Exercise 5.8

1. Sketch the graph of each of the following curves, showing clearly the asymptotes.
   (a) $\frac{x^2}{16} - \frac{y^2}{9} = 1$
   (b) $x^2 - 2y^2 = 10$
   (c) $xy = 9$
   (d) $2xy = 15$
   (e) $x = \frac{2}{y-1}$
   (f) $y = \frac{2}{x-2}$
   (g) $xy = -16$
   (h) $(x-1)(y-2) = 1$

2. Find the Cartesian equations of the curves having the following parametric equations.
   (a) $x = 3t,\ y = \frac{3}{t}$
   (b) $x = \frac{5}{2}t,\ y = \frac{5}{2t}$
   (c) $x = 2t,\ y = -\frac{2}{t}$
   (d) $x = -\frac{2}{3}t,\ y = \frac{2}{3t}$
   (e) $x = 1 + t,\ y = \frac{1}{t}$
   (f) $x = 4t,\ y = 1 - \frac{4}{t}$

3. Write down the parametric equations of the curves having the following Cartesian equations.
   (a) $xy = 16$
   (b) $9xy = 25$
   (c) $16xy = -1$
   (d) $y = \frac{64}{x}$
   (e) $x - 1 = \frac{4}{y}$
   (f) $(x-2)y = 1$

4. If the gradient of the tangent at the point $(ct, \frac{c}{t})$ on the hyperbola $xy = c^2$ is $-\frac{1}{t^2}$, find the equations of the tangents and normals at each of the given points on the given hyperbola below.
   (a) $(3t, \frac{3}{t})$; $xy = 9$
   (b) $(3, \frac{4}{3})$; $xy = 4$
   (c) $(\frac{3}{2}t, \frac{3}{2t})$; $4xy = 9$
   (d) $(\frac{9}{2}, \frac{3}{4})$; $8xy = 27$

5. Find the equation of the tangent to the hyperbola $xy = 4$ satisfying the following:
   (a) it has a gradient $-\frac{3}{4}$
   (b) it passes through the point $(2, 0)$

6. The normal at the point $P(6, \frac{3}{2})$ which lies on the hyperbola $x = 3t$ and $y = \frac{3}{t}$ meets the curve again at Q. Find the coordinates of Q and the length of PQ.

7. The tangent and normal at the point $P(3, 4)$ which lies on the curve $xy = 12$, meet the x and y-axes respectively at Q, R and Q', R'. Find the length of QR and $Q'R'$.

8. P and Q are two variable points lying on the hyperbola $x = 3t,\ y = \frac{3}{t}$. The tangents at P and Q meet at T. If PQ passes through the point $(6, 2)$, find the equation of the locus of T as PQ varies.

9. The line $2y = x + 7$ intersects the curve $x = 2t,\ y = \frac{2}{t}$ at A and B. Find the respective values of t corresponding to A and B. Hence, find the coordinates of the point of intersection of the tangents at A and B.

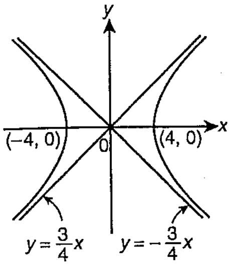
10. Prove that for all values of k, the line $k^2x + y = 8k$ is a tangent to the hyperbola $xy = 16$. Find the coordinates of the point of contact.

11. By finding the gradients of the tangents from the point $(2, -3)$ to the hyperbola $4xy = 25$, find the acute angle between the two tangents.

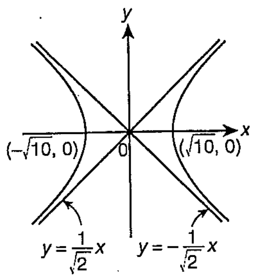
12. The normal to the hyperbola $xy = c^2$ at the point $Q(cq, \frac{c}{q})$ intersects the straight line $y = x$ at R. If O is the origin, show that $OQ = QR$. If the tangent to the hyperbola at Q intersects OR at P, prove that $OP \cdot OR = 4c^2$.

Exercise 5.8

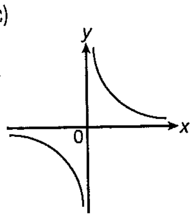
1. (a)



(b)



(c)



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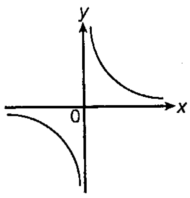

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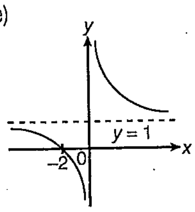

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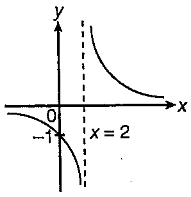
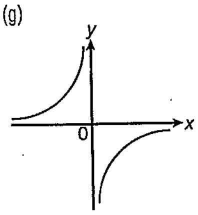
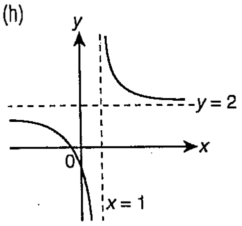


(g)



(h)



2. (a) $xy = 9$ (b) $4xy = 25$
(c) $xy + 4 = 0$ (d) $9xy + 4 = 0$
(e) $xy = y + 1$ (f) $xy - x + 16 = 0$

3. (a) $x = 4t,\ y = \frac{4}{t}$ (b) $x = \frac{5}{3}t,\ y = \frac{5}{3t}$
(c) $x = \frac{1}{4}t,\ y = -\frac{1}{4t}$ (d) $x = 8t,\ y = \frac{8}{t}$
(e) $x = 1 + 2t,\ y = \frac{2}{t}$ (f) $x = 2 + t,\ y = \frac{1}{t}$

4. (a) $t^2y + x = 6t;\ ty - t^3x = 3(1 - t^4)$
(b) $9y + 4x = 24;\ 12y - 27x + 65 = 0$
(c) $t^2y + x = 3t;\ 2ty - 2t^3x = 3(1 - t^4)$
(d) $6y + x = 9;\ 4y - 24x + 105 = 0$

5. (a) $4y + 3x + 8\sqrt{3} = 0,\ 4y + 3x - 8\sqrt{3} = 0.$
(b) $y + 4x = 8$

6. $\left(-\frac{3}{8}, -24\right),\ \frac{51}{8}\sqrt{17}$

7. $QR = 10;\ Q'R' = \frac{35}{12}$

8. $x + 3y = 9$

9. $\frac{1}{2}, -4; \left(\frac{16}{7}, -\frac{8}{7}\right)$

10. $\left(\frac{4}{k}, 4k\right)$

11. $\tan^{-1}\left(\frac{35}{13}\right)$ or 69°37'