

Three-dimensional Trigonometry

Trigonometry can be applied to solid, three-dimensional shapes such as cuboids, pyramids and triangular prisms.

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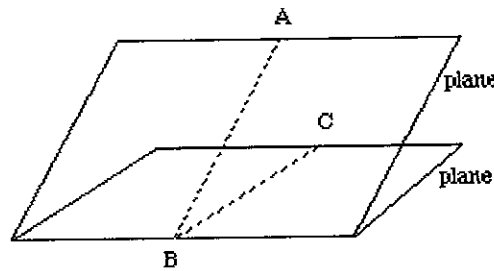
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Definitions

- A **plane** is a flat surface.
- Two lines or planes are **perpendicular** if they meet at right angles.
- **Collinear** points lie on the same line.
- **Coplanar** points lie on the same plane.
- **Concurrent** lines pass through the same point.
- A **polyhedron** is a solid shape having several faces. e.g. a pyramid.
- A **prism** is a polyhedron with a regular cross-section. e.g. a cuboid.
- An **edge** is the intersection of two faces.
- A **vertex** is the point of intersection of three or more faces of a polyhedron.

Intersecting Planes

Two planes intersect at a line. To find the angle between two planes, draw lines on each plane that meet on, and are perpendicular to, the line of intersection.



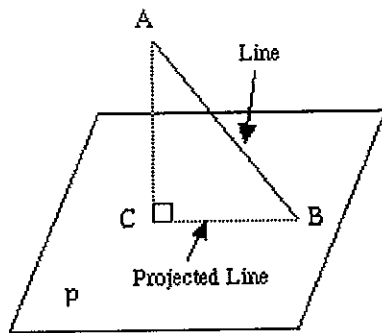
e.g. Angle ABC is the angle between the two planes.

Line and plane

To find the angle between a line and a plane, a perpendicular line is dropped from any point on the line. The point where this perpendicular meets the plane is then joined to the point where the line meets the plane. This line is called the projection of the line on the plane.

The angle required is the angle between the projection and the line.

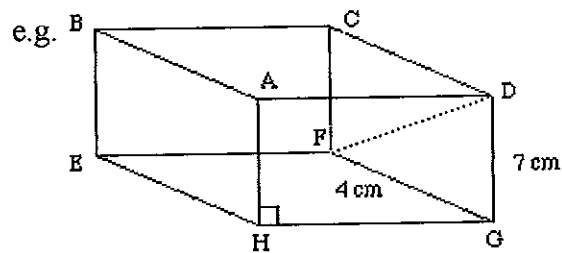
e.g.



CB is the projection of line AB on the plane.
The angle between line AB and plane p is $\angle ABC$

Problems

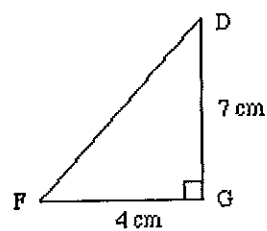
When a three-dimensional problem has to be solved, the best way is to try and isolate and redraw the triangles involved.



To find the length of FD

Redraw the triangle FDG
Now use Pythagoras' Theorem:

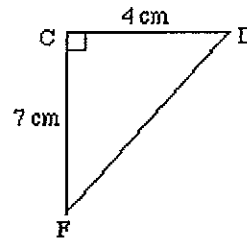
$$\begin{aligned} FD^2 &= FG^2 + DG^2 \\ FD^2 &= 4^2 + 7^2 \\ FD^2 &= 16 + 49 \\ FD^2 &= 65 \\ FD &= 8.1 \text{ (to 2 sig. fig.)} \end{aligned}$$



To find angle CFD

Redraw the triangle CFD
Now use SOH/CAH/TOA ☺

$$\begin{aligned}\tan \hat{C}FD &= \frac{4}{7} \\ &= 0.5714 \\ \hat{C}FD &= 29.7^\circ \text{ (to 1 d.p.)}\end{aligned}$$



12

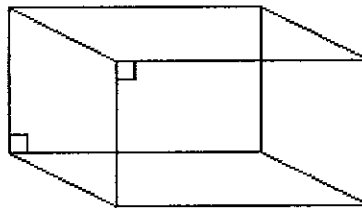
Three-dimensional Trigonometry

1. The figure shows a rectangular prism (cuboid).

(a) How many planes are there in this figure?

(b) How many vertices?

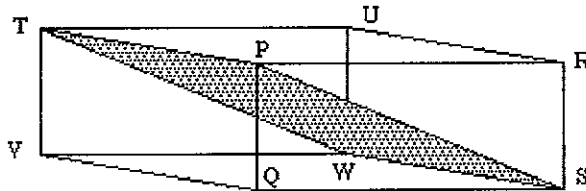
(c) How many edges does the figure have?



(d) How many vertical lines?

(e) How many of the angles between the edges are right angles?

2. The figure shows a rectangular box.



(a) Name the angle between plane PTWS and QVWS?

(b) What is the intersection of line UW and plane VWSQ?

(c) What is the intersection of plane PTUR and plane TUVW?

(d) Which point is the intersection of lines TW and PT?

(e) What is the angle between plane PSWT and plane TUWV?

3. The figure shows a pyramid on a horizontal square base MNOP.

AB is vertical and C is the midpoint of NO.

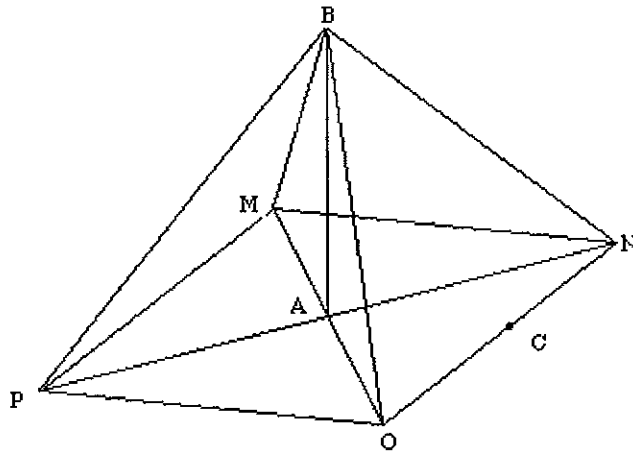
(a) Copy the figure.

(b) Mark the angle between BP and the plane $MNOP$. Label this angle x .

(c) Draw lines on the figure to show the angle between plane BNO and plane $MNOP$. Label this angle y .

(d) If AB is 9 cm and BP is 11 cm, find the length of AP .

(e) If $\angle BOA$ is 67° and BO is 11 cm, find the length of OA .



Three-dimensional Trigonometry

1. (a) 6 (b) 8 (c) 12 (d) 4 (e) 24

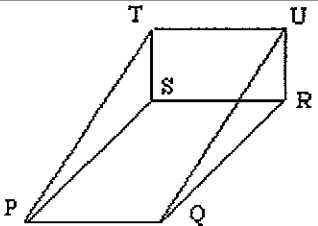
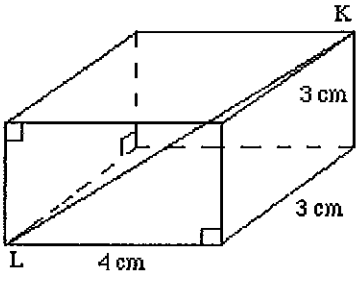
2. (a) \angle PSQ or \angle TWV
(b) W
(c) TU
(d) T
(e) 90°

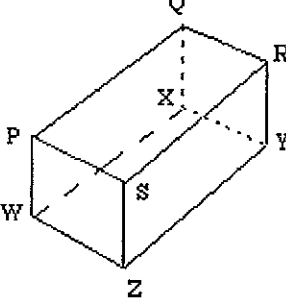
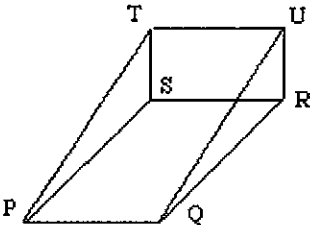
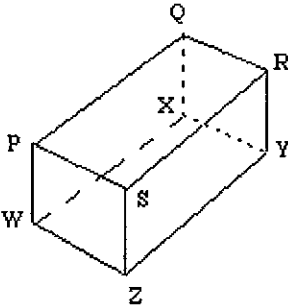
3. (d) 6.3 cm (to 2 sig. fig.)
(e) 6.3 cm (to 2 sig. fig.)

Three-dimensional Trigonometry

Unit Test #45

Select your answers to the following 10 questions from the pop-up menus in the right hand column. When you are satisfied with your answers, fill in your name in the space provided below the test, and click the "Submit Test" button. Clicking the "Begin Test Again" button will clear all the answers.

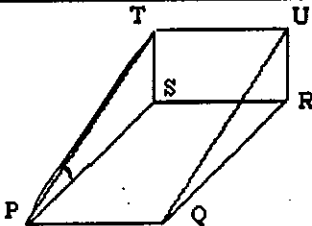
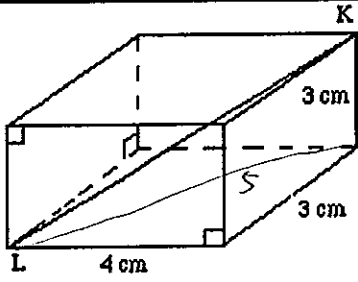
Q1:	How many faces has a cuboid?	A. 8 B. 4 C. 12 D. 6	Answer 1:	<input type="text"/>
Q2:	Lines that pass through the same point are called:	A. coplanar B. collinear C. congruent D. concurrent	Answer 2:	<input type="text"/>
Q3:	 <p>The figure shows half of a cuboid.</p> <p>The angle between the planes PQRS and PQUT in the diagram is</p>	A. angle SPT B. angle RQT C. angle SQT D. angle QPT	Answer 3:	<input type="text"/>
Q4:	 <p>The length of the diagonal KL is</p>	A. greater than 7 cm B. 7 cm C. between 5 cm and 7 cm D. 5 cm	Answer 4:	<input type="text"/>
Q5:	How many vertices has a triangular prism?	A. 4 B. 6 C. 8 D. 12	Answer 5:	<input type="text"/>


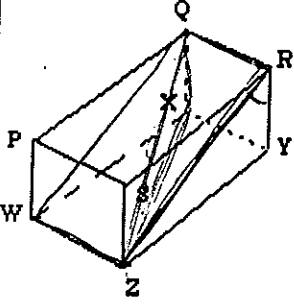


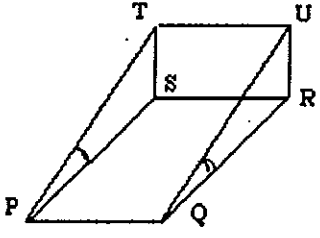

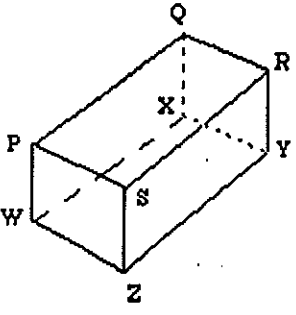
Q6:	How many vertices has a rectangular prism (a cuboid)?	A. 4 B. 6 C. 8 D. 12	Answer 6:	<input type="checkbox"/>
Q7:	 <p>The figure above is a cuboid.</p> <p>The angle between planes QWZR and QXYR is</p>	A. angle ZQX B. angle ZRX C. angle ZQY D. angle ZRY	Answer 7:	<input type="checkbox"/>
Q8:	<p>A cuboid has dimensions of 3cm by 4cm by 5cm.</p> <p>The length of the longest diagonal on any of the faces is:</p>	A. 5 cm B. $\sqrt{34}$ cm C. $\sqrt{41}$ cm D. $\sqrt{50}$ cm	Answer 8:	<input type="checkbox"/>
Q9:	<p>The diagram shows half of a cuboid.</p>  <p>The angle between the plane TURS and plane SRQP is:</p>	A. angle RUQ B. angle UQR C. 30° D. 90°	Answer 9:	<input type="checkbox"/>
Q10:	<p>The figure shows a cuboid.</p>  <p>What is the angle between plane QRYX and QRPS?</p>	A. 45° B. 90° C. angle SRZ D. angle YRZ	Answer 10:	<input type="checkbox"/>

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Q2:	Lines that pass through the same point are called:	A. coplanar B. collinear C. congruent (D) concurrent ✓	Answer 2: <input type="checkbox"/>
Q3:	 <p>The figure shows half of a cuboid.</p> <p>The angle between the planes PQRS and PQUT in the diagram is</p>	(A) angle SPT ✓ B. angle RQT C. angle SQT D. angle QPT	Answer 3: <input type="checkbox"/>
Q4:	 <p>The length of the diagonal KL is</p> $\frac{25}{\sqrt{34}}$	A. greater than 7 cm B. 7 cm (C) between 5cm and 7 cm ✓ D. 5 cm	Answer 4: <input type="checkbox"/>
Q5:	How many vertices has a triangular prism?	A. 4 (B) 6 ✓ C. 8 D. 12	Answer 5: <input type="checkbox"/>

Q6:	How many vertices has a rectangular prism (a cuboid)?	A. 4 B. 6 C. 8 ✓ D. 12	Answer 6:	
Q7:	 <p>The figure above is a cuboid.</p> <p>The angle between planes QWZR and QXYR is</p>	A. angle ZQX B. angle ZRX C. angle ZQY D. angle ZRY ✓	Answer 7:	
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