available coins are 1c, 2c, 5c, 10c, 20c, 50c, \$1 and \$2.)

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

26. Mary's brother and grandmother both died young. The sum of their lifespans equalled 66 years. Mary's brother died 93 years after their grandmother was born. How many years after their grandmother died was Mary's brother born?

(A) 37 (B) 33 (C) 30 (D) 27 (E) 17

27. A shop buys 40 pens of three different types at a cost of \$40. If the pens cost 25c, \$1 and \$5 each, and there are more \$1 pens than \$5 pens, how many 25c pens were bought?

(A) 20 (B) 12 (C) 24 (D) 16 (E) 18

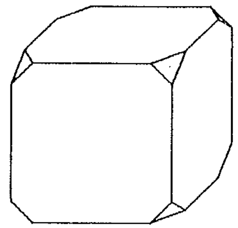
28. How many whole numbers between 100 and 999 have their digits in strictly decreasing order? (e.g. 321, 961, but not 322)

(A) 36 (B) 84 (C) 112 (D) 120 (E) 898

29. Three sets of twin girls are each married to twin boys in such a way that each girl's twin sister is married to her husband's twin brother. If all twelve of them enter a mixed doubles tennis tournament, in how many ways can they be arranged as six mixed pairs (i.e. in each pair there is one girl and one boy) so that no one is paired with their spouse or their twin's spouse?

(A) 80 (B) 16 (C) 48 (D) 32 (E) 96

30. The corners of a cube are cut off in such a way as to form triangles. If all 24 corners of the figure formed are joined by diagonals to each other, how many of these diagonals pass through the interior of the figure?



(A) 84 (B) 108 (C) 120 (D) 142 (E) 240

JUNIOR DIVISION – 1991

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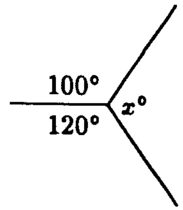

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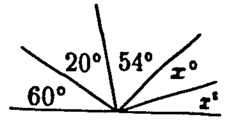

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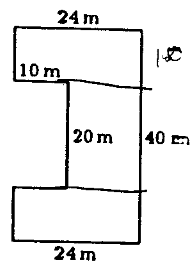

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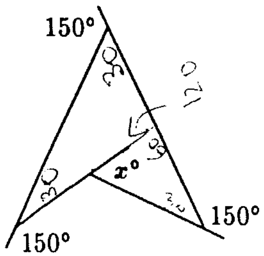

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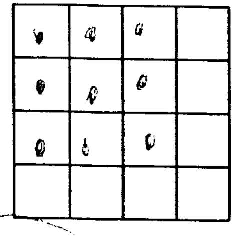

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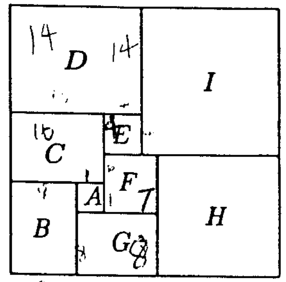

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