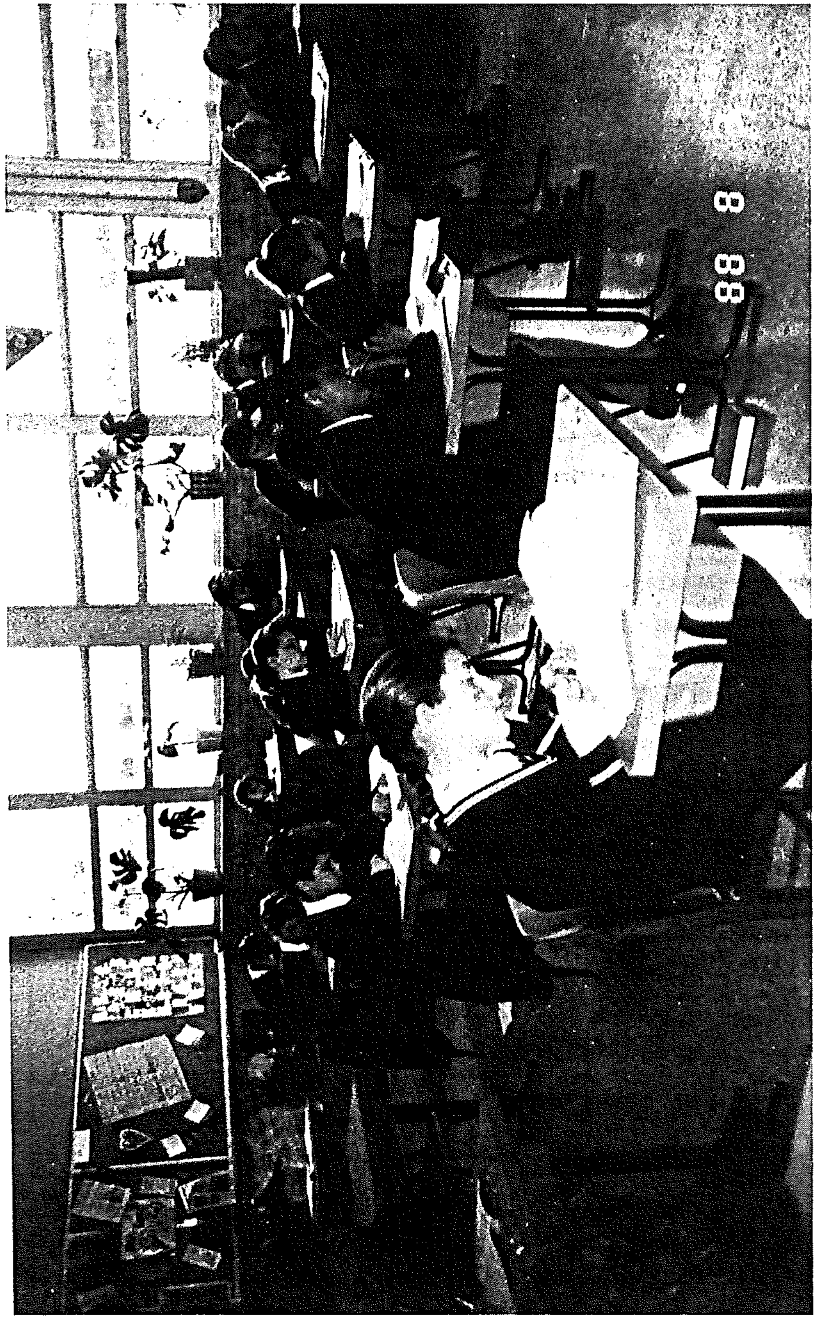


Year 8 students at Annesley College, South Australia, answering the questions in the Junior division of the Competition.

Questions - Junior Division - '88

Questions 1-10, 3 marks each

1. $2 + 2.2$ equals

(A) 4.84 (B) 4.44 (C) 4.4 (D) 4.2 (E) 2.4

2. $\frac{2}{3} + \frac{3}{2}$ equals

(A) $\frac{13}{6}$ (B) 1 (C) $\frac{6}{5}$ (D) $\frac{5}{6}$ (E) $\frac{13}{5}$

3. The value of $8 \times (23 - 9)$ is

(A) 175 (B) 96 (C) 176 (D) 256 (E) 112

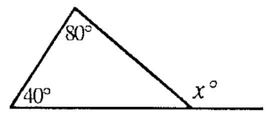
4. The value of $\frac{0.3}{5}$ is

(A) 0.6 (B) 0.15 (C) 0.02 (D) 0.05 (E) 0.06

5. In the diagram, x equals

(A) 100 (B) 120 (C) 160

(D) 140 (E) 130



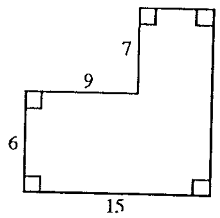
6. Two thirds of a number is equal to 36. The number is

(A) 54 (B) 24 (C) 45 (D) 12 (E) 57

7. The average of 0.03, 0.3 and 3.0 is

(A) 0.3 (B) 3.33 (C) 1.11 (D) 9.99 (E) 0.33

8. In the diagram lengths are shown in centimetres. The perimeter of the figure, in centimetres, is



(A) 195 (B) 56 (C) 50 (D) 28 (E) 37

9. Which of the following is 24% of \$150?

(A) \$34 (B) \$36 (C) \$63 (D) \$84 (E) \$120

10. This year Australia is celebrating 200 years of European settlement. Some historians believe that Australia has been inhabited for about 40 000 years. The fraction of this time that Australia has been settled by people of European descent is closest to

(A) $\frac{1}{20}$ (B) $\frac{1}{40}$ (C) $\frac{1}{100}$ (D) $\frac{1}{200}$ (E) $\frac{1}{2000}$

Questions 11-20, 4 marks each

11. Given that $3.84 \times 2.75 = 10.56$ the value of $1.056 \div 0.00275$ is

(A) 0.384 (B) 3.84 (C) 38.4 (D) 384 (E) 3840

12. The apple tree in my back yard was planted 3 years and 4 months before its fruit was picked in February 1988. The tree was planted in

(A) October 1985 (B) June 1984 (C) October 1984 (D) November 1984 (E) September 1984

13. If the length of a rectangle is doubled and the width is tripled, the area of the rectangle has been multiplied by

(A) 2 (B) 3 (C) $2\frac{1}{2}$ (D) 8 (E) 6

14. My recipe for Anzac biscuits states that the ingredients needed to make 35 biscuits include two cups of rolled oats. I want to make 210 Anzac biscuits for my party. One packet of rolled oats contains 5 cups of oats. How many packets of rolled oats do I need?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

15. The product $(1+\frac{1}{5})(1+\frac{1}{6})(1+\frac{1}{7})(1+\frac{1}{8})(1+\frac{1}{9})$ is equal to

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

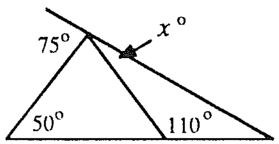
16. One may define a 'micro-century' as one millionth of a century. This is approximately

(A) 5 seconds (B) 50 seconds (C) 5 minutes (D) 50 minutes (E) 5 hours

17. Two identical jars are filled with mixtures of water and vinegar in the ratio of 2 to 1 and 3 to 1 respectively. If both jars are emptied into another container then the ratio of water to vinegar in the mixture is

(A) 5 to 1 (B) 12 to 5 (C) 17 to 7 (D) 6 to 5 (E) 5 to 2

18. In the diagram x equals



(A) 25 (B) 30 (C) 35 (D) 40 (E) 45

19. The total mass of a bottle and its contents of 20 identical tablets was 180 grams. When the bottle contained 15 tablets it was found that the total mass was 165 grams. The mass of the bottle, in grams, was

(A) 103 (B) 115 (C) 120 (D) 125 (E) 146

20. Amongst the children in a family a boy has as many sisters as brothers, but each sister has only half as many sisters as brothers. The number of children in the family is

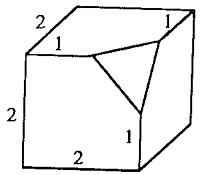
(A) 7 (B) 5 (C) 6 (D) 4 (E) 9

Questions 21-30, 5 marks each

21. If p and q are positive integers (whole numbers), and $p + q < 10$, how many different values can the product pq take?

(A) 36 (B) 20 (C) 12 (D) 15 (E) 16

22. A hollow cube has had one of its corners cut off, leaving a triangular hole. The dimensions in metres are shown. What is the area, in square metres, of the outside of the remainder of the cube?



(A) $14\frac{1}{2}$ (B) $30\frac{1}{2}$ (C) 21 (D) $22\frac{1}{2}$ (E) $24 - \frac{\sqrt{6}}{2}$

23. Con leaves the outskirts of Melbourne at 8 am and travels north to Sydney at a constant speed of 80 km/h. Maria leaves from the same place at 8.30 am and takes the same route travelling at a constant speed of 100 km/h. At what time will Maria overtake Con along the road to Sydney?

(A) 10.15 am (B) 10.30 am (C) 10.45 am (D) 11.00 am (E) 11.45 am

24. Four points P, Q, R and S are spaced at one metre intervals, in that order, along a straight line. The length of the shortest path from P to S, in metres, keeping at least one metre from Q and R, is

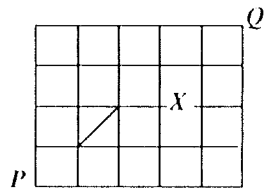
(A) $1 + \pi$ (B) $\frac{4\pi}{3}$ (C) 5 (D) $1 + 2\pi$ (E) $1 + 2\sqrt{2}$

25. If m is a number between 15 and 30 and n is a number between 3 and 8, then $\frac{m}{n}$ must be a number between

(A) $\frac{8}{15}$ and 5 (B) $1\frac{7}{8}$ and 10 (C) $3\frac{3}{4}$ and 10

(D) 5 and 10 (E) $3\frac{3}{4}$ and 15

26. The accompanying diagram is a road plan of a city. All the roads go east-west or north-south, with the exception of the one short diagonal road shown. The road marked X is being repaired and is impassable in the middle. Of all the possible routes from P to Q, there are several shortest routes. How many such shortest routes are there?



(A) 4 (B) 7 (C) 9 (D) 14 (E) 16

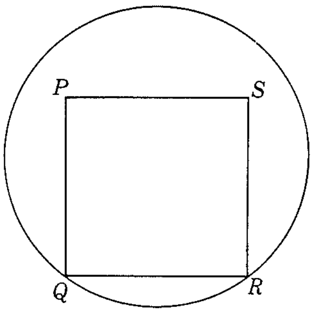
27. The sum of all the four-digit numbers that can be obtained by using the digits 1, 2, 3 and 4 once in each is

(A) 66660 (B) 11110 (C) 9999 (D) 33330 (E) 5555

R DIVISION

e

S



$+ \frac{\sqrt{2}}{2}) \pi$ (C) $\sqrt{2}\, \pi$

(E) $(1 + \frac{\sqrt{2}}{2}) \pi$

t of the equation

$\sqrt{3}$ (C) $11 - 2\sqrt{30}$

(E) $17 - 12\sqrt{2}$

xhibition tennis match between two
andom from a pool of N players,
wo names and before announcing
ch involve at least one Australian?"
"Yes". The chance of both players

$\frac{-1}{-1}$ (C) $\frac{n+1}{2N-n+1}$

(E) $\frac{2n-1}{2N-1}$

ther in two distinct points, are in a
he same point. They divide the plane
cluding the exterior of all circles, is

0 (D) 90 (E) 92

ANSWERS

Junior		Intermediate		Senior	
1	E	1	E	1	B
2	E	2	A	2	B
3	A	3	C	3	C
4	B	4	C	4	E
5	D	5	D	5	D
6	C	6	E	6	E
7	C	7	C	7	D
8	E	8	E	8	D
9	B	9	A	9	C
10	D	10	C	10	B
11	E	11	E	11	A
12	C	12	E	12	B
13	E	13	D	13	B
14	A	14	C	14	B
15	E	15	B	15	C
16	A	16	B	16	E
17	E	17	E	17	E
18	A	18	C	18	B
19	E	19	C	19	C
20	A	20	C	20	D
21	D	21	A	21	D
22	C	22	B	22	B
23	D	23	D	23	D
24	B	24	C	24	C
25	C	25	B	25	D
26	C	26	D	26	B
27	D	27	A	27	D
28	A	28	D	28	E
29	D	29	B	29	A
30	D	30	B	30	E

1. $4.8 - 2.5 = 2.3$, **hence (E)**

2. (This is also question 1 of the Intermediate paper)
$\frac{1}{4} + \frac{1}{6} = \frac{6+4}{24} = \frac{10}{24} = \frac{5}{12}$, **hence (E)**

3. (Also I.2)
The number smallest in value is 0.078 , **hence (A)**

4. $\frac{5}{17}$ of $34 = \frac{5}{\cancel{17}} \times \frac{\overset{2}{\cancel{34}}}{1} = 5 \times 2 = 10$, **hence (B)**

5. The exterior angle equals (the sum of the opposite interior angles), because both equal (180° minus the third angle in the triangle).
$\therefore\ x = 90 + 36 = 126$, **hence (D)**

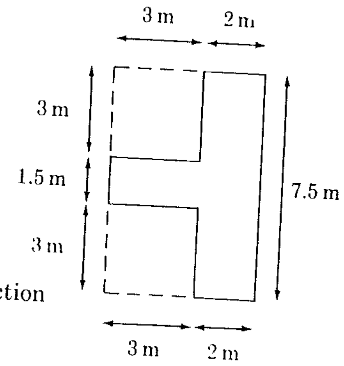
6. $5x - 2 + 4 - 3x = 2x + 2$, **hence (C)**

7. $\dfrac{2}{1 - \frac{2}{3}} = \dfrac{2}{\frac{1}{3}} = 2 \times 3 = 6$, **hence (C)**

8. $7\frac{1}{2}\%$ of $80 = \dfrac{7\frac{1}{2}}{100} \times 80 = \dfrac{\overset{3}{\cancel{15}}}{\cancel{2}} \times \dfrac{1}{\underset{5}{\cancel{100}}} \times \dfrac{\overset{2}{\cancel{\overset{4}{\cancel{80}}}}}{1} = 3 \times 2 = 6$, **hence (E)**

9. Six apples at 17¢ ea = \$1.02
Eight oranges at 23¢ ea = \$1.84
Total cost = \$2.86
Thus change from \$5 = \$2.14 , **hence (B)**

10. (Also I.5 and S.5)
Total height = $(1\,000\,000 \times 0.25)$ mm = $1\,000 \times 0.25$ m = 250 m , **hence (D)**



11. All measurements are shown in the diagram
Alternative 1
Total area = area of vertical section
+ area of additional horizontal section
$= (2 \times 7.5)\,m^2 + (3 \times 1.5)\,m^2$
$= (15 + 4.5)\,m^2$
$= 19.5\,m^2$,
hence (E)

Alternative 2
Total area = area of large rectangle − area of two corner squares
$= (5 \times 7.5)\,m^2 - [2 \times (3 \times 3)]\,m^2$
$= (37.5 - 2 \times 9)\,m^2$
$= (37.5 - 18)\,m^2$
$= 19.5\,m^2$,
hence (E)

12. Actual playing time

$= (2\frac{45}{60} - 1\frac{52\frac{1}{2}}{60})$ hours $= (\dfrac{165 - 112\frac{1}{2}}{60})$ hours

$= \dfrac{52\frac{1}{2}}{60}$ hours $= \dfrac{\overset{7}{\cancel{\overset{21}{\cancel{105}}}}}{2 \times \underset{4}{\cancel{\underset{12}{\cancel{60}}}}}$ hours $= \frac{7}{8}$ hours , **hence (C)**

13. (Also I.8)
The bottle should last

$\frac{150}{1.25}$ days, i.e. $\dfrac{\overset{30}{\cancel{150}} \times 4}{\cancel{5}}$ days, i.e. 120 days, or about 4 months , **hence (E)**

14. Each floor measures $(35 \times 16)\,m^2$, i.e. $560\,m^2$.
To have $5600\,m^2$ one needs 10 floors ,
hence (A)

15. The speedometer reads $(1 + \frac{10}{100})$ times actual speed, i.e. 1.1 times actual speed.

If the speedometer reads 100 km/h the actual speed is $\frac{100}{1.1}$ km/h,

i.e. $\frac{1000}{11}$ km/h, i.e. $90\frac{10}{11}$ km/h ,
hence (E)

16. If there are only 8 pigs the food should last $16 \times \frac{14}{8}$ days,
i.e. 28 days , **hence (A)**

17. (Also I.6 and S.4)

$\frac{2}{15} = \frac{1}{8} + \frac{1}{x}$

$\therefore \frac{1}{x} = \frac{2}{15} - \frac{1}{8} = \frac{16-15}{120} = \frac{1}{120}$

$\therefore x = 120$, **hence (E)**

18. Let the length, breadth and height of the figure be a, b and c cm respectively.

$\therefore ab = 12$

$bc = 6$

$ac = 8$

$\therefore (ab)(bc)(ac) = (12)(6)(8)$

$= (2^2 \times 3)(2 \times 3)(2^3)$

$\therefore a^2 b^2 c^2 = 2^6 3^2$

$= (2^3 \times 3)^2$

$(abc)^2 = (24)^2$

Since the volume is positive, volume = 24 cm^3 , **hence (A)**

19. (Also I.11)
Mary's score in 10 tests totals $68 \times 10 = 680$ marks. If her average over 11 tests is to be 70 her total over 11 tests must be 770 marks. Thus she must score $770 - 680$, i.e. 90 marks in her next test , **hence (E)**

20. Suppose $5x$ units of volume of the mixture is combined with $5y$ units of pure olive oil, to give the required combination:
The original mixture contains $2x$ units of volume of olive oil and $3x$ units of volume of vinegar.
We then have $3x = 2x + 5y$

$\therefore x = 5y$

$\therefore \frac{x}{y} = \frac{5}{1}$

thus a 5 to 1 combination is required , **hence (A)**

21. Let the three numbers be x, y and z so that

$x + y = 38$ ... (1)

$y + z = 44$ ... (2)

$x + z = 52$ ... (3)

Subtracting (1) from (2) gives

$z - x = 6$ ... (4)

Adding (3) and (4) gives

$2z = 58$,

$\therefore z = 29$.

Clearly z is the largest of the three numbers , **hence (D)**

22. (Also I.19 and S.15)
On one side of my house there are $\frac{137-1}{2} = \frac{136}{2} = 68$ houses.
On the other side are $\frac{85-1}{2} = \frac{84}{2} = 42$ houses.
The total number of houses on my side of the street, including my own, is
$68 + 1 + 42 = 111$, **hence (C)**

23. Numbers which when divided by 6 give remainder 1 are 1, 7, 13, 19, 25, 31, 37, 43, 49, 55, 61, . . .
Numbers which when divided by 11 give remainder 6 are 6, 17, 28, 39, 50, 61, . . .
The smallest number in common is 61 , **hence (D)**

24. (Also I.16 and S.10)
With obvious notation

C + L = A ... (1)

A = S + 1 ... (2)

L = C − 5 ... (3)

F + 5 = 2P ... (4)

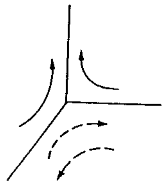
L = F + 8 ... (5)

P = 50 ... (6)

Now (6) ⇒ P = 50,
hence (4) ⇒ F = 95,
(5) ⇒ L = 103,
(3) ⇒ C = 108,
(1) ⇒ A = 211,
(2) ⇒ S = 210 ,

hence (B)

25. (Also I.20 and S.19)
There are two types of vertex in any configuration of paths:
(1) There is an uncontested edge: In this case the diagram shows that there must then be exactly two uncontested edges at that vertex.
(2) All three edges are contested.



Suppose that in some configuration of paths there are r vertices of type (2).
Then the number of contested edges on the cube is

$\frac{1}{2}[3r + (8 - r)] = 4 + r \geqslant 4$

($\frac{1}{2}$ because each edge is contested twice)

with the minimum occurring when $r = 0$, i.e. every vertex is of type (1).
Such a configuration is possible. For example, arrange the paths so that, viewed from inside the cube, those on a pair of opposite faces are clockwise and the other four are anticlockwise , **hence (C)**

26. Let the distance to Goulburn be d units and let t be the number of minutes the slow train has been travelling when it is passed by the express.

The speed of the slow train = $\frac{d}{165}$ units/min.

The speed of the express = $\frac{d}{100}$ units/min.

They meet when they have both covered equal distances.

$\therefore \frac{dt}{165} = \frac{d(t-39)}{100}$

$\therefore 100t = 165t - (165)(39)$

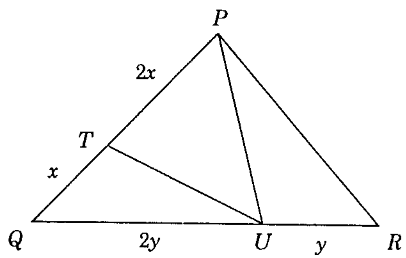
$\therefore 65t = (165)(39)$

$\therefore t = \frac{\overset{33}{\cancel{165}} \times \overset{3}{\cancel{39}}}{\underset{5}{\cancel{65}}} = 99$

9 h 17 min + 1 h 39 min = 10 h 56 min , **hence (C)**

27. (Also S.21)
Note that each time the hands cross each other they twice make a right angle. They cross 11 times in half a day. Thus they make 44 right angles in a day ,
hence (D)

28. First consider Δs PQU and PUR, which have a common altitude from their bases to P.



Area $\Delta PQU = 2 \times$ area ΔPUR
since base $\Delta PQU = 2 \times$ base ΔPUR

$\therefore$ Area $\Delta PQU = \frac{2}{3} \times$ area ΔPQR
$= 60\,\text{cm}^2$

Similarly Δs UPT and UTQ have a common altitude, with base $\Delta UPT = 2 \times$ base ΔUTQ

$\therefore$ Area $\Delta UPT = \frac{2}{3} \times$ area $\Delta UPQ = \frac{2}{3} \times 60\,\text{cm}^2 = 40\,\text{cm}^2$, **hence (A)**

29. (Also I.26 and S.20)
Let the free allowance per person be x kg. Let the excess charge be \$$y$ per kilogram greater than x. With Peter and Lois both travelling we have

$60 + 100 = 160$
= Total charge
= (Number of "free" kilograms) × (Cost per "free" kilogram)
  + (Number of "charged" kilograms) × (Cost per "charged" kilogram)
$= (2x)(0) + (52 - 2x)y$

Similarly, if Peter had travelled alone, we would have $340 = (x)(0) + (52 - x)y$

$\therefore y = \frac{160}{52 - 2x} = \frac{340}{52 - x}$

$\therefore 160(52 - x) = 340(52 - 2x)$

$\therefore (160)(52) - 160x = (340)(52) - 680x$

$\therefore x = \frac{(340 - 160)(52)}{680 - 160} = \frac{(180)(52)}{520} = 18$, **hence (D)**

30. (Also I.28 and S.23)
Let the first and last numbers in the string be f and l respectively.
Then 75 = (number of numbers in string) × (average)
$= n\left(\frac{f + l}{2}\right)$

$\therefore 150 = n(f + l)$

Now n, f and l are all integers
$\therefore n$ must divide $150 = 2 \times 3 \times 5^2$
The only possible values are thus $n = 2, 3, 5, 6, 10, 15, \ldots$
Enumerating possibilities,

n	$f + l$	*Sum*
2	75	75 = 37 + 38
3	50	75 = 24 + 25 + 26
5	30	75 = 13 + 14 + 15 + 16 + 17
6	25	75 = 10 + 11 + 12 + 13 + 14 + 15
10	15	75 = 3 + 4 + 5 + . . . + 11 + 12

For $n = 15$ and higher the lowest number must be negative, since, e.g. for $n = 15$ the average (and thus middle) number must be 5.
There are 5 possible values of n ,
hence (D)