# Inequalities

An **inequality** (or **inequation**) is a statement that one quantity is larger (or smaller) than another. A **linear inequality** is an inequality in which the highest power of the pronumeral is 1. For example, $m < 2$ and $x + 5 \geqslant 2$ are linear inequalities but $t^2 + 1 > 4$ is not. Examples of inequality statements are common in everyday life.
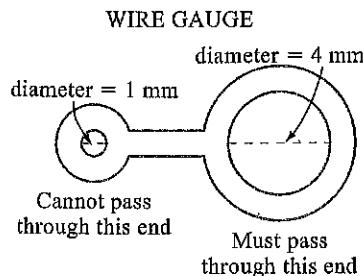
LOW 3.5 m CLEARANCE	MAX LOAD 1.4 tonnes	Minimum height for ride is 1.2 m
$H \leqslant 3.5$ where H is height of vehicle.	$m \leqslant 1.4$ tonnes where m is mass of the load.	$h \geqslant 1.2$ m where h is the height

WIRE GAUGE

diameter = 4 mm

diameter = 1 mm

Cannot pass through this end

Must pass through this end

NO ENTRY UNLESS YOU ARE 18 YEARS OR OLDER

$A \geqslant 18$ where
A is the required age.

If d is the diameter of the wire, then d must be between 1 mm and 4 mm. We write this as $d > 1$ and $d < 4$. We can combine them to write $1 < d < 4$.

> is greater than $\geqslant$ is greater than or equal to

< is less than $\leqslant$ is less than or equal to

Interpreting an inequality statement

Inequality statements can be read from left to right and also from right to left.

Example

$5 < 8$ can be read from left to right as *5 is less than 8* or it can be read from right to left as *8 is greater than 5*. So $8 > 5$ is the same as $5 < 8$.
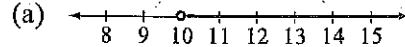
Similarly, $x > 4$ and $4 < x$ are equivalent statements.

Representing inequalities on a number line

Examples
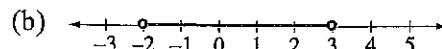
| Number line	Statement of inequality
(a) 8 9 10 11 12 13 14 15	$x > 10$
(b) −3 −2 −1 0 1 2 3 4 5	$-2 < x < 3$
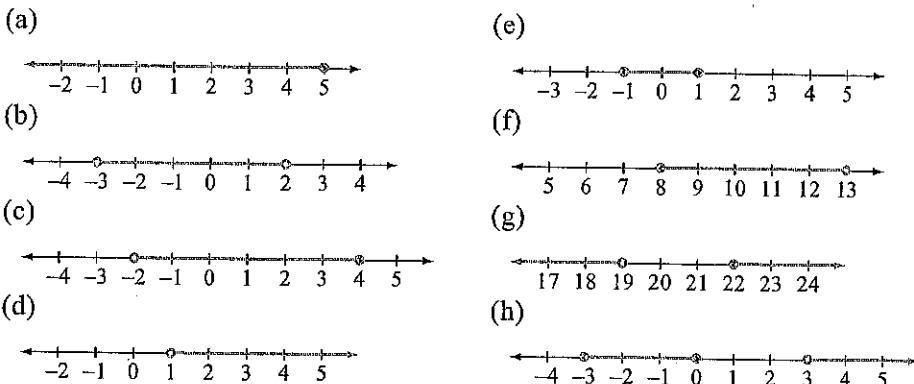
(c)

$$27 \leqslant x \leqslant 30$$

(d)

$$x \leqslant 3$$

(e)

$$x < -1 \text{ and } x \geqslant 2$$

Exercise 108

1. Write an inequality statement for x to match each number line.

   (a)

   (b)

   (c)

   (d)

   (e)

   (f)

   (g)

   (h)

2. Draw a number line to show each inequality statement.

   (a) $12 \leqslant x < 15$
   (b) $x > -1$
   (c) $4 < x$
   (d) $-2 \leqslant x \leqslant 1$

   (e) $x > 0$ and $x \leqslant -3$
   (f) $6 < x < 10$
   (g) $-8 < x \leqslant 2$
   (h) $x \leqslant -4$ and $x \geqslant 3$

Group investigation: Operations on inequalities

1. Write an inequality statement that is true, e.g. $5 < 8$.
   (a) Add 6 to both sides. Is the new statement true?
   (b) Subtract 10 from both sides. Is the new statement true?
   (c) Multiply both sides by 3. Is the new statement true?
   (d) Divide both sides by 2. Is the new statement true?
   (e) Multiply both sides by -4. Is the new statement true?
   (f) Divide both sides by -10. Is the new statement true?

2. What operations caused your true inequality statement to be no longer true?

3. To maintain the truth of an inequality when you divide or multiply both sides of it by a negative number, what must happen to the $>$ or $<$ symbol? Discuss and compare your answer with other students.

Solving inequalities

A linear equation has only one solution, but a linear inequality may have many solutions. These solutions can easily be represented on a number line.

> **When solving linear inequalities, the rules are the same as for linear equations except when multiplying or dividing both sides by a negative number; then you must reverse the inequality sign.**

Example 1   Solve the inequality $2x - 1 \geqslant 11$ and graph the solution on a number line.
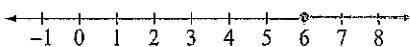
$$2x - 1 \geqslant 11$$

$$2x - 1 + 1 \geqslant 11 + 1$$

$$2x \geqslant 12$$

$$\frac{2x}{2} \geqslant \frac{12}{2}$$

$$x \geqslant 6$$

Example 2   Solve and graph the inequality $-11 > 1 - 2x$.
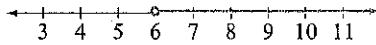
$$-11 > 1 - 2x$$

$$-11 - 1 > 1 - 2x - 1$$

$$-12 > -2x$$

$$\frac{-12}{-2} < \frac{-2x}{-2} \qquad \text{(dividing by a negative number reverses the inequality sign)}$$

$$6 < x$$

$$\therefore x > 6$$

Example 3   Solve and graph the inequality $\dfrac{3x}{4} - \dfrac{2x}{5} < 14$.

$$\frac{3x}{4} - \frac{2x}{5} < 14$$
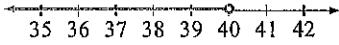
Multiply both sides by 20.

$$20 \times \frac{3x}{4} - 20 \times \frac{2x}{5} < 20 \times 14$$

$$15x - 8x < 280$$

$$7x < 280$$

$$x < 40$$

1. Find, by substitution, which of the following is *not* a possible solution to the inequality given.

    (a) $4x + 5 \geqslant 21$

        A $x = 2$ B $x = 7$ C $x = 10$ D $x = 4$

    (b) $3a + 2 > 14$

        A $a = 7$ B $a = 4$ C $a = 5$ D $a = 9$

    (c) $-2x - 1 > 16$

        A $x = -10$ B $x = -8$ C $x = -20$ D $x = -12$

    (d) $\dfrac{5x - 1}{4} \leqslant 2x - 1$

   A $x = 1$ B $x = 0$ C $x = 4$ D $x = 6$

2. For each of the following, solve the inequality and graph the solution on a number line.

    (a) $x + 5 < 3$

    (b) $x - 4 > 4$

    (c) $x + 1 \geqslant 5$

    (d) $x - 3 \leqslant -2$

    (e) $7 < x + 6$

    (f) $-5 \leqslant x - 2$

    (g) $4 \geqslant x - 4$

    (h) $-1 < x - 1$

    (i) $x + 2 < -5$

    (j) $x - 3 \geqslant -3$

3. Solve the following inequalities.

    (a) $3x \geqslant 6$

    (b) $2x < 5$

    (c) $\dfrac{x}{4} > 2$

    (d) $\dfrac{x}{3} \leqslant -2$

    (e) $6x < 10$

    (f) $3x \geqslant -9$

    (g) $-15 \leqslant 5x$

    (h) $20 > 4x$

    (i) $8 < \dfrac{x}{3}$

    (j) $-4 \geqslant \dfrac{x}{5}$

4. Solve the following inequalities. Graph each solution on a number line.

    (a) $-2x > 4$

    (b) $-3x \leqslant -6$

    (c) $-\dfrac{x}{4} \geqslant -1$

    (d) $-\dfrac{x}{2} < \dfrac{1}{2}$

    (e) $1 - x > 4$

    (f) $-3 \leqslant 1 - x$

    (g) $-2x + 1 > -3$

    (h) $1 - 3x \leqslant 7$

    (i) $-\dfrac{3x}{2} > 6$

    (j) $-7 \geqslant -\dfrac{2x}{3}$

5. For each of the following, solve the inequality and graph the solution on a number line.

    (a) $x - 1 > 4$

    (b) $2x \geqslant 6$

    (c) $3x - 1 < 19$

    (d) $7 < x + 1$

    (e) $8 \geqslant 2x - 6$

    (f) $3 \leqslant \dfrac{x - 4}{4}$

    (g) $5(x - 2) \geqslant 20$

    (h) $-6x < 12$

    (i) $1 - 4x < 17$

    (j) $\dfrac{2p}{-3} \leqslant -6$

    (k) $1 + \dfrac{5a}{3} \leqslant -9$

    (l) $-3w + 10 > 4$

6. Solve the following inequalities and graph each solution on a number line.

(a) $\dfrac{2y + 4}{3} > 12$

(b) $2 - 5y \leqslant 22$

(c) $y + 5 \leqslant 2y - 2$

(d) $\dfrac{8a + 4}{2} > 6$

(e) $9 \geqslant \dfrac{5m - 2}{2}$

(f) $-3(2 - y) < 6$

(g) $8(2m - 2) \geqslant -6$

(h) $7 - \dfrac{y}{6} > 1$

(i) $-2 \leqslant \dfrac{3a}{4} + 6$

(j) $\dfrac{m - 2}{3} + 4 > 1$

(k) $2y + 3(1 - 2y) \geqslant 15$

(l) $\dfrac{2a + 1}{4} > \dfrac{1 - 3a}{2}$

7. Solve, graphing each solution on a number line.

(a) $\dfrac{2a}{5} - \dfrac{3a}{2} > 6$

(b) $\dfrac{-5(y - 2)}{4} \leqslant 2\dfrac{1}{2}$

(c) $\dfrac{7 - 2p}{3} > \dfrac{3p - 4}{4}$

(d) $8x - 2 < 2 - 8x$

(e) $-2 \leqslant 2x < 6$

(f) $-14 \leqslant 3x - 2 \leqslant 7$

(g) $\dfrac{1 - y}{2} + \dfrac{2 - y}{3} < 2$

(h) $1 - \dfrac{2t}{3} < 2 - \dfrac{t}{4}$

(i) $\dfrac{-7p}{3} - 8 > -10$

(j) $1 - 2x > \dfrac{3x}{4}$

(k) $\dfrac{6 - 4(1 + 2x)}{-5} > 6$

(l) $\dfrac{8 + m}{4} - \dfrac{m - 2}{3} < 4$

▶ Group activity: The language of inequalities

(Work in pairs.)

1. You have $20 to spend on items which cost $4.80 each.
   (a) How does the inequality $4.8n \leqslant 20$ relate to this situation? What does n stand for?
   (b) Solve $4.8n < 20$ in the context of the problem. Graph your solutions on a number line. Consider the following:
      (i) Can n be negative?   (ii) Can n be a fraction?
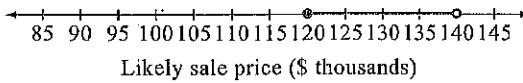
2. Describe a situation where the inequality $4n + 6p < 100$ would apply, given that n and p are positive whole numbers. Compare your answer with those of other groups.

3. A real estate agent selling your property at auction summarises her expectations of the sale price on the number line below.

   85 90 95 100 105 110 115 120 125 130 135 140 145
   Likely sale price ($ thousands)

   (a) What do you think is the lowest price acceptable for the property?
   (b) You will be satisfied with a sale price of $137 000. Does this seem likely to happen?
   (c) If you are offered $142 000 for the property, should you accept the offer? Explain.

4. Make up some inequalities or number lines and swap them with other groups.

EQUATIONS AND INEQUALITIES

(h) $a = \sqrt{\dfrac{A - \pi h}{\pi}}$

(i) $a = \dfrac{u - up}{kp - w}$

(j) $a = \dfrac{-Nb - Mb}{M - N}$ or $a = \dfrac{-b(N + M)}{M - N}$ or $a = \dfrac{b(M + N)}{N - M}$

5. (a) $D = 3.5$ (c) $V = \dfrac{M}{D}$; $V = \dfrac{1}{6}$
   (b) $M = 620$

6. (a) $C = 45.2448$ (b) $r = \dfrac{C}{2\pi}$ (c) $r = 20.5$

7. (a) $C = 290$ (b) $d = \dfrac{C - 80}{4.2}$ (c) $d = 76$

8. (a) $V = 4.6 \times 10^2$ (b) $t = \dfrac{2V}{c}$ (c) $t = 5$

9. (a) $r = 1 - \dfrac{a}{S}$ (b) $r = \dfrac{29}{32}$

10. (a) $S = -620$ (b) $l = \dfrac{2S}{n} - a$ (c) $l = 20$

11. (a) (i) $P = 3Q + 1$ (ii) $P = 61$ (iii) $Q = 8$
    (b) (i) $P = \dfrac{Q}{5} + 1$ (ii) $P = 5$ (iii) $Q = 120$
    (c) (i) $P = 11 - 2Q$ (ii) $P = -29$ (iii) $Q = -7$

12. (a) 182 cm (b) 39 cm

13. (a) $196.77 (c) $712

Exercise 9.06

1. (a) $x \leqslant 5$ (e) $-1 \leqslant x \leqslant 1$
   (b) $-3 < x < 2$ (f) $8 \leqslant x < 13$
   (c) $-2 < x \leqslant 4$ (g) $x < 19$ and $x \geqslant 22$
   (d) $x > 1$ (h) $-3 \leqslant x \leqslant 0$ and $x > 3$

2. (a) — number line
   (b) — number line
   (c) — number line
   (d) — number line
   (e) — number line
   (f) — number line
   (g) — number line
   (h) — number line

Exercise 9.07

1. (a) A (b) B (c) B (d) B

2. (a) $x < -2$ (c) $x \geqslant 4$
   (b) $x > 8$ (d) $x \leqslant 1$
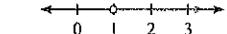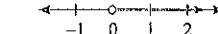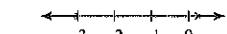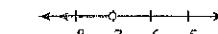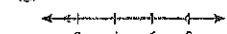
(e) $x > 1$ (h) $x > 0$
(f) $x \geqslant -3$ (i) $x < -7$
(g) $x \leqslant 8$ (j) $x \geqslant 0$
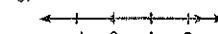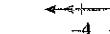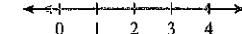
3. (a) $x \geqslant 2$ (f) $x \geqslant -3$
   (b) $x < 2\frac{1}{2}$ (g) $x \geqslant -3$
   (c) $x > 8$ (h) $x < 5$
   (d) $x \leqslant -6$ (i) $x > 24$
   (e) $x < 1\frac{2}{3}$ (j) $x \leqslant -20$
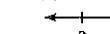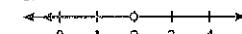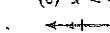
4. (a) $x < -2$ (f) $x \leqslant 4$
   (b) $x \geqslant 2$ (g) $x < 2$
   (c) $x \leqslant 4$ (h) $x \geqslant -2$
   (d) $x > -1$ (i) $x < -4$
   (e) $x < -3$ (j) $x \geqslant 10\frac{1}{2}$
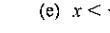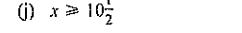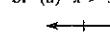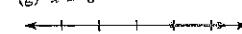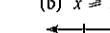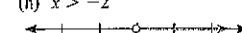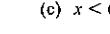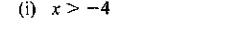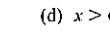
5. (a) $x > 5$ (g) $x \geqslant 6$
   (b) $x \geqslant 3$ (h) $x > -2$
   (c) $x < 6\frac{2}{3}$ (i) $x > -4$
   (d) $x > 6$ (j) $p \geqslant 9$
   (e) $x \leqslant 7$ (k) $a \leqslant -6$
   (f) $x \geqslant 16$ (l) $w < 2$
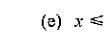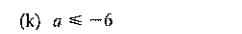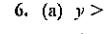
6. (a) $y > 16$ (d) $a > 1$
   (b) $y \geqslant -4$ (e) $m \leqslant 4$
   (c) $y \geqslant 7$ (f) $y < 4$

(g) $m \geqslant \frac{5}{8}$

(j) $m > -7$

(h) $y < 36$

(k) $y \leqslant -3$

(i) $a \geqslant -10\frac{2}{3}$

(l) $a > \frac{1}{8}$

1. (a) $14g^2 - g - 3$
   (b) $6m^2 - 13m + 5$
   (c) $3y^2 + 16y + 5$
   (d) $4d^2 - 20d + 25$
   (e) $25 + 40w + 16w^2$
   (f) $16x^2 - 9y$
   (g) $25 - \frac{k^2}{25}$
   (h) $t^2 + 6rt + 9t^2 - 16$
   (i) $x^4 - 8x^2 + 16$
   (j) $\frac{a^2}{b^2} + 2a + b^2$

2. (a) $4(w - 4)$
   (b) $x(x - y)$
   (c) $5n(3m - n)$
   (d) $-3(3y + 1)$
   (e) $(g + 2)(2 - g)$
   (f) $(4x - 3)(y + 7)$
   (g) $(1 - k)(k^2 - 1) = (1 - k)(k - 1)(k + 1)$
   (h) $3a(2b - 1 + 4a)$

3. (a) $(d - 3)(d + 3)$
   (b) $(5w - 4)(5w + 4)$
   (c) $(x - \frac{1}{2})(x + \frac{1}{2})$
   (d) $25(h - 2)(h + 2)$
   (e) $(x^2 - 2)(x^2 + 2)(x^4 + 4)$
   (f) $(t + 4)(t + 5)$
   (g) $(u + 2)(u - 1)$
   (h) $(y + 9)(y - 2)$
   (i) $(m - 15)(m + 1)$
   (j) $(7 + a)(6 - a)$

4. (a) $(2x + 1)(x + 9)$
   (b) $(3g - 7)(3g + 4)$
   (c) $(2m - 5)(2m - 3)$
   (d) $(2d + 3e)(d + e)$
   (e) $(7y - 4)(2y - 3)$
   (f) $(2c - 5)^2$

5. (a) $\frac{k}{k - 3}$
   (b) $\frac{m + b}{m + c}$
   (c) $\frac{h - 7}{h - 3}$
   (d) $\frac{w - 5}{w - 4}$

6. (a) $\frac{-2}{x^2 - 1}$
   (b) $\frac{11}{(3p - 1)(2p + 3)}$
   (c) $\frac{2d}{d^2 - 1}$
   (d) $\frac{4m}{m^2 - n^2}$
   (e) $\frac{5q - 12}{q(q - 2)(q - 3)}$
   (f) $\frac{2r^2 - 4}{(r - 2)(r + 2)(r + 1)(r - 1)}$

7. A census is a survey of the entire population, while a sample is only a part of the population that has been selected in order to find information about the whole population.

8. (a) stratified (b) stratified

10. Yes, biased in favour of older people, who are more likely to be anti-republic.

11. (a) 2.90 (c) about 10c
    (b) between 1991–1992 (d) $2044

12. (a) range = 6, mode = 6, median = 5.5, mean = $5.1\dot{6}$
    (b) range = 6, mode = 46 and 48, median = 48, mean = 48.5

Skills review

1. (a) $p = 3$
   (b) $p = -2$
   (c) $y = -11$
   (d) $y = 8\frac{2}{3}$
   (e) $a = -25$
   (f) $y = -\frac{1}{4}$
   (g) $m = \frac{4}{7}$
   (h) $a = 5\frac{10}{17}$
   (i) $y = -3\frac{5}{7}$
   (j) $n = -1\frac{8}{13}$

2. (a) $80°, 80°, 130°, 70°$ ($x = 40$) (b) 10 metres

3. (a) $h = \frac{3V}{\pi r^2}$; $h = 2.8$
   (b) (i) $S = 100$ (ii) $t = 12$
   (c) $P = \frac{Q}{5} - 2$; $P = 23$

4. (a) $a < 10$

(e) $y \leqslant 2\frac{5}{9}$

(b) $y < -4$

(f) $p > 60$

(c) $x \leqslant 4$

(g) $y > 4\frac{1}{4}$

(d) $y > -2\frac{1}{2}$

(h) $-3\frac{1}{2} \leqslant x \leqslant 12\frac{1}{2}$

7. (a) $a < -5\frac{5}{11}$

(g) $y > -1$

(b) $y \geqslant 0$

(h) $t > -2\frac{2}{5}$

(c) $p < 2\frac{6}{17}$

(i) $p < \frac{6}{7}$

(d) $x < \frac{1}{4}$

(j) $x < \frac{4}{11}$

(e) $-1 \leqslant x \leqslant 3$

(k) $x > 4$

(f) $-4 \leqslant x \leqslant 3$

(l) $m > -16$

13. (a)
    (b)

(c)

(d)
(e)
(f)
(g)

14. (a)
    (b)

15.

16. (a)
    (b)
    (c)
    (d)
    (e)

17. (a)

18. (a)